

Validation of the IDEA Tracking Reconstruction for Prompt and Displaced Muon Signatures at FCC-ee

Advisors: Dr. Juliette Alimena | Dr. Matteo Presilla |
Sofia Giappichini

Reviewers: Prof. Dr. Markus Klute |
Prof. Dr. Jan Kieseler

Obedience Munkombwe | 2026-08-24

ETP – Institut für experimentelle Teilchenphysik



Motivation: Why displaced tracking?

Question: Can the current IDEA Tracking Reconstruction chain find and fit prompt and displaced charged-particle tracks.

- The study is performed on a fully simulated detector response.
- The goal is to identify possible reconstruction blind spots or failure regions.
- Physics interpretation depends on efficiency, resolution, and systematic effects.

Physics Benchmarks

For prompt tracks

$$e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-,$$

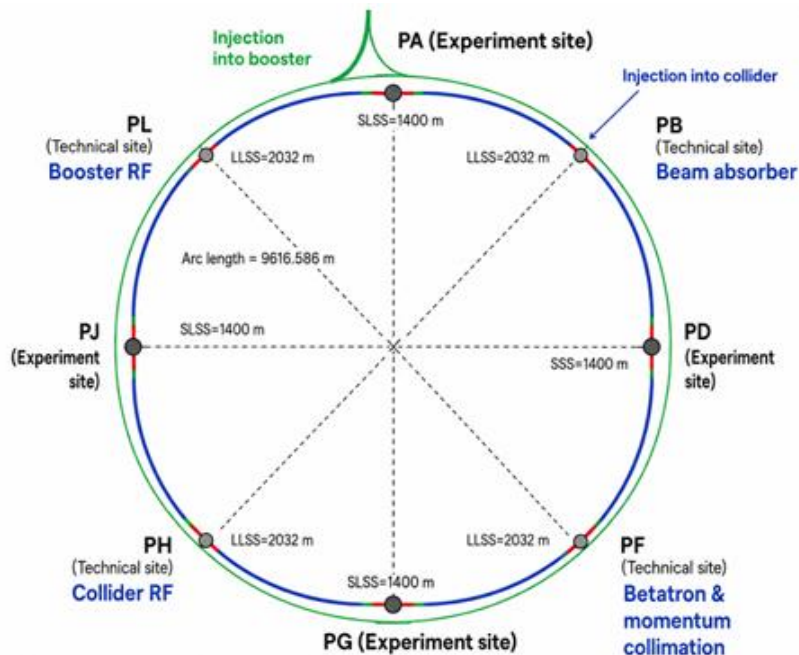
For Displaced tracks

$$e^+e^- \rightarrow Z \rightarrow N_\mu \nu_\mu, \quad N_\mu \rightarrow \mu^+\mu^-\nu_\mu.$$

Heavy Neutral Lepton with muon mixing

The Future Circular Collider(FCC-ee)

- FCC-ee is a proposed circular collider based at CERN
- FCC-ee circumference around 90 km
- ~2028 Project Approval by CERN Council
- Expected large number of Z bosons
- Clean environment (precisely known initial state)
- high luminosity



Working point	Z pole	WW threshold	ZH	$t\bar{t}$	
$\sqrt{s}$ [GeV]	88, 91, 94	157, 163	240	340–350	365
Lumi/IP [$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$]	140	20	7.5	1.8	1.4
Lumi/year [ab^{-1}]	68	9.6	3.6	0.83	0.67
Run time [years]	4	2	3	1	4
Integrated luminosity [ab^{-1}]	205	19.2	10.8	0.42	2.70
Number of events	6×10^{12} Z	2.4×10^8 WW	2.2×10^6 ZH + 65k WW $\rightarrow$ H	2×10^6 $t\bar{t}$ + 370k ZH + 92k WW $\rightarrow$ H	

The IDEA Detector Concept - Tracker

Motivation

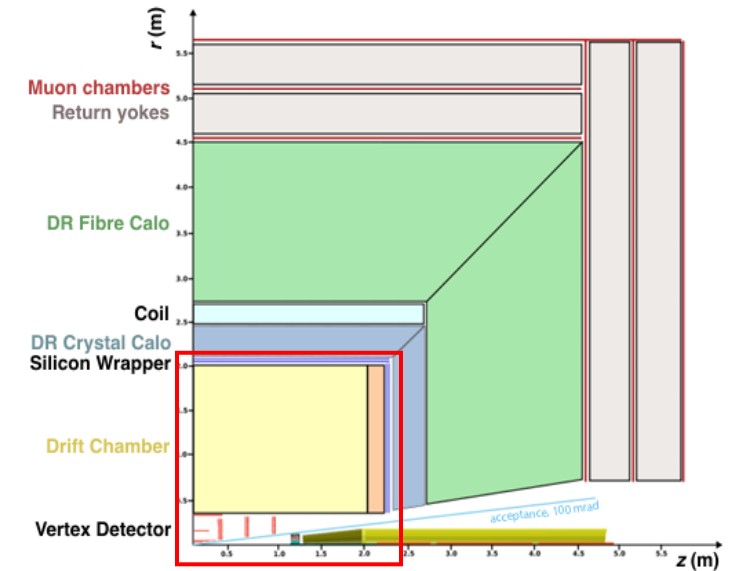
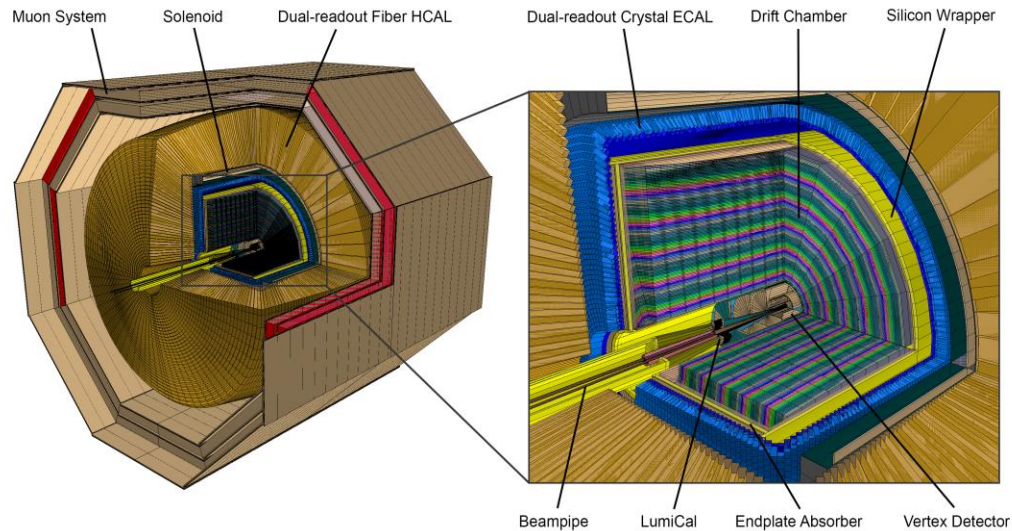


Figure 3: Overview of the IDEA detector layout.

VTX — Silicon vertex detector

- $R \approx 14\text{--}36\text{ mm}$ inner layers
- outer VTX: $R \approx 130\text{--}315\text{ mm}$
- Precise vertex and impact-parameter measurements

DCH — Drift chamber

- $R \approx 0.35\text{--}2.0\text{ m}, |z| < 2.0\text{ m}$
- 112 measurement layers, $B \approx 2\text{ T}$
- Gas mixture: 90% He and 10% $i\text{C}_4\text{H}_{10}$ (isobutane)
- Main tracking lever arm

SiWr — Silicon wrapper

- $R \approx 2.04\text{--}2.08\text{ m}$
- Precise outer constraint on the fitted trajectory

→ The large tracking volume also gives sensitivity to tracks produced away from the interaction point.

Reconstruction framework



Source: [FCC-config](#)

Track Finder

- digitized tracker hits are encoded as graph / geometric features
- a pre-trained GNN model exported in ONNX → maps hits to an embedding space
- clustering in this space defines track candidates in Pre-fitTracks

Track Fitter

- fits candidate hits with a deterministic annealing filter
- provides track parameters and covariance matrices
- may reject candidates with no usable state or invalid fit quality

Validation strategy: Sample

1. Single-muon guns

Controlled single-track tests

- Prompt and displaced production points
- Fixed momentum and polar-angle scans
 - Isolates detector and kinematic effects

2. $Z \rightarrow \mu^+ \mu^-$

Prompt two-track physics topology

- **MadGraph5_aMC@NLO + Pythia8** at $\sqrt{s} = 91.188\text{GeV}$
- events = 918320
 - Tests pair reconstruction and candidate ambiguity

3. Heavy Neutral Leptons (HNLs)

Displaced two-track physics topology

$$Z \rightarrow N_\mu \nu_\mu \quad N_\mu \rightarrow \mu^+ \mu^- \nu_\mu$$

- **MadGraph5_aMC@NLO + Pythia8** at $\sqrt{s} = 91.188\text{GeV}$
- events = 921k
- $M_N = 40\text{GeV}$ $U_{\mu N} = 2 \times 10^{-6}$
 - Tests reconstruction away from the interaction point

Validation strategy: truth matching and efficiencies

Reconstructable truth muons:

- $|PDG| = 13$ and stable final state.
 - At least four truth-linked tracker sim hits.
 - $|\cos \theta| \leq 0.99$.
-

Candidate match:

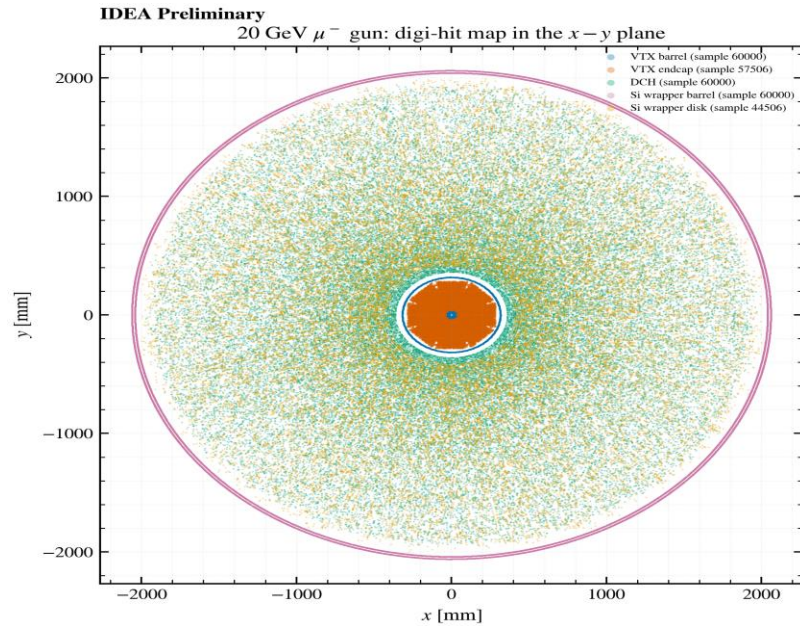
$$P(t, c) = \frac{Nhits(c \leftarrow t)}{Nhits(c)}$$

A candidate passes for truth muon t when $P \geq 0.75$ and it contains at least four reconstructed hits.

