Identification of Patterns in Cosmic-Ray Arrival Directions using **Dynamic Graph Convolutional Neural Networks**

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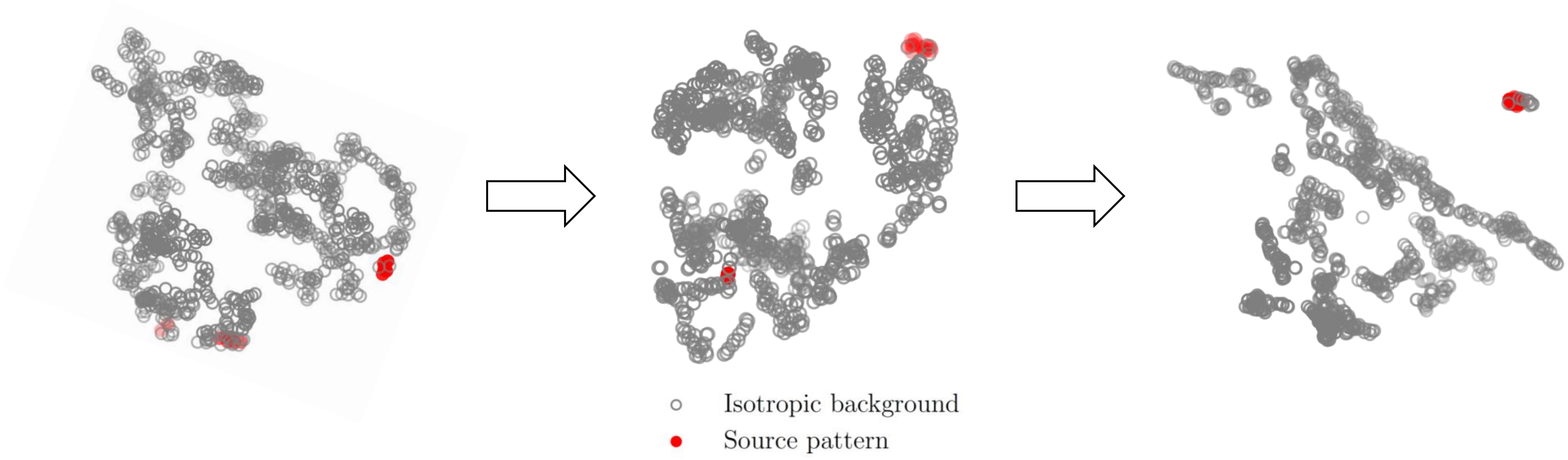
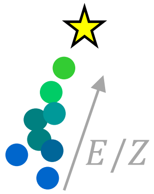
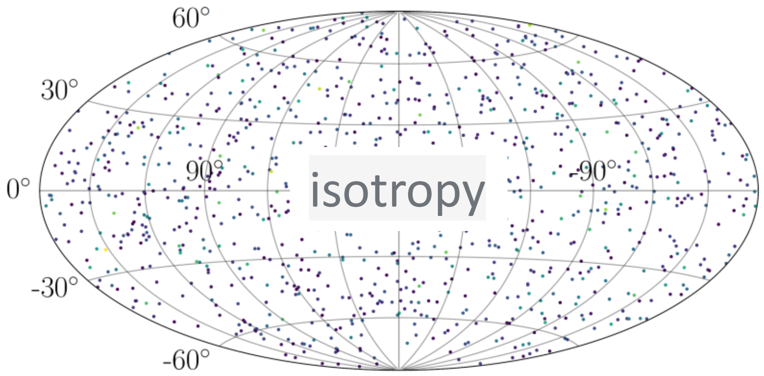


Motivation

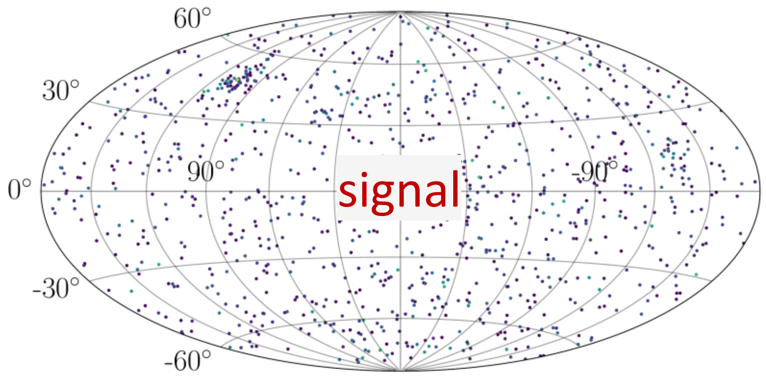
Ultra-high energy cosmic rays from source → Passage through GMF → **Energy- and charge**-dependent deflection → **Pattern** in arrival directions



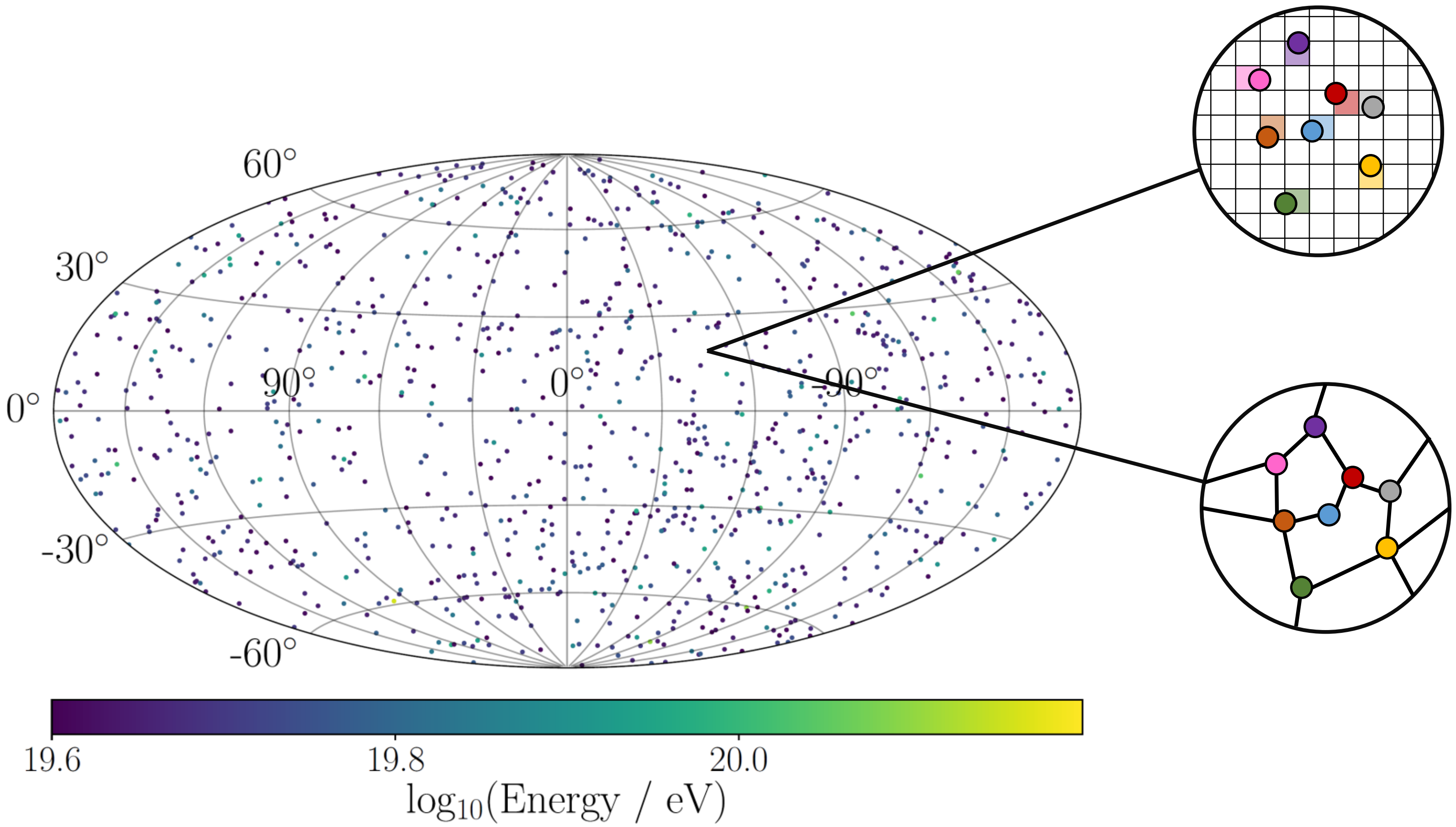
- ➞ Identify patterns to identify sources
- ➞ Pattern recognition task
- ➞ Use **convolutional neural networks**