Reconstruction efficiencies

- **Finder efficiency:** $\epsilon_{finder} = \frac{N(\text{truth muons matched to finder})}{N(\text{reconstructable truth muons})}$
- **Fitted-track efficiency:** $\epsilon_{fit} = \frac{N(\text{truth muons matched to fitted tracks})}{N(\text{reconstructable truth muons})}$
- **Fit retention:** $\epsilon_{ret} = \frac{N(\text{finder-matched muons retained after fitting})}{N(\text{finder-matched muons})}$
- **Two-track efficiency:** $\epsilon_{\mu\mu} = \frac{N(\text{events with both muons fitted})}{N(\text{events with reconstructable pair})}$

Detector response: 20 GeV muon gun

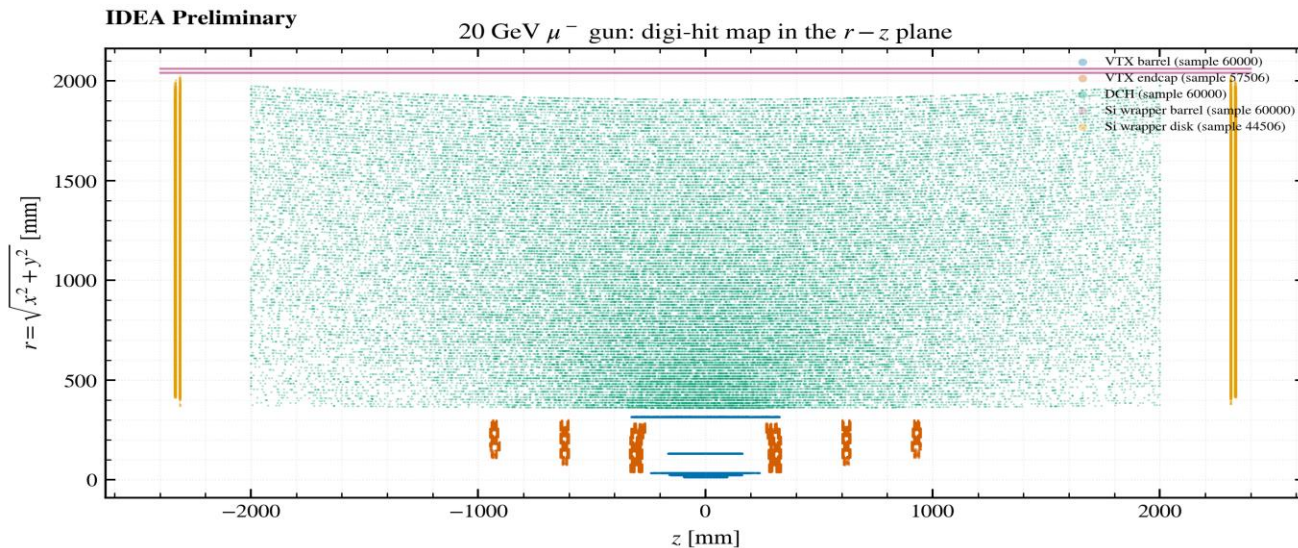


Sample

- 50,000 μ^- -events
- $10^\circ < \theta < 170^\circ$.
- $p = 20 \text{ GeV}$.
- Uniform azimuth.

Checks:

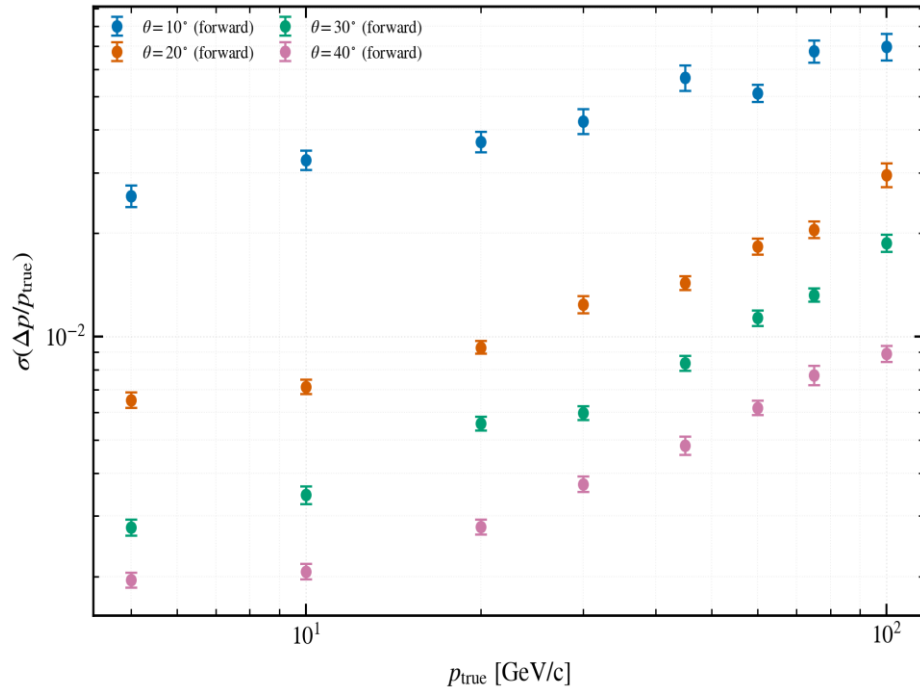
- VTX, DCH, and silicon-wrapper structures are populated.
- All truth muons have at least four linked tracker sim hits.
- This establishes detector coverage and acceptance.



Controlled prompt-muon performance

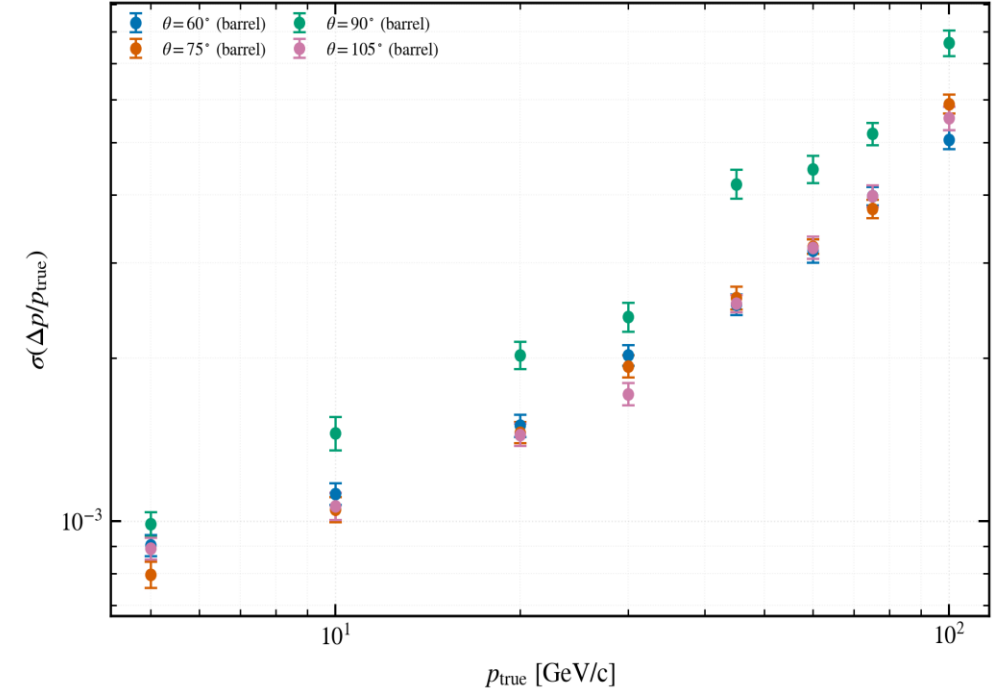
IDEA Preliminary

Forward fixed polar-angle muon gun: $\sigma(\Delta p/p_{\text{true}})$ vs p_{true}



IDEA Preliminary

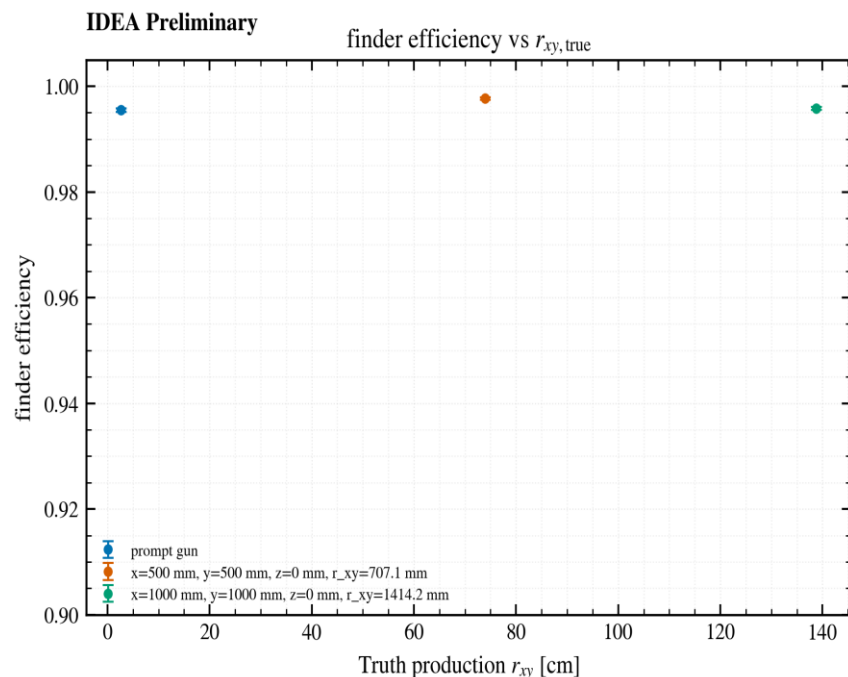
Barrel fixed polar-angle muon gun: $\sigma(\Delta p/p_{\text{true}})$ vs p_{true}



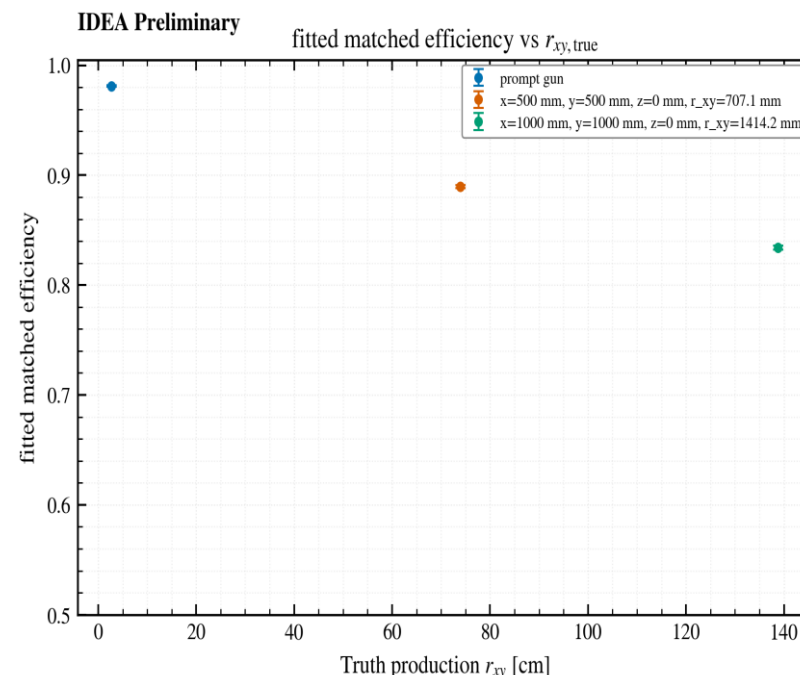
Region	Polar angle	p = 10GeV	p = 100GeV
forward	$10^\circ - 40^\circ$	0.21-3.26%	0.89-6.98%
barrel	$60^\circ - 105^\circ$	0.105-0.145%	0.506-0.764%

- **Angle:** Forward tracks $\rightarrow$ shorter transverse lever arm $\rightarrow$ poorer resolution.
- **Momentum :** Higher p $\rightarrow$ smaller curvature $\rightarrow$ poorer resolution.

Prompt and displaced single-muon



- $R_{xy} = 0$ mm
- $R_{xy} = 707$ mm
- $R_{xy} = 1414$ mm
- Keeping $z = 0$



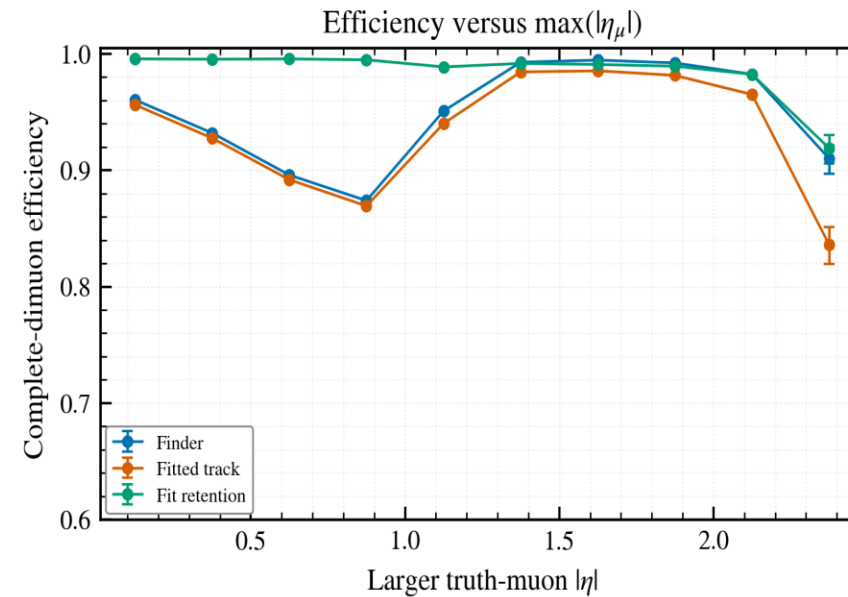
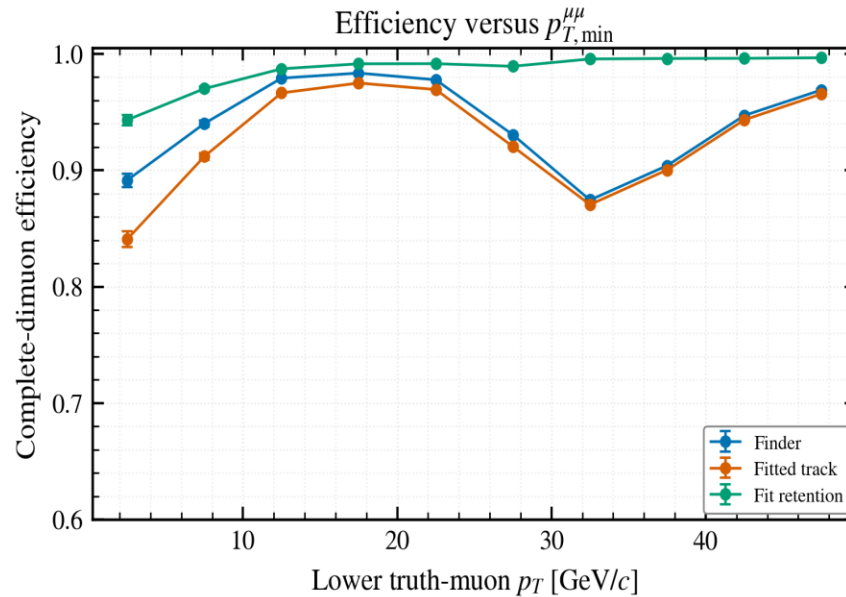
R_{xy} [mm]	finder [%]	fitted [%]	fit retention [%]
0	99.549 ^{+0.030} _{-0.032}	98.104 ^{+0.062} _{-0.064}	98.549 ^{+0.054} _{-0.056}
707	99.768 ^{+0.021} _{-0.023}	88.970 ^{+0.144} _{-0.145}	89.177 ^{+0.143} _{-0.144}
1414	99.583 ^{+0.029} _{-0.031}	83.409 ^{+0.171} _{-0.172}	83.758 ^{+0.170} _{-0.171}

- **Finder >99.5%:** displaced muons still leave enough hits to form candidates..
- **Fitted efficiency drops with radius:** fewer inner hits and a shorter lever arm make fitting harder..
- **Main loss:** occurs during fitting, not candidate finding.

Results: $Z \rightarrow \mu + \mu -$

IDEA Preliminary

$Z \rightarrow \mu^+ \mu^-$ complete-pair reconstruction efficiency



- Once both prompt muons are found, the fitter retains more than 99.3% of the pairs
- The dominant event-level loss is therefore associated with finding rather than fitting.
- The loss is strongly localized “transition” detector region

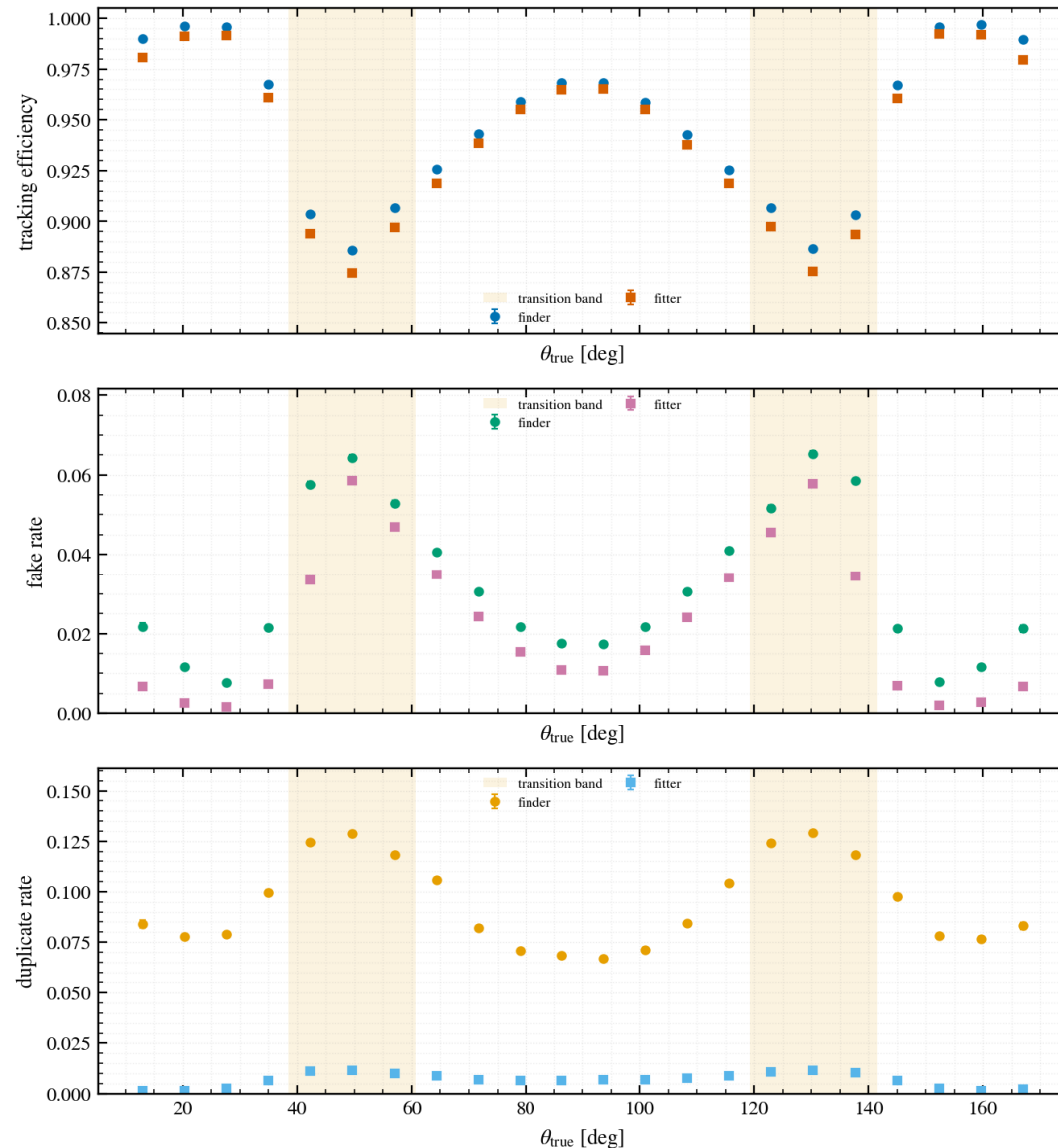
Inclusive performance

Quantity	Efficiency
Both muons found	$93.596^{+0.026}_{-0.026} \%$
Both muons fitted	$92.981^{+0.027}_{-0.027} \%$
Fit retention	$99.344^{+0.009}_{-0.009} \%$

The angular dips are a finder-stage feature

IDEA Preliminary

$e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$ angular diagnostic vs θ_{true}