Classification task



Approach

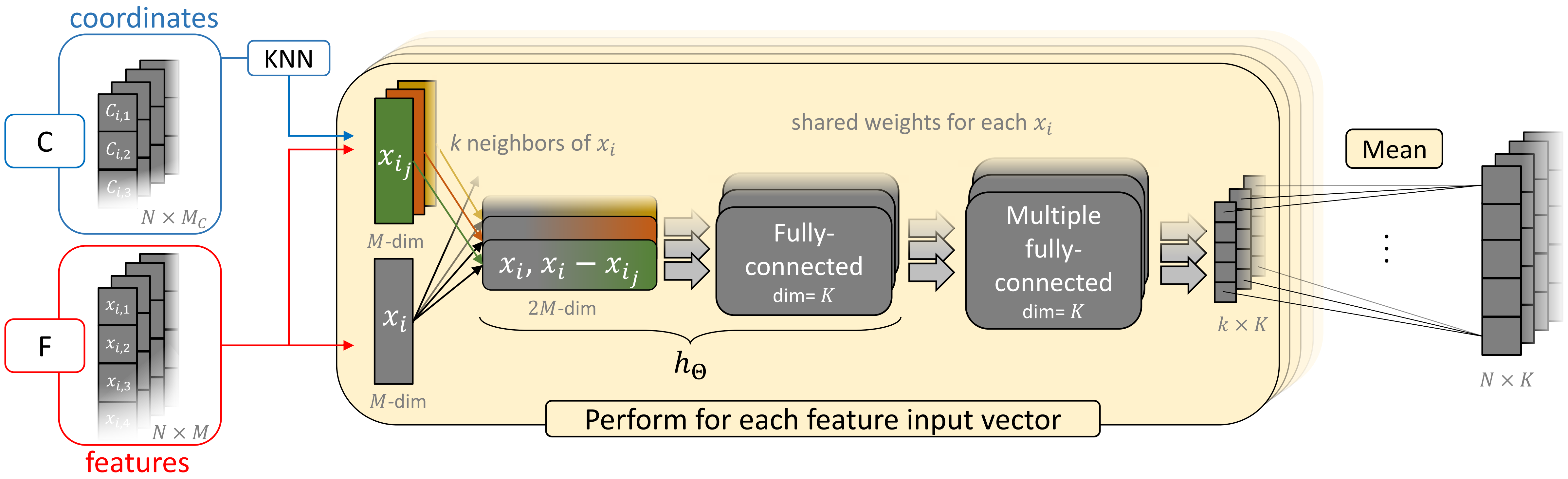


Continuously distributed on **sphere**

- ↯ Not well-suited for classical pixel-grid-based CNNs
- ➞ **Use Graph Convolutional Neural Network!**

EdgeConv[1] **Layer**

$$h_\Theta^a\left(x_{i,c}, x_{i_j,c}\right) = \sum_{c=1}^{M} \theta_c^a\, x_{i,c} + \sum_{c=1}^{M} \theta_c'^a\left(x_{i_j,c} - x_{i,c}\right) \qquad a = 1 \ldots K \text{ (number of filters)}$$

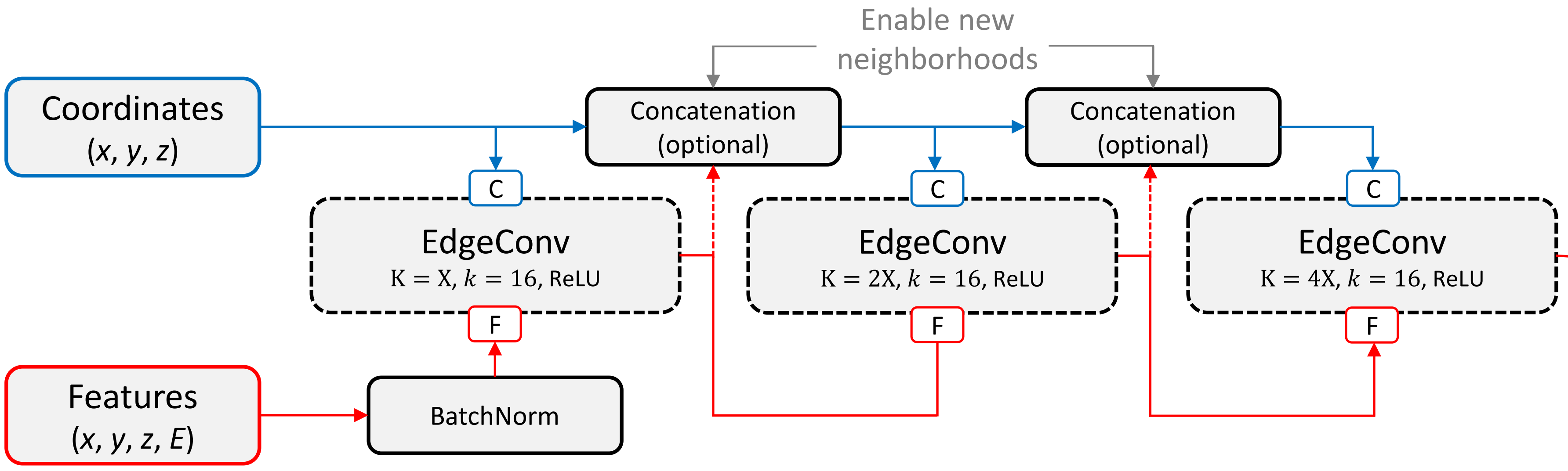


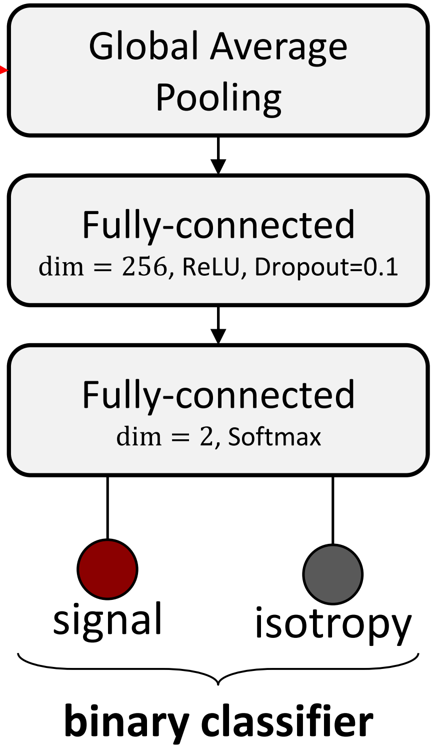
- **Weight sharing** for each pair of cosmic rays
- **Mean** over neighbors j