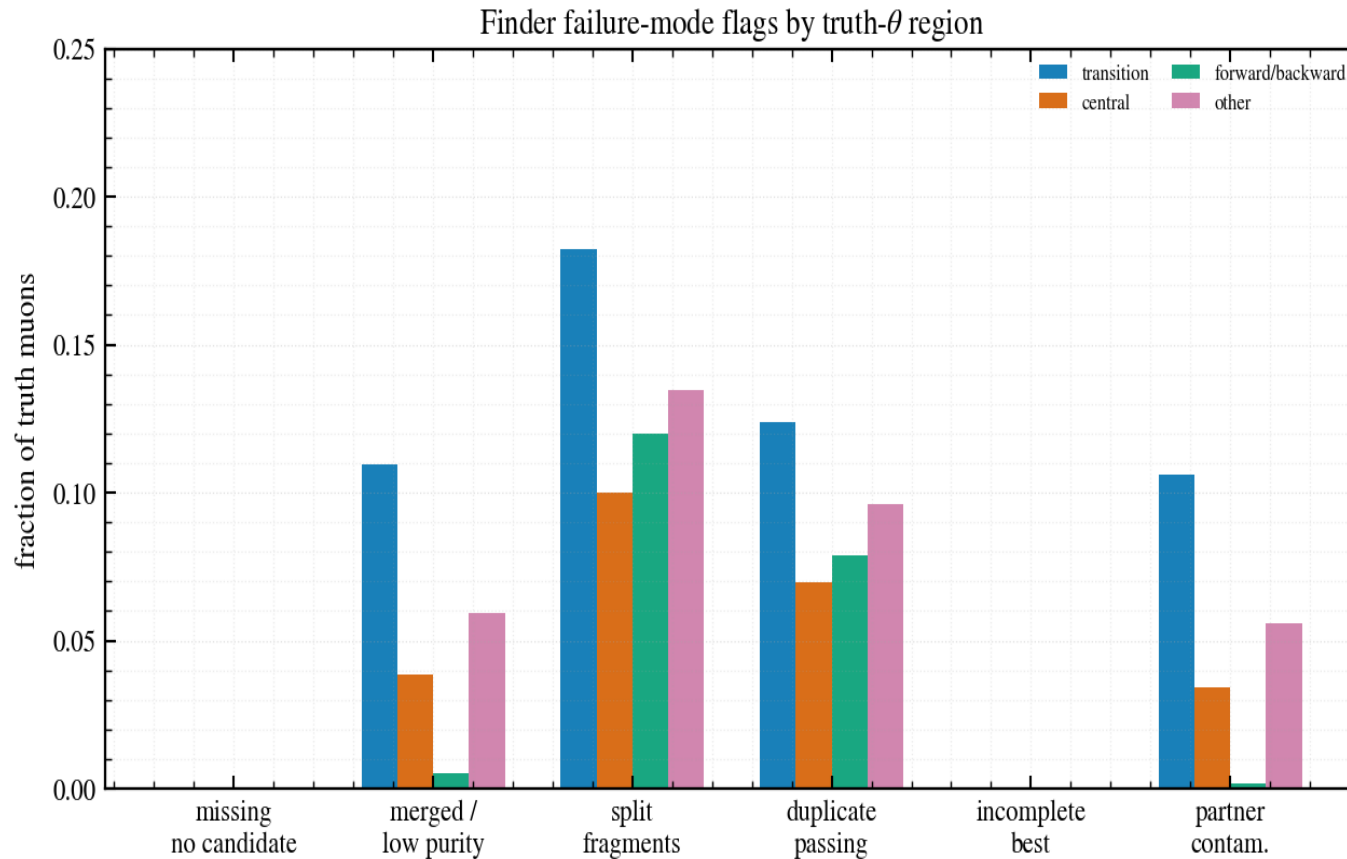


- The mirrored low-efficiency bins coincide with enhanced fake & duplicate rates
- Local efficiency reductions occur near the barrel–forward transition bands.
- Finder and fitted-track efficiencies follow the same structure.
- The pair fit-retention remains high once both tracks are found.

→ The angular inefficiency originates mainly during track finding, not fitting.

Why? Candidate ambiguity in the “transition regions”

IDEA Preliminary



Transition region

Duplicate passing:	12.38%
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Partner hits in best candidate:	10.62%
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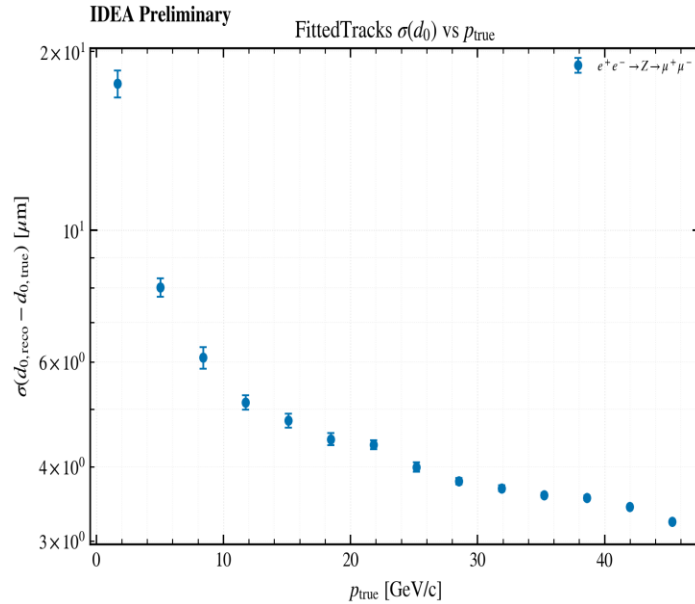
Low-purity best candidate:	10.97%
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- Low-purity / merged candidates increase strongly in the transition region.
- Duplicate candidates and partner-muon contamination also increase.
- The problem is therefore **hit assignment during pattern recognition**, not simply missing detector hits.

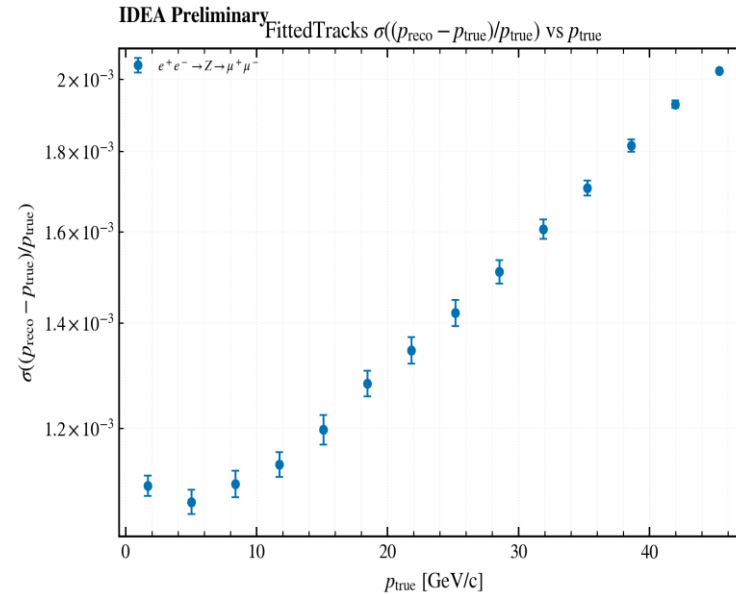
→ Localized GNN pattern-recognition limitation - not a detector-acceptance loss.

Fitted-parameter precision on the prompt sample

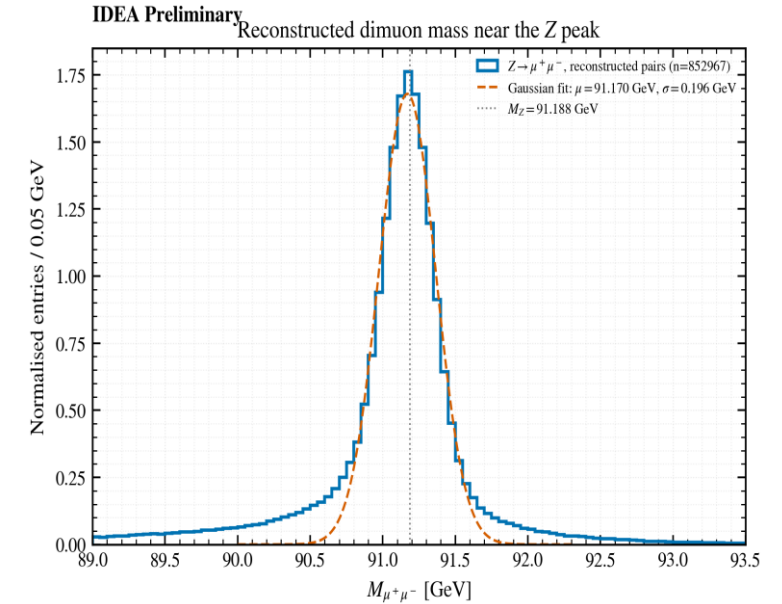
Impact parameter resolution



Relative Momentum



Dimuon peak



- **Track origin:** d_0 resolution reaches a few μm at high momentum.
- **Momentum:** relative momentum resolution is at the 10^{-3} level and worsens with momentum.
- **Event check:** the fitted tracks reconstruct the Z peak at 91.170 GeV , close to the expected value 91.188 GeV .

HNLs as a displaced-track benchmark

What is an HNL?

- A hypothetical heavy neutral fermion that mixes weakly with SM neutrinos.
- Small mixing can give a long lifetime.
- The HNL can therefore travel a measurable distance before decaying, producing displaced charged tracks.

Process:

$$e^+e^- \rightarrow Z \rightarrow N_\mu \nu_\mu$$

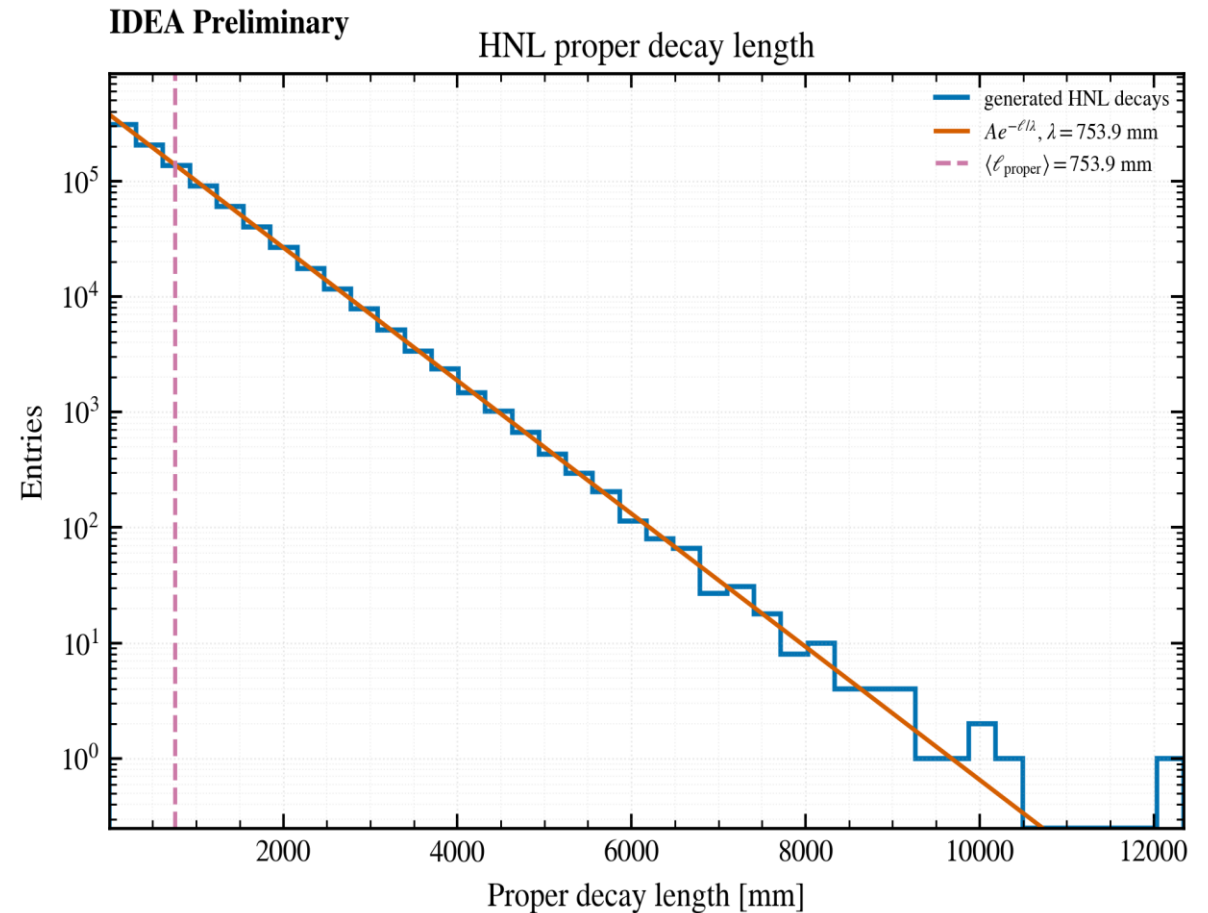
$$N_\mu \rightarrow \mu^+ \mu^- \nu_\mu$$

Benchmark:

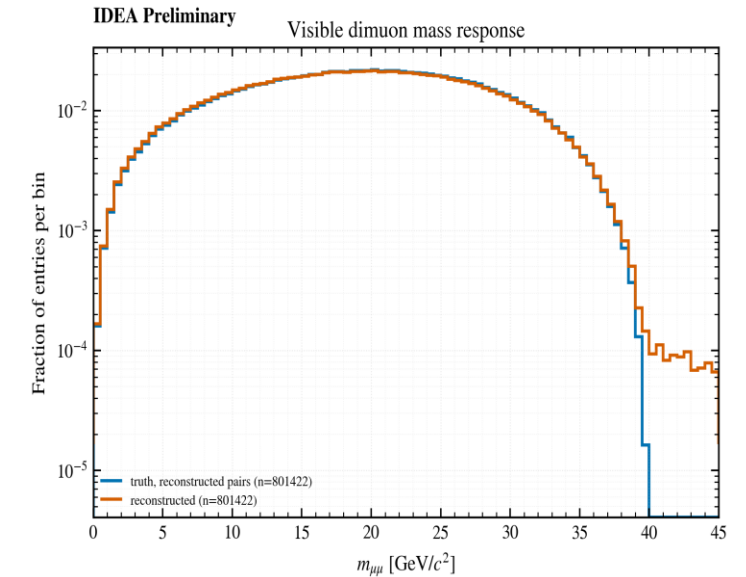
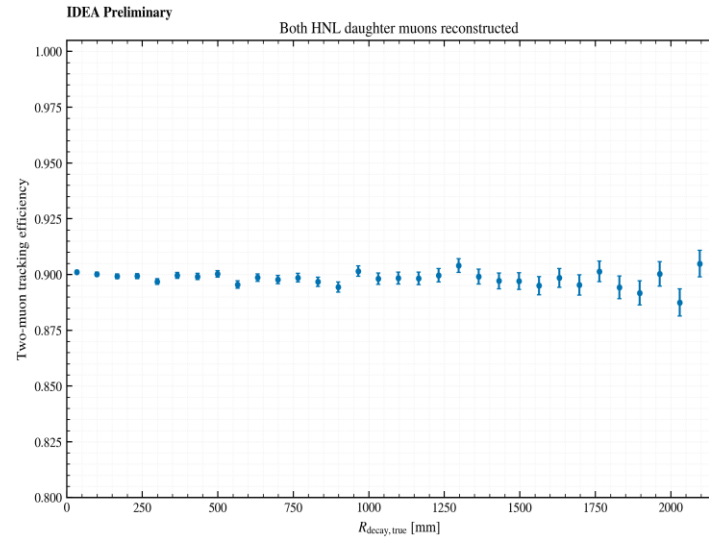
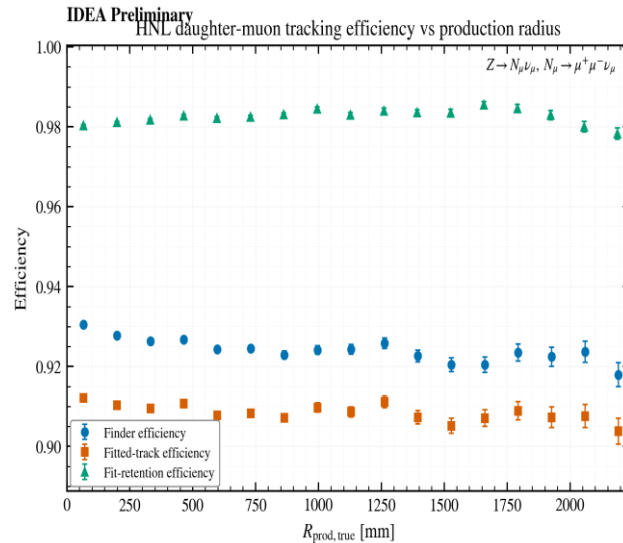
$$M_N = 40\text{GeV} \quad U_{\mu N} = 2 \times 10^{-6}$$

$$\text{nominal } c\tau = 754.6\text{mm}$$

$$\text{generated mean} = 753.9\text{mm}$$



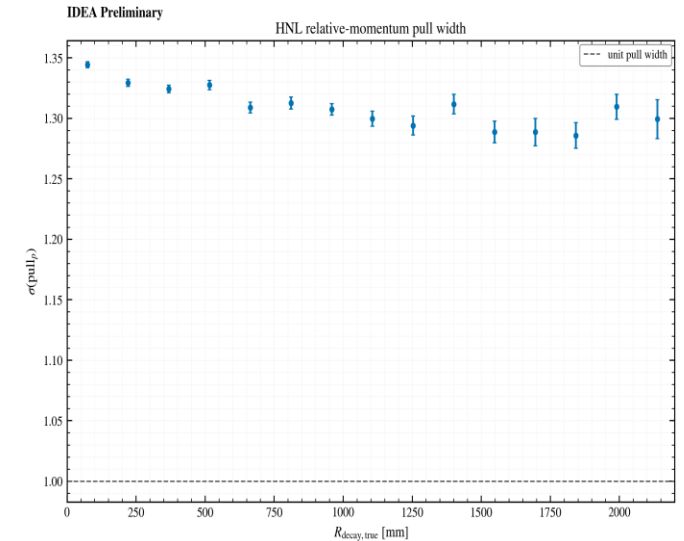
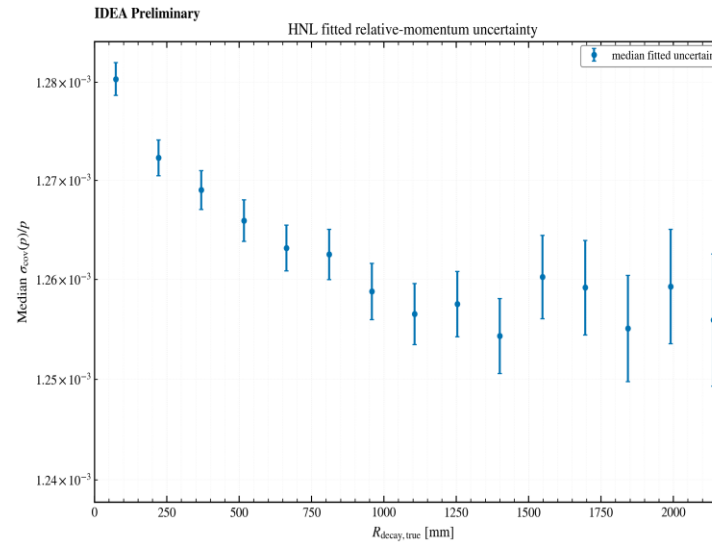
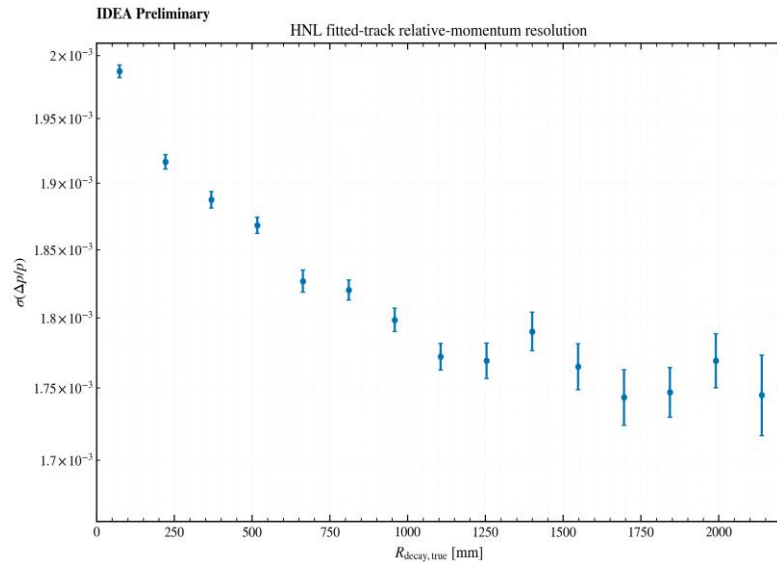
Visible HNL decay: event-level reconstruction



- **Fit retention:** 98.19% of finder-matched muons remain matched after fitting.
- **Pair efficiency:** 89.89% of reconstructable HNL pairs have both fitted tracks.
- **Radius dependence:** diagnostic only
- **Visible mass:** no 40 GeV peak because the neutrino is not reconstructed.

Efficiency definition	Efficiency [%]
Finder, per reconstructable muon	$92.6703^{+0.0193}_{-0.0194}$
Fitted track, per reconstructable muon	$90.9885^{+0.0212}_{-0.0213}$
Fit retention	$98.1852^{+0.0103}_{-0.0103}$
Both fitted tracks, per reconstructable pair	$89.8930^{+0.0319}_{-0.0320}$

Track-parameter performance: residuals versus covariance



- Momentum resolution is broadly stable with decay radius.
- Residual widths describe the selected successfully fitted population
- The pull width is 1.27–1.35, above the ideal value of 1; the fitted uncertainties require an approximate scale factor of 1.3.

Summary and Outlook

Candidate finding

- The finder performs efficiently for isolated prompt and displaced muon guns
- In $Z \rightarrow \mu + \mu$ –events, angular losses are associated with candidate ambiguity and mixed truth-hit content.
- The fitted-track angular dependence largely inherits the finder response.