} **permutation invariance**

[1] https://arxiv.org/abs/1801.07829

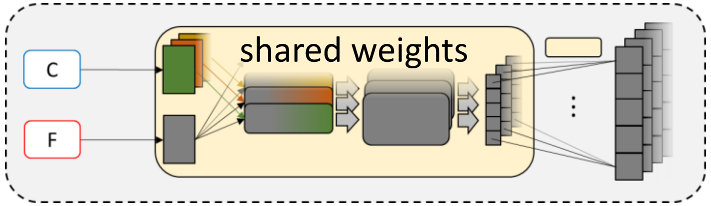
Network Architecture





Architecture inspired by
network used in particle physics
(*ParticleNet*[1])

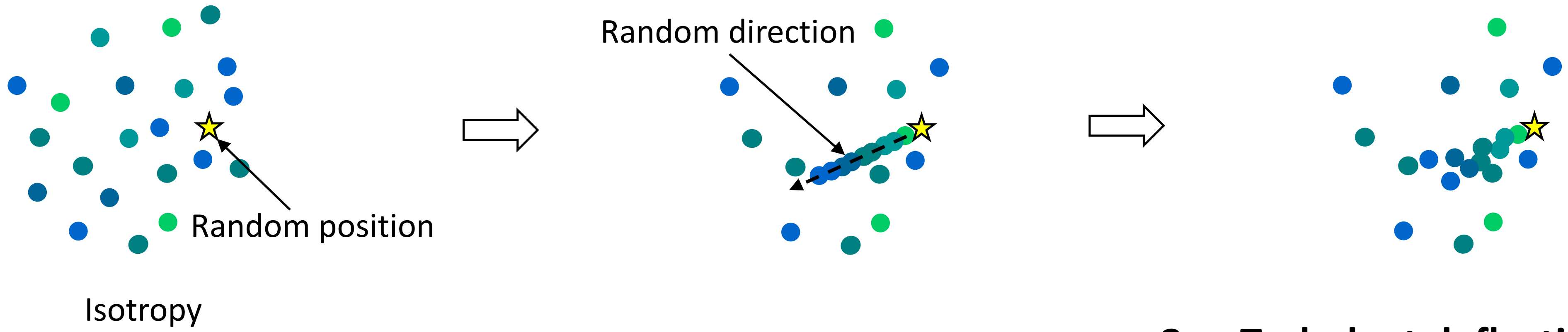
EdgeConv Layer



[1] https://arxiv.org/abs/1902.08570

Simulation of a Single Source

Simplified scenario: **one source pattern** of N_S cosmic rays + isotropic background



1. **Coherent deflection**
   Rotation in random direction with rotation angles

$$\delta_{\text{coh}}(R = E/Z) = \frac{D}{R/EV}\,\text{rad}$$

2. **Turbulent deflection:**
   Scattering according to Fisher distribution of width

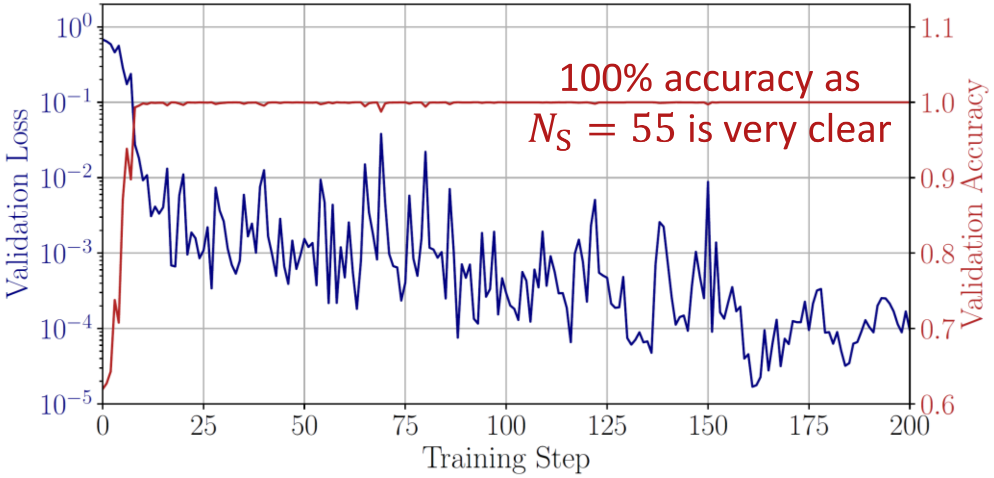
$$\sigma_{\text{turb}}(R = E/Z) = \frac{T}{R/EV}\,\text{rad}$$

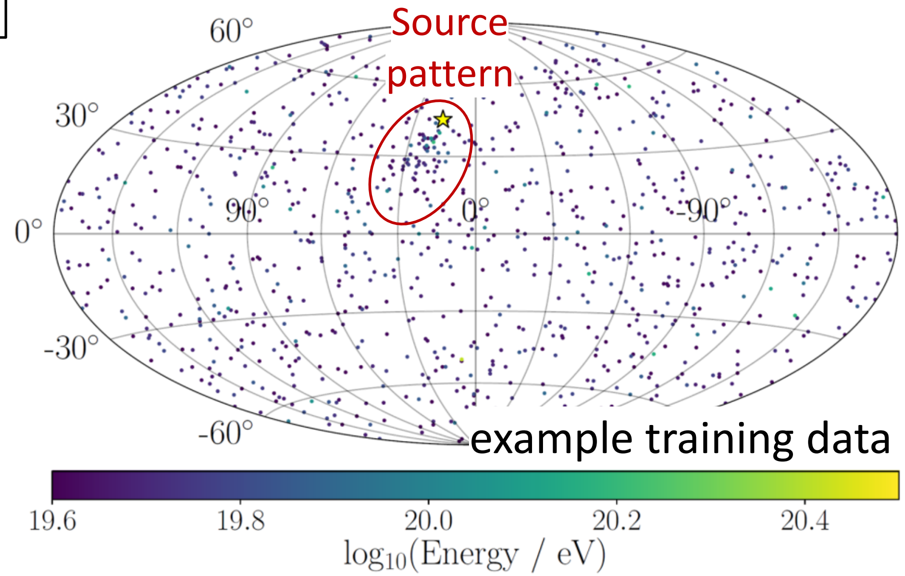
Training

- **1000 cosmic rays** with $\boldsymbol{E > 40}$ **EeV**, spectrum similar to measurements of Pierre Auger Observatory
- Simulate on the fly during training → **no overfitting**
- Train on **strong multiplets** and let the network generalize

Composition	Turb. deflection T	Coherent deflection D	Source CRs
Pure Helium	50% of JF12 maximum in train values from JF12 in validation	Typical values from JF12 but larger than turbulent	55