- The finder model and reconstruction configuration were evaluated as produced; no model retraining or parameter optimization was performed
- The $Z \rightarrow \mu + \mu$ – “transition-region” deficit correlates with mixed candidates, but the available evidence does not establish a unique detector-geometry cause.
- The HNL result is a tracking study. A displaced-vertex fit and a complete HNL event selection are outside its scope.

Outlook:

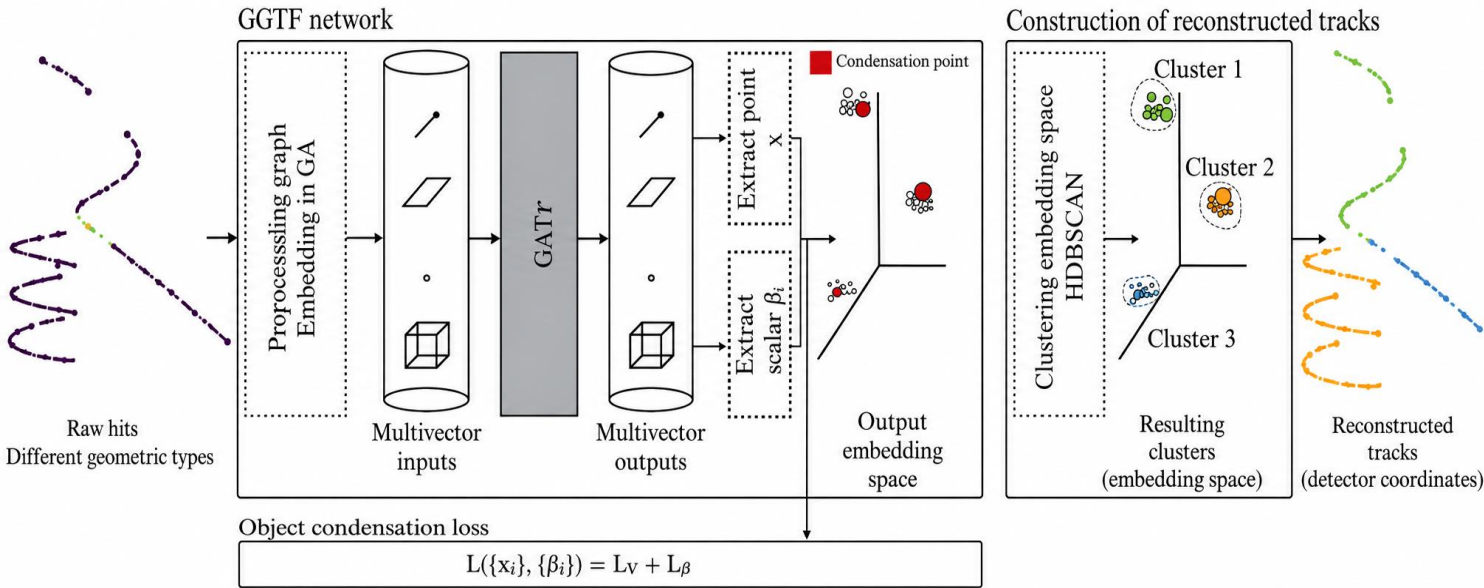
- A dedicated study on model retraining and parameter optimization is needed

Track fitting

- Fit retention is high for the prompt dimuon sample.
- Additional post-finder losses become important for tracks produced far from the interaction point.
- Pull widths above unity show that the fitted covariance underestimates the measured core spread in the studied samples.

BACKUP

GGTF



- Silicon and drift-chamber hits are encoded as geometric-algebra multivectors and processed together by GATr.
- For each hit i , the network predicts a three-dimensional learned embedding x_i and a condensation score β_i .
- The highest-score unassigned hit with $\beta_i > T\beta$ seeds a candidate. Unassigned hits satisfying $\|x_j - x_i\| < Td$ are attached to it; the procedure is repeated to form the finder output.

Configured post-inference thresholds

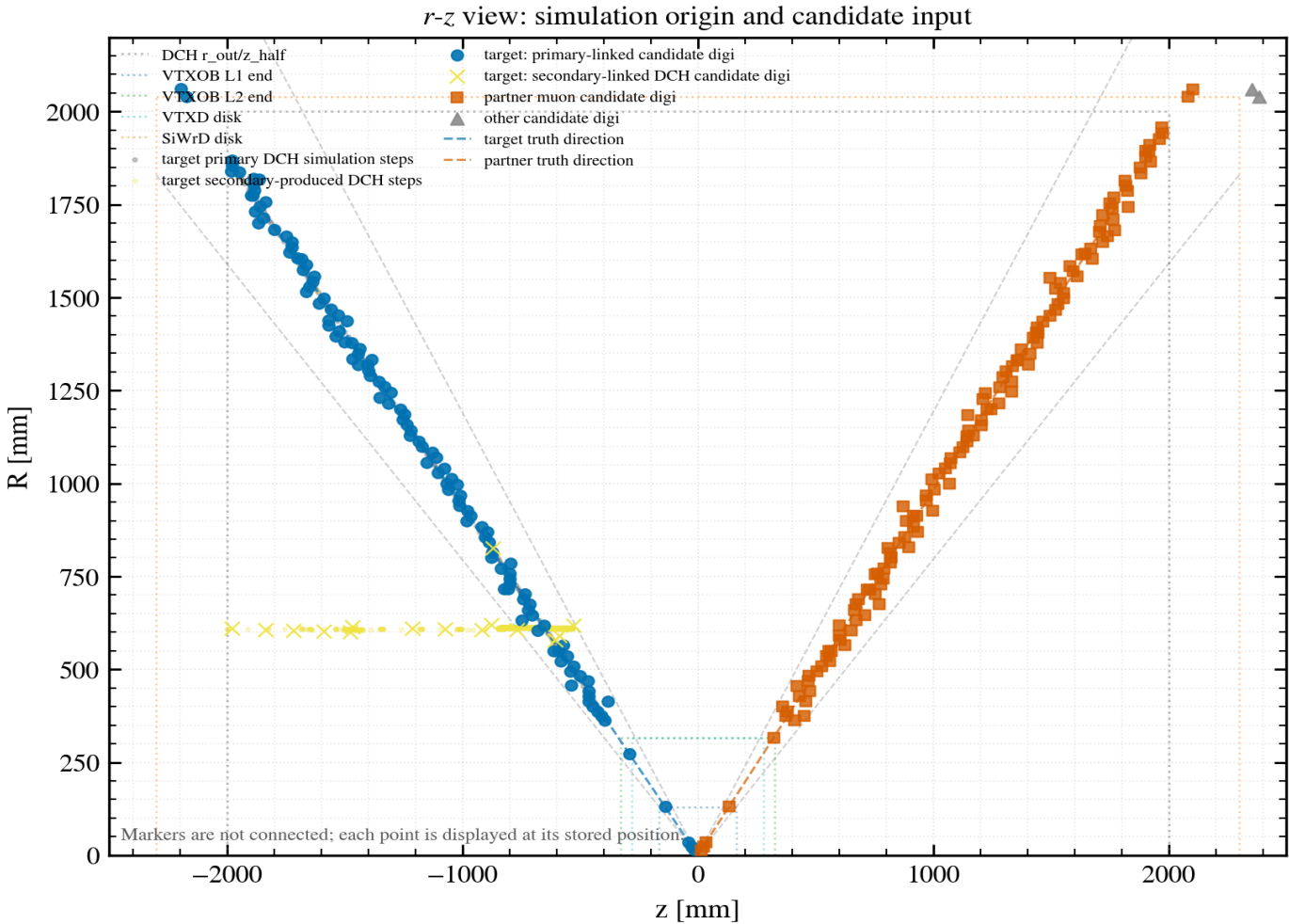
- Model was trained on $Z \rightarrow qq$ no background
- $T\beta = 0.6$ $Td = 0.3$ (learned embedding space).
- Raising $T\beta$ accepts fewer seeds: fake or duplicate candidates may decrease, but genuine tracks can be lost. Lowering it has the opposite tradeoff.
- Decreasing Td makes hit association tighter and can omit hits or split a trajectory. Increasing it recovers a broader neighborhood but can mix or merge nearby trajectories.

→ No retraining or parameter optimization was done!!!

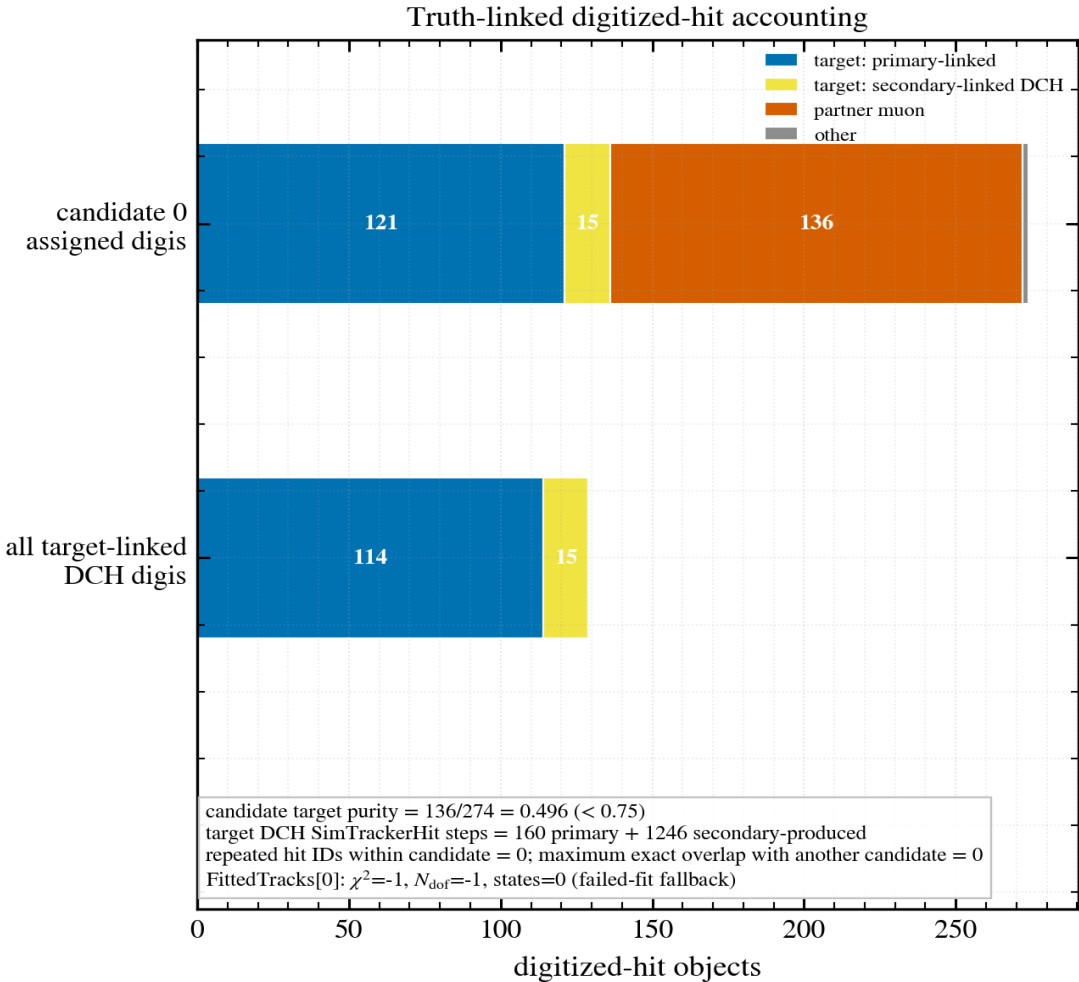
Event Display

IDEA Preliminary

$Z \rightarrow \mu^+ \mu^-$ event 148413: secondary DCH activity and finder-candidate accounting



Simulation-step counts are energy-deposition records, not counts of independent detector measurements or particles.



BACKUP

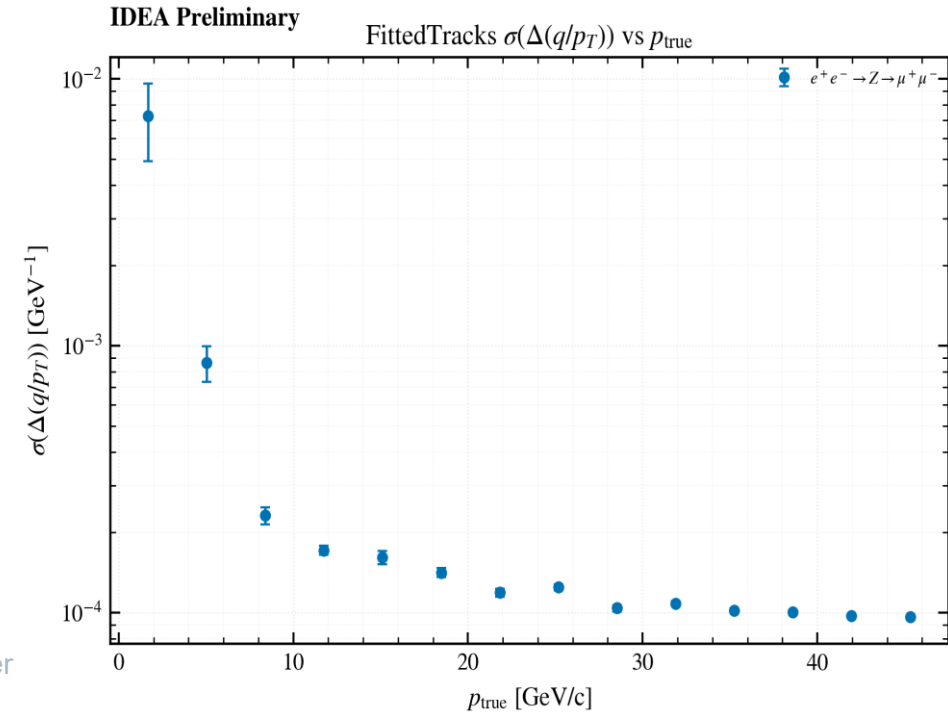
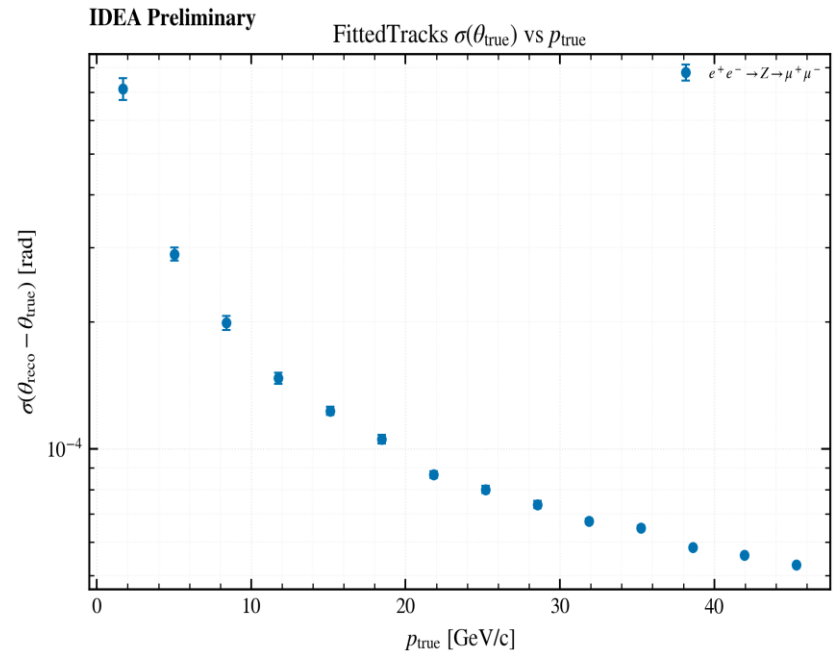
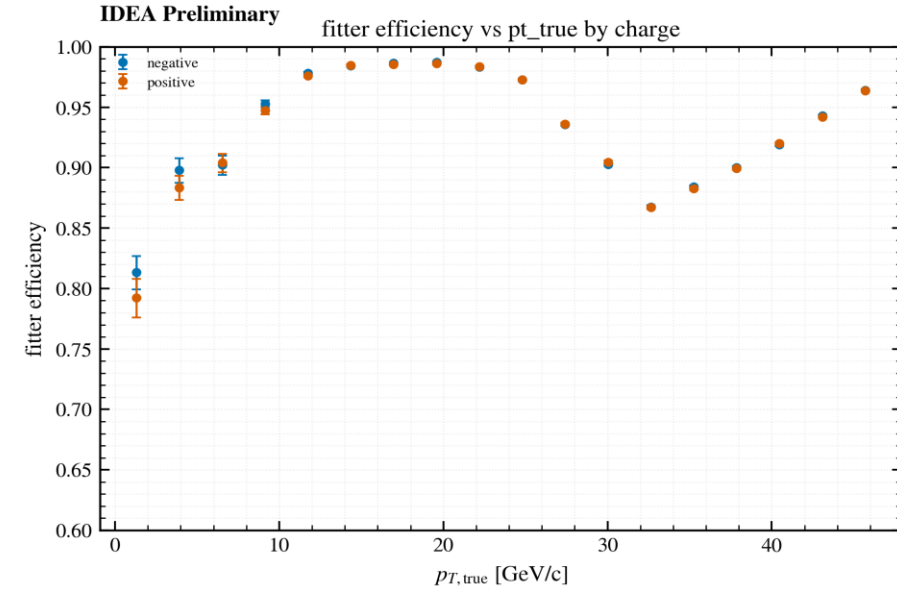
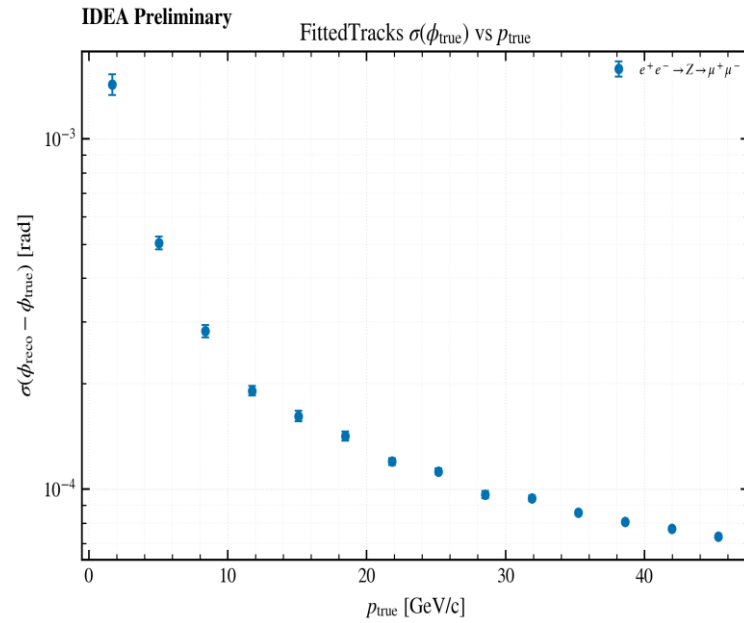
region	finder efficiency	fake rate	duplicate rate
transition bands	0.899	0.058	0.124
central control	0.964	0.020	0.069
forward/backward control	0.995	0.011	0.079

Transition: $38.5^\circ < \theta < 60.7^\circ$ and $119.3^\circ < \theta < 141.5^\circ$