EdgeConv dims	Loss	Optimizer	Concatenation
16/32/64	Categorical cross entropy	Adam	No

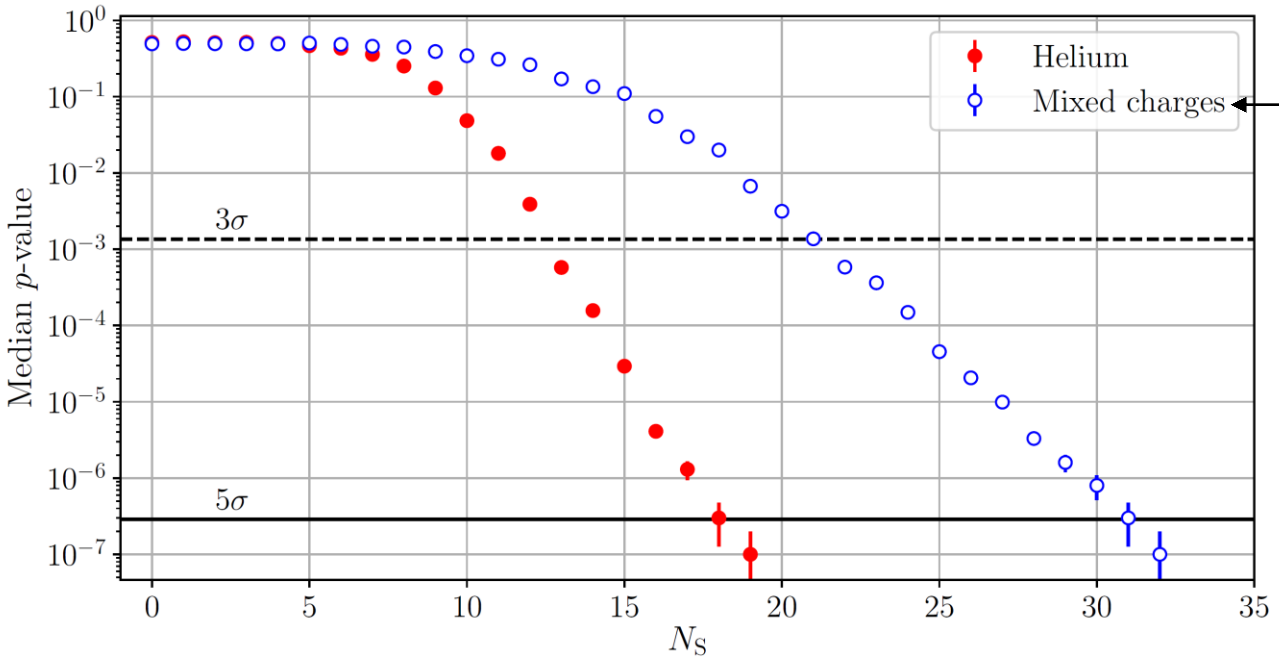




Sensitivity

Analyze cosmic-ray skies simulated using position-dependent deflection strengths from JF12

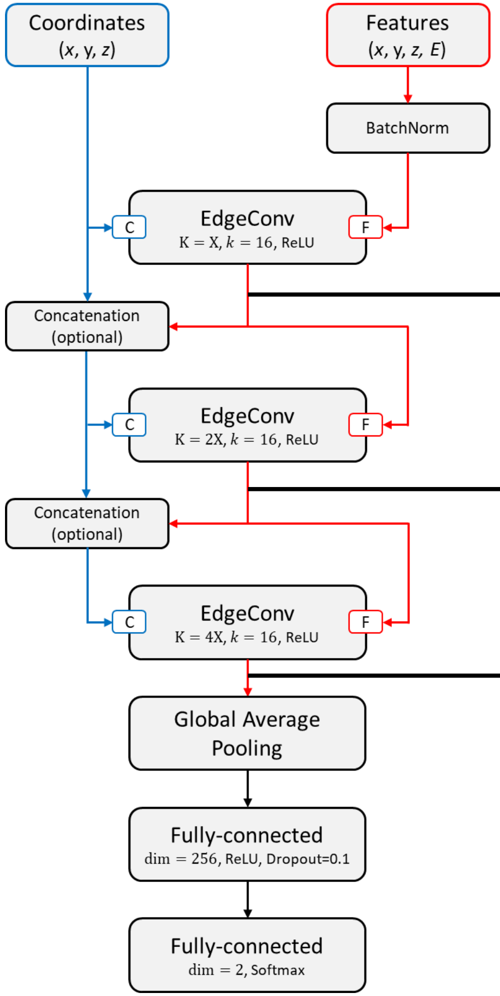
- Determine the **'signal'-output** of
  - 10^3 cosmic-ray skies for **varying** $\boldsymbol{N_S}$ (x_{sig})
  - 10^7 **isotropic** cosmic-ray skies
- Calculate the relative amount of **'signal'-outputs from isotropy** $\geq \boldsymbol{x_{\mathrm{sig}}}$



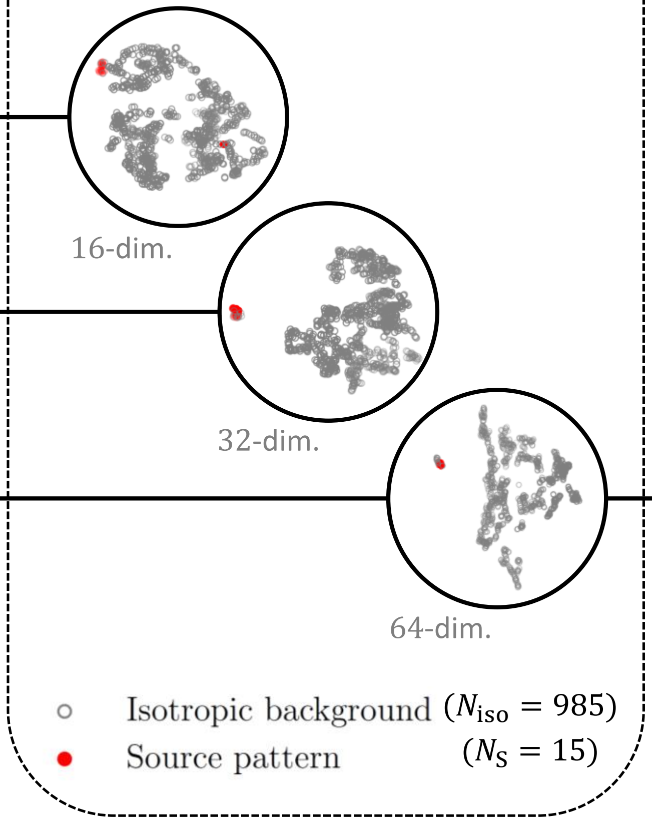
15% H
45% He
40% C, N, O

Works great, but what does it do?

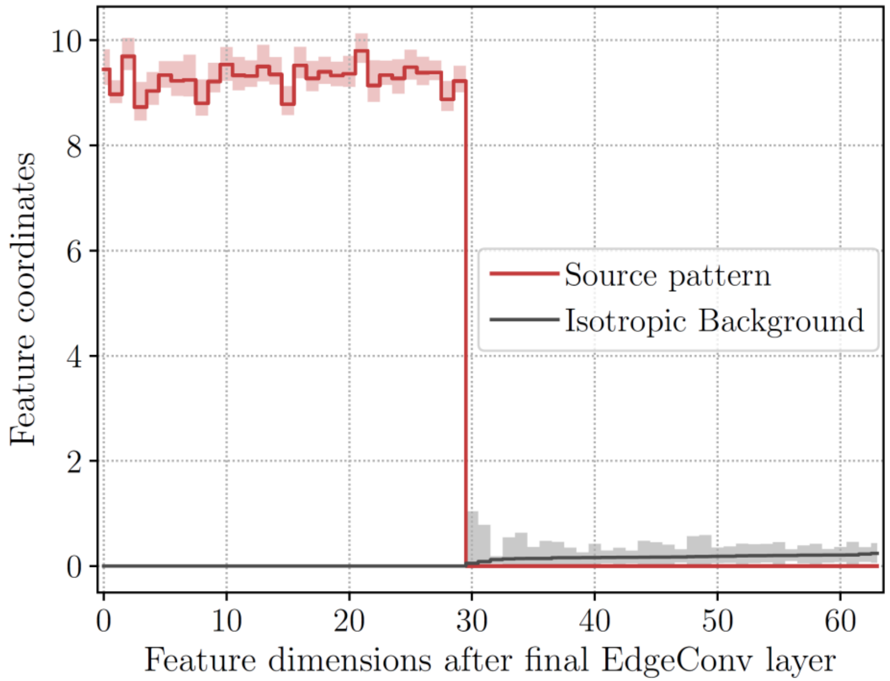
The Network‘s Working Principle



Visualize **high-dimensional point clouds** using t-SNE:

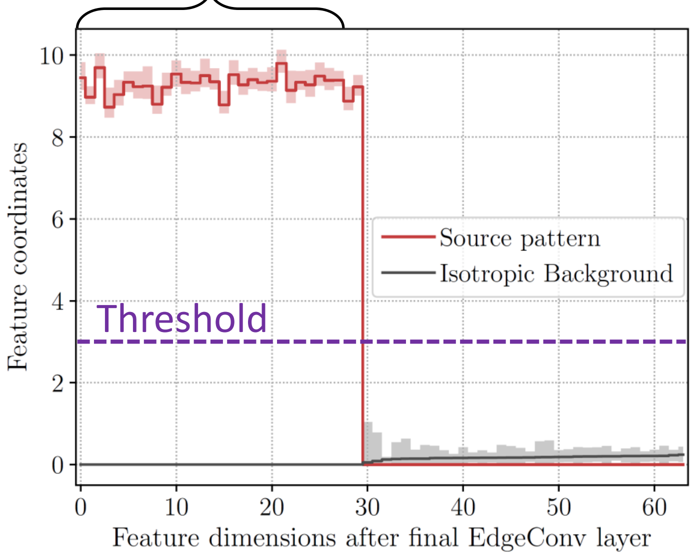


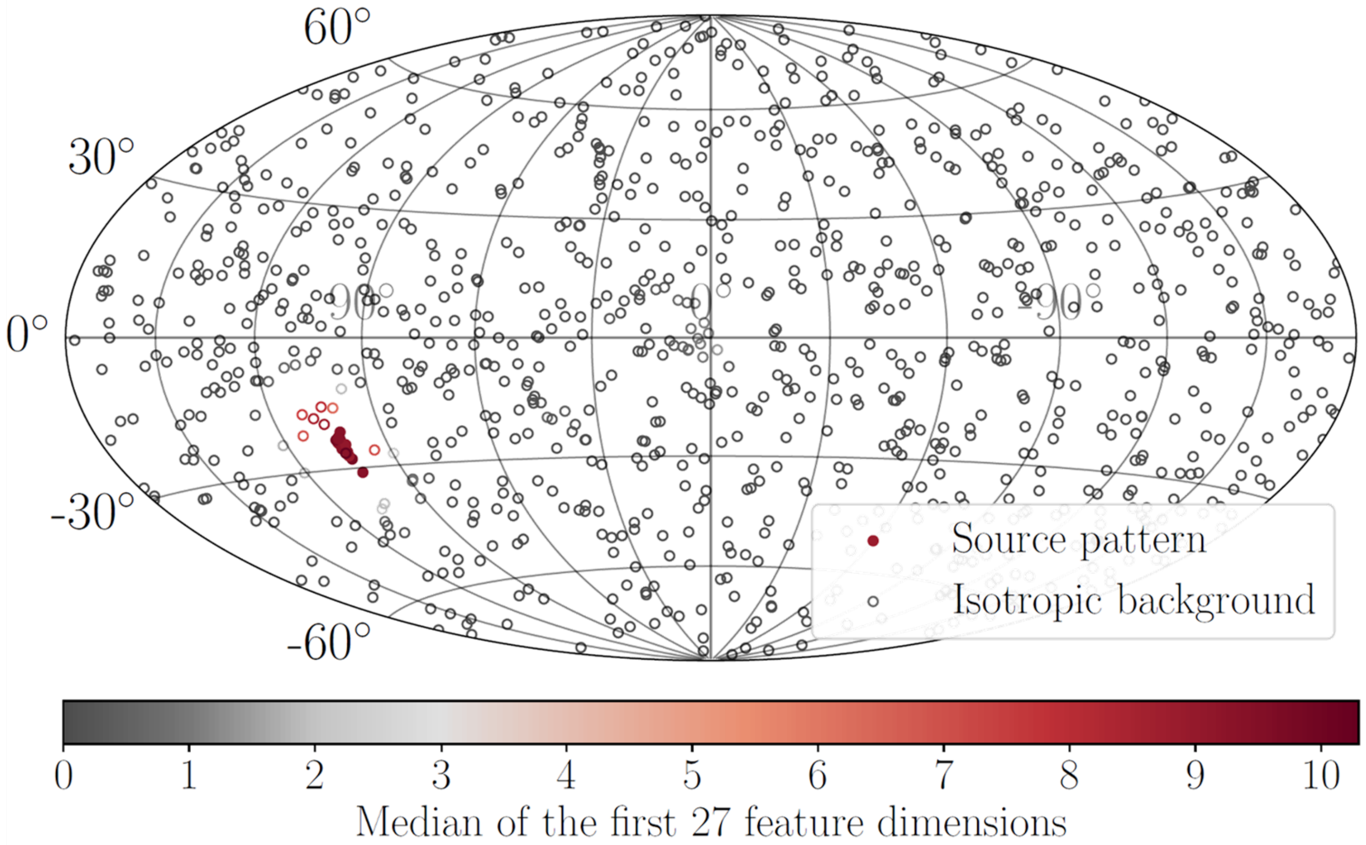
One cosmic-ray sky with $\boldsymbol{N_S = 15}$
(Shaded: 68% percentile)