Central control: $75^\circ < \theta < 105^\circ$

Fwd/bwd: $9^\circ < \theta < 31.5^\circ$ and $148.5^\circ < \theta < 171^\circ$

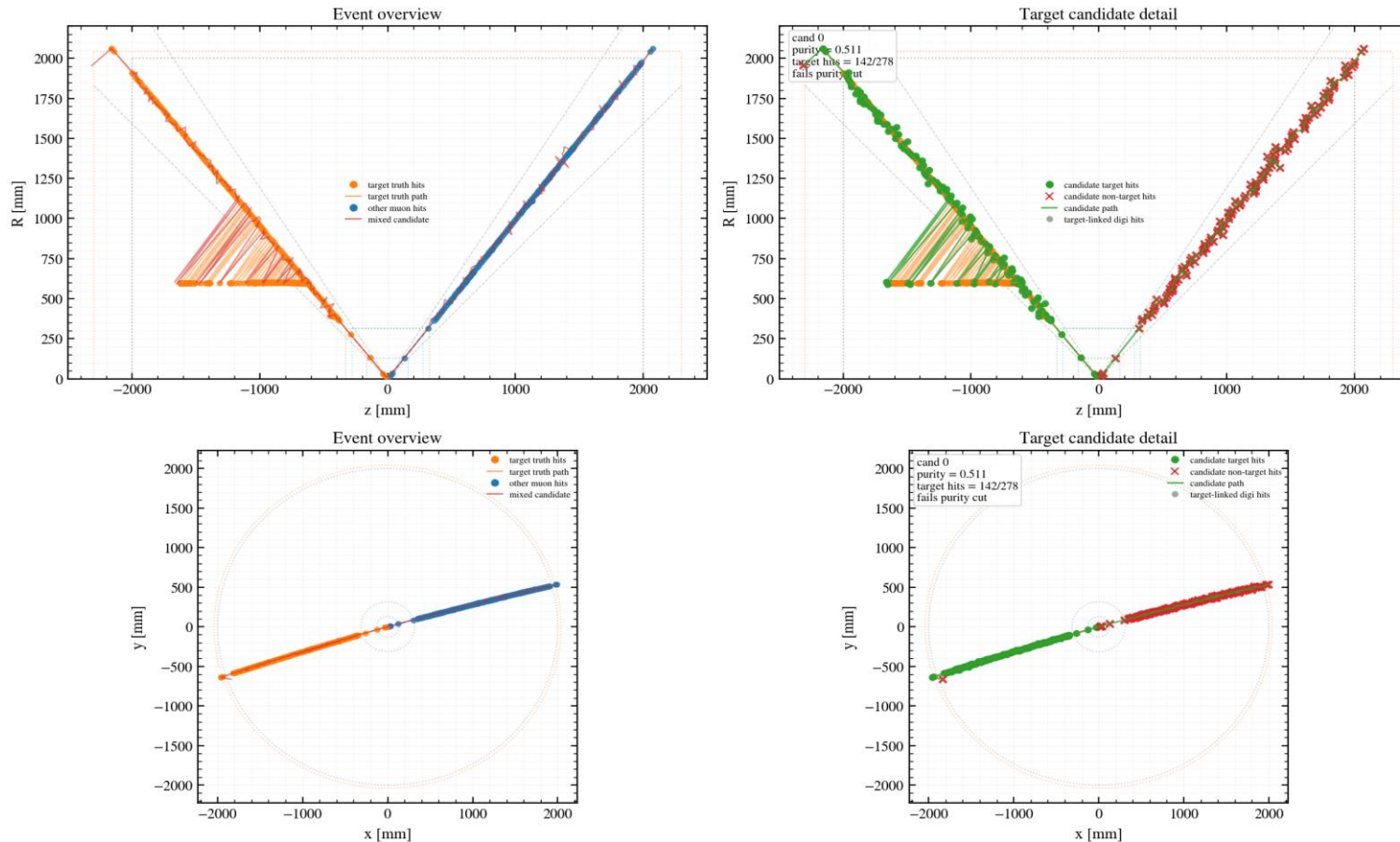
Track parameters Z Sample



One event makes the mechanism visible

IDEA Preliminary

Z->mumu transition-region event display: event 134839, truth 6 (not matched by validation cuts)
theta=136.35 deg, eta=-0.915, pT=31.16 GeV, track collection=PrefitTracks



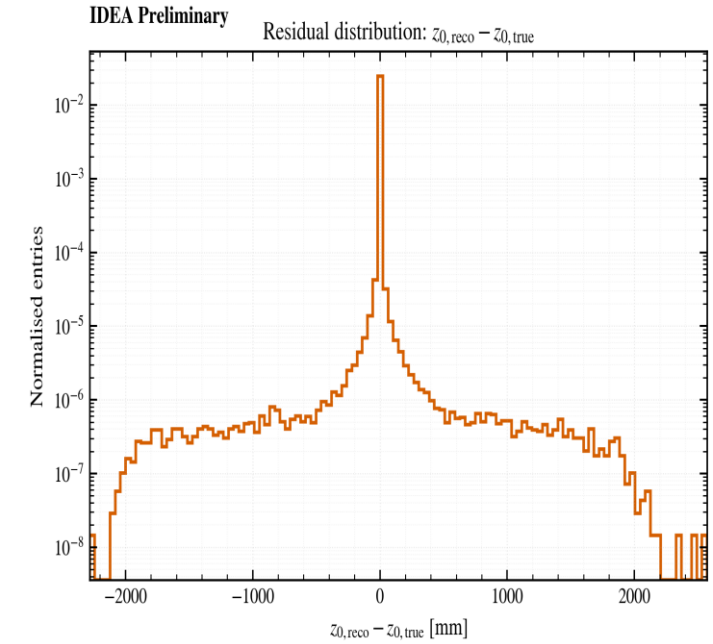
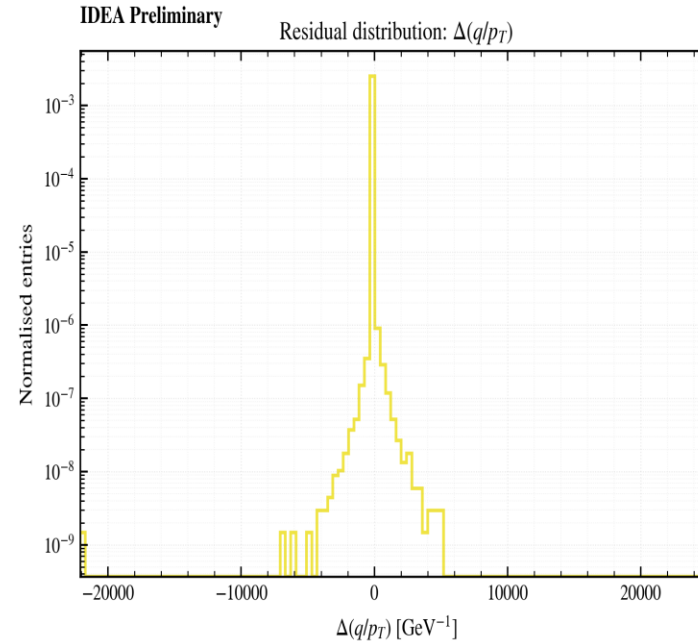
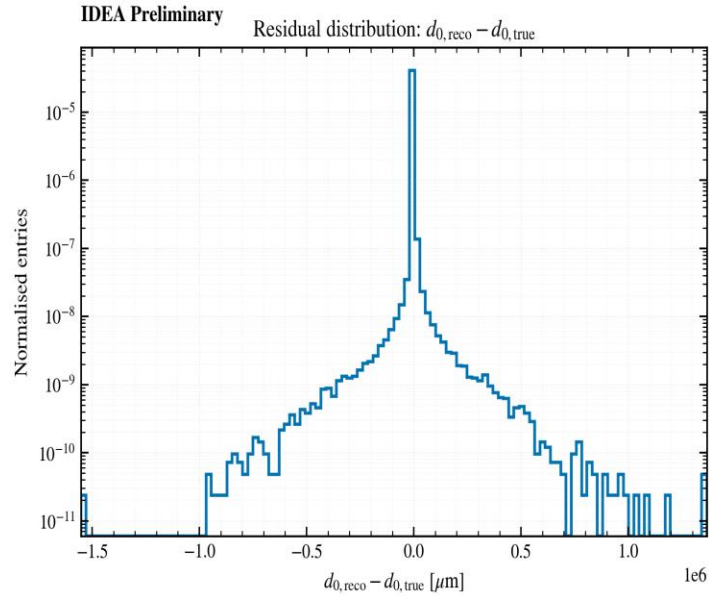
Event 134839

For the selected transition-region truth muon, candidate 0 contains:

- 142 *hits* from the target muon;
- 135 *hits* from the partner muon;
- 278 *assigned hits* in total; target *purity* = 0.511.

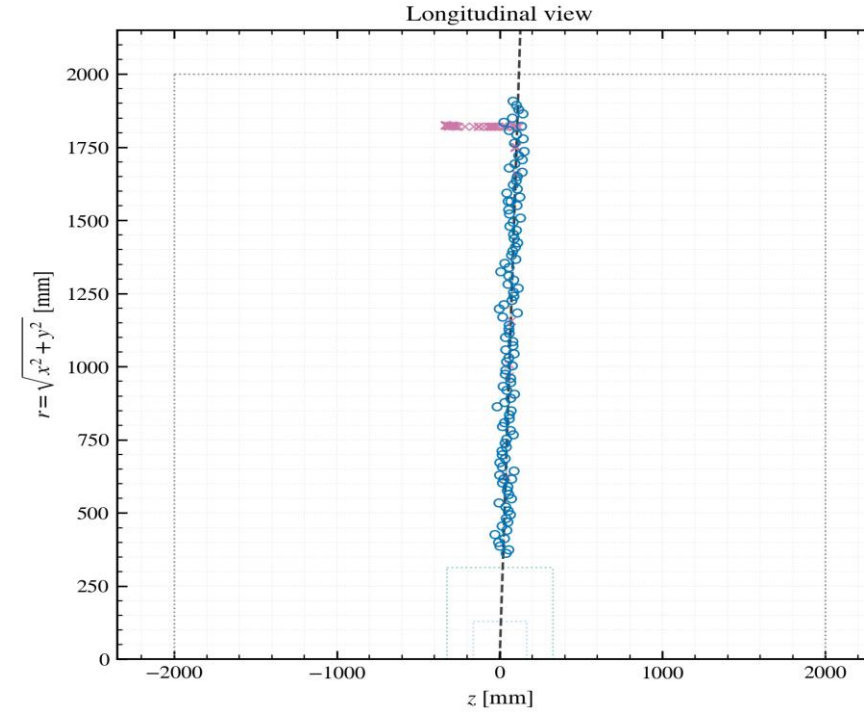
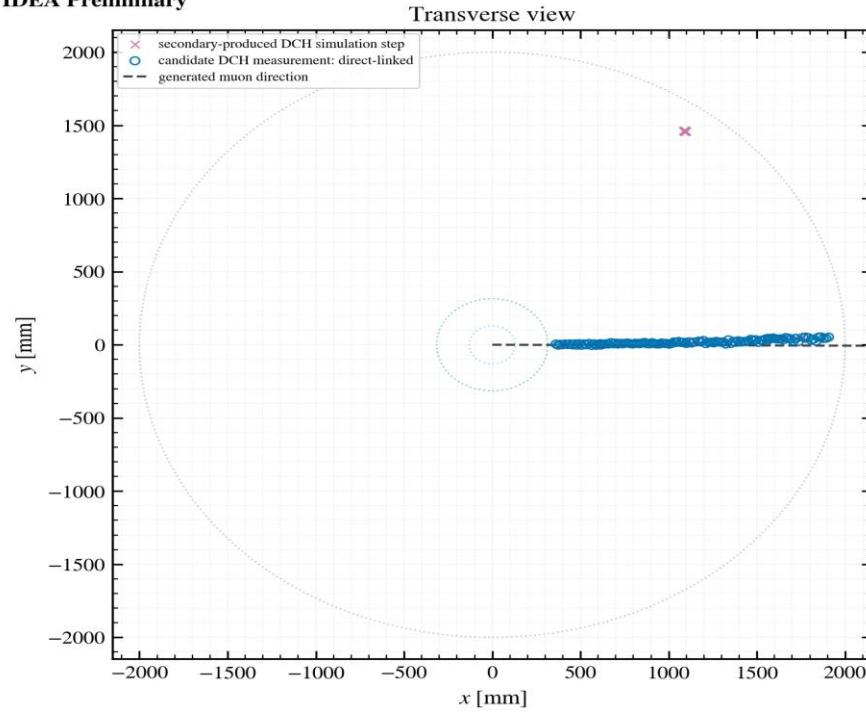
The finder has combined two truth contributions. The strict per-muon match therefore rejects this candidate.

Residuals



Secondary activity

IDEA Preliminary



Event contains at least one

Events

Fraction [%]

Secondary-produced DCH simulation step

48 425

96.850^{+0.077}_{-0.079}

Secondary-linked digitized DCH measurement

39 551

79.102^{+0.181}_{-0.182}

Finder candidate containing a secondary-linked DCH measurement

39 551

79.102^{+0.181}_{-0.182}