Individual Classification of Cosmic Rays

Take **median** of first 27 dimensions and classify via **threshold** of 3





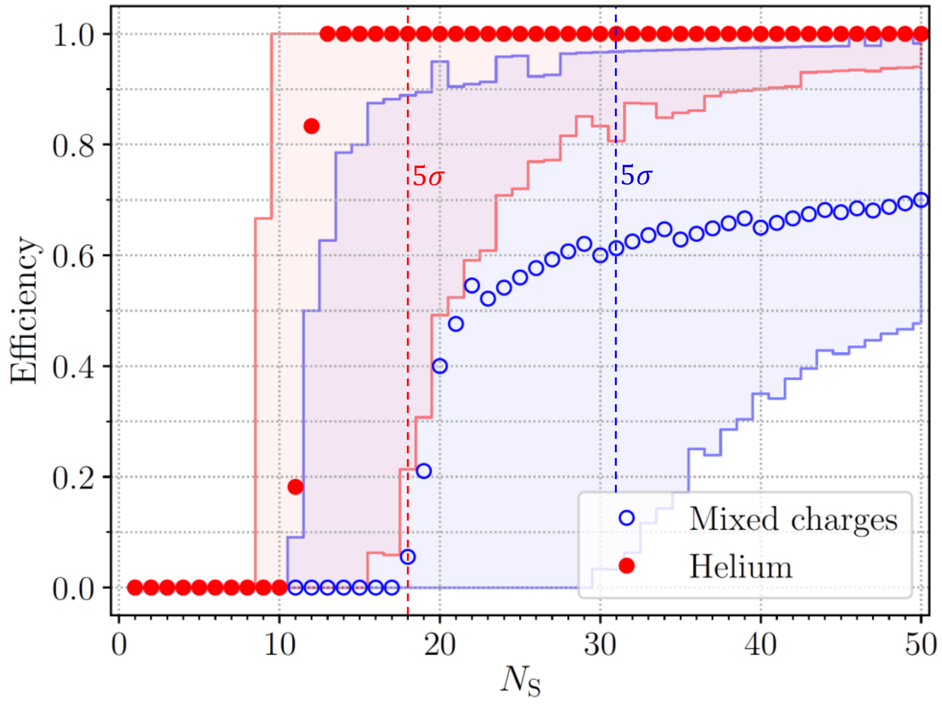
Individual Classification of Cosmic Rays

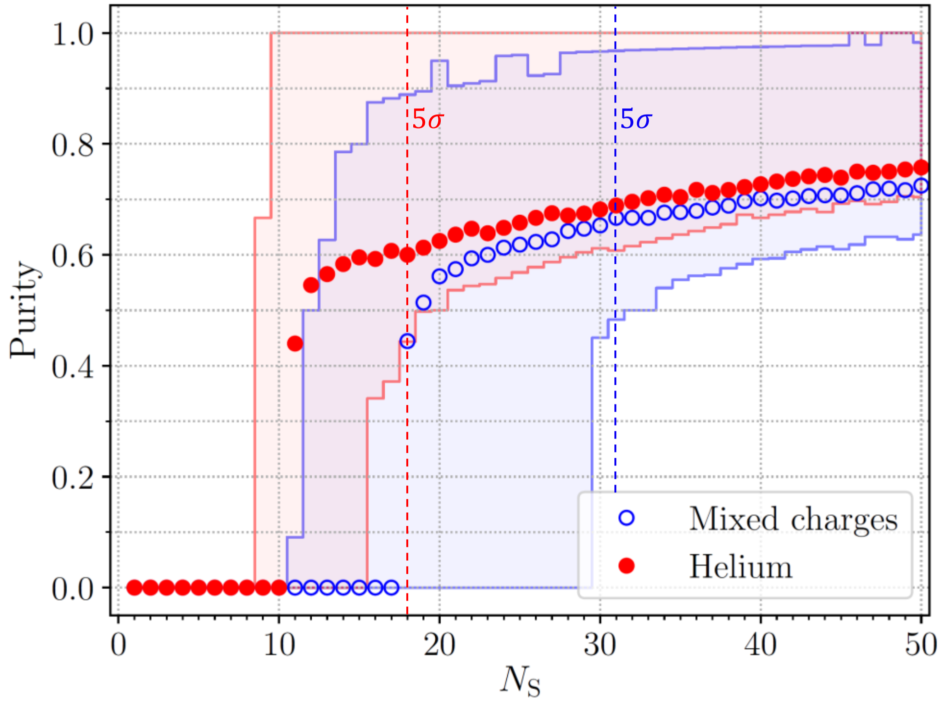
Efficiency:

$$\epsilon = \frac{\text{\# correctly identified signal cosmic rays}}{\text{\# signal cosmic rays}}$$

Purity:

$$\rho = \frac{\text{\# correctly identified signal cosmic rays}}{\text{\# identified signal cosmic rays}}$$

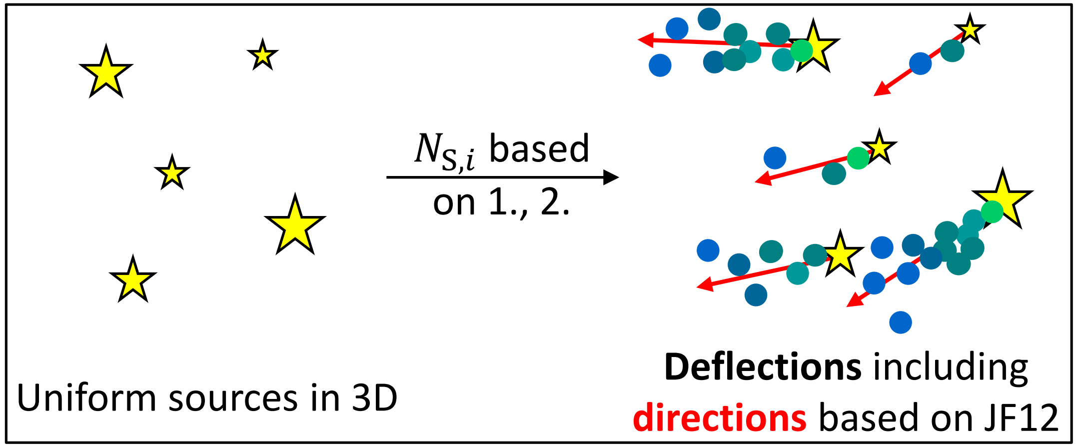




(of 1000 cosmic rays)

Multiple Sources: Simulation & Training

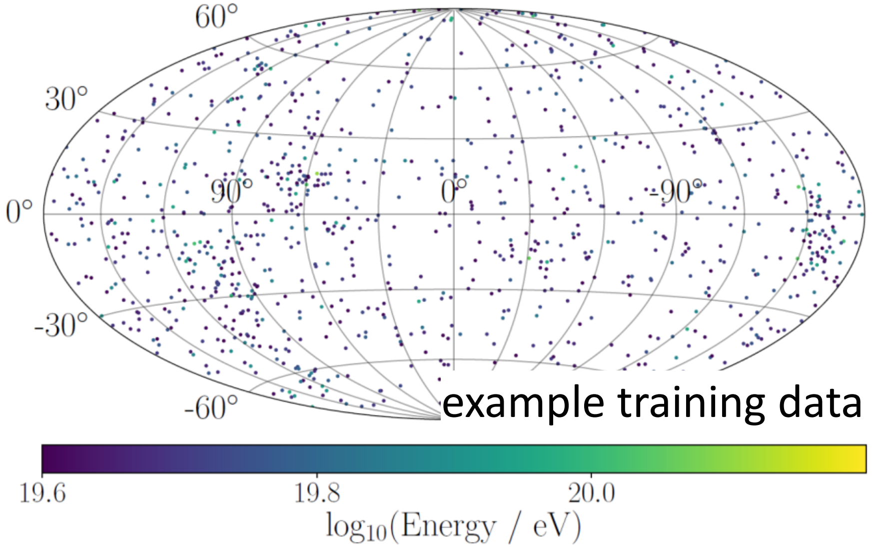
- Realistic **3D universe**: **uniformly distributed** and identical sources, accounting for:
  1. **Geometrical effect** on fluxes: $f_i \propto d_i^{-2}$
  2. **Interactions**, e.g. with photon fields
- Use parameters from **Auger Combined Fit**[1]
- Only free parameter: **source density ρ_S**
- **Apply deflection** based on JF12



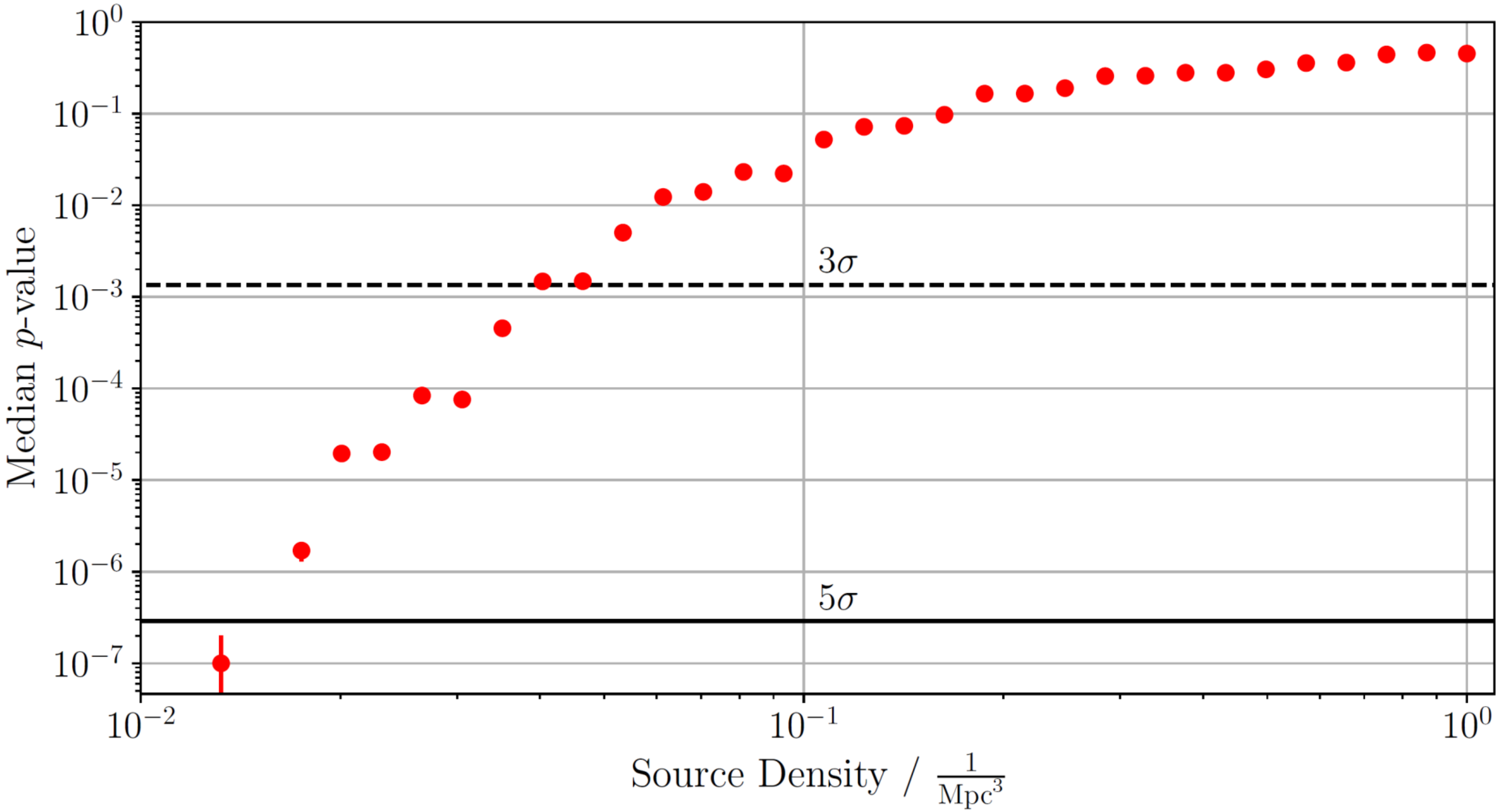
[1] https://arxiv.org/abs/1612.07155

Adjust network to new simulation:

- EdgeConv-dimensions: 64/128/256
- **Concatenation: Yes**
- Train on $\rho_S = 10^{-3}/\text{Mpc}^3$
- **Deflection** strengths and directions **maps** from JF12 **randomly rotated** and used for all cosmic rays to achieve global consistency

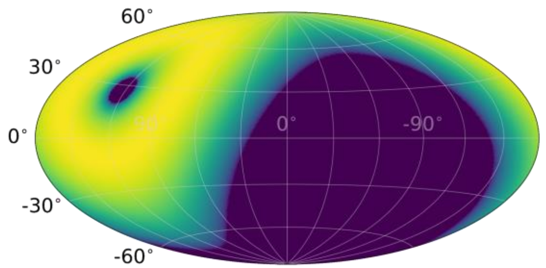


Sensitivity

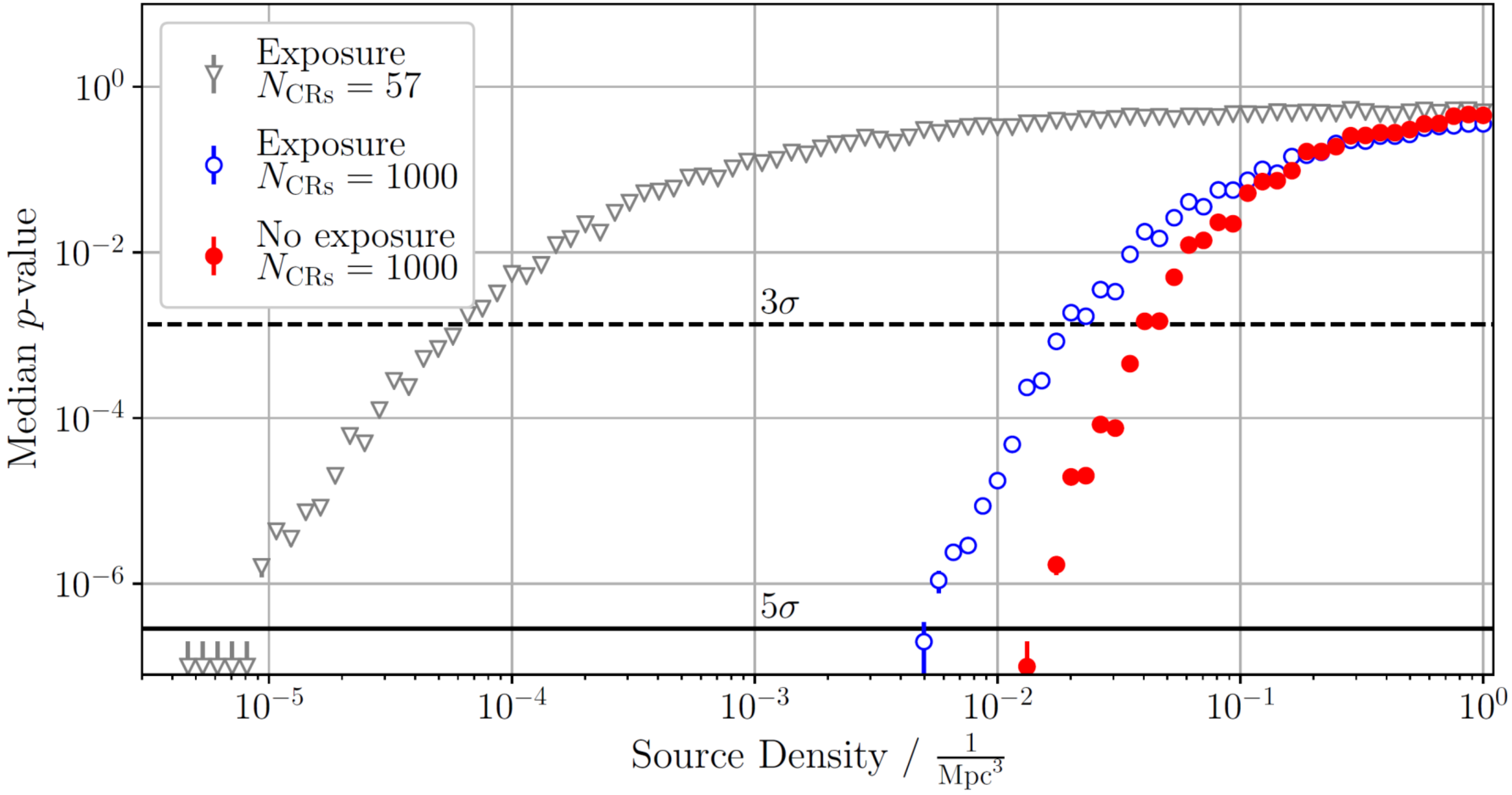


5σ at $\rho_S \approx 1.5 \cdot 10^{-2}/\mathrm{Mpc}^3$

Sensitivity: Effect of Limited Sky Coverage



Examplary exposure of observatory in galactic coordinates



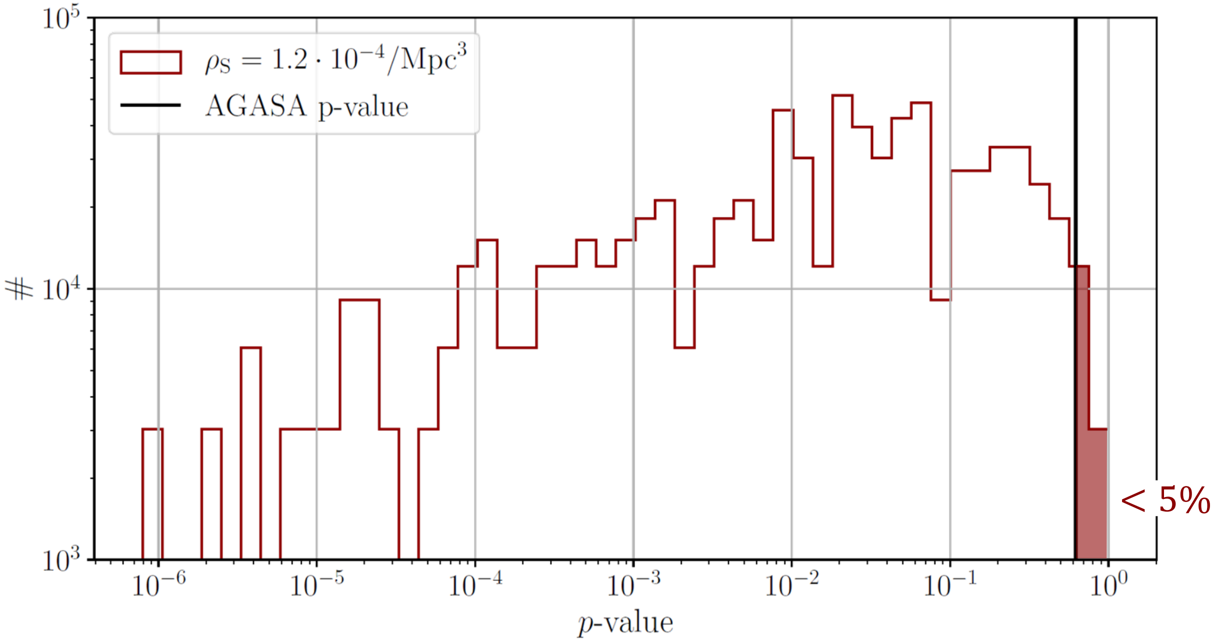
Source density limit from AGASA data

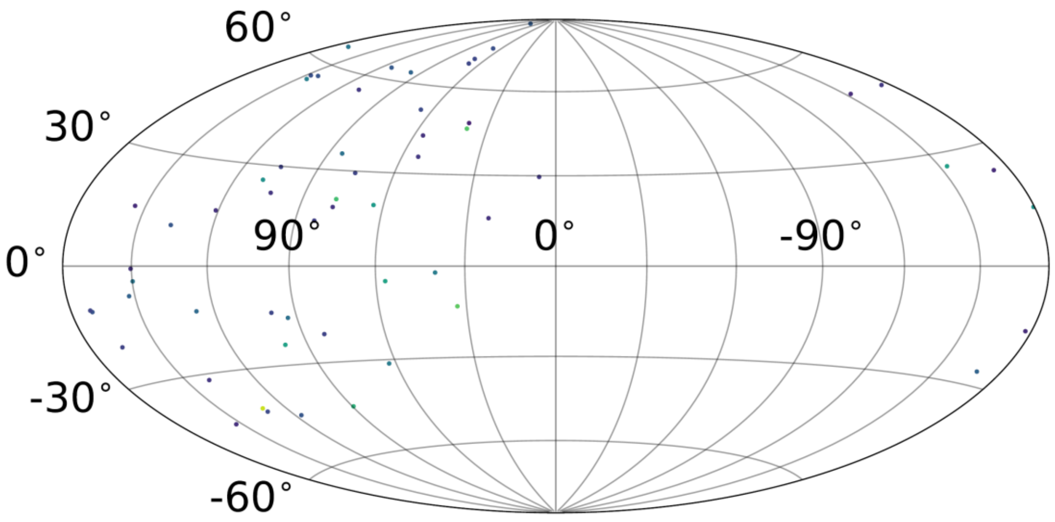
Akeno Giant Air Shower Array (**AGASA**): cosmic ray observatory, in operation from 1990 to 2007

1. Calculate ***p*-value** using equatorial scrambling as isotropy

   $\rightarrow \mathbf{p = 0.63}$

2. Take ***p*-value distribution** for varying ρ_S and determine the **probability of** $\mathbf{p \geq 0.63}$



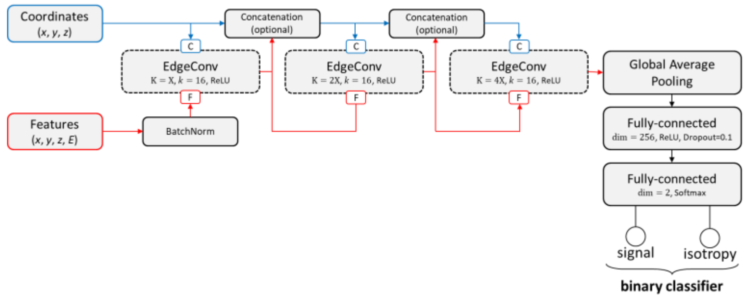


public AGASA dataset

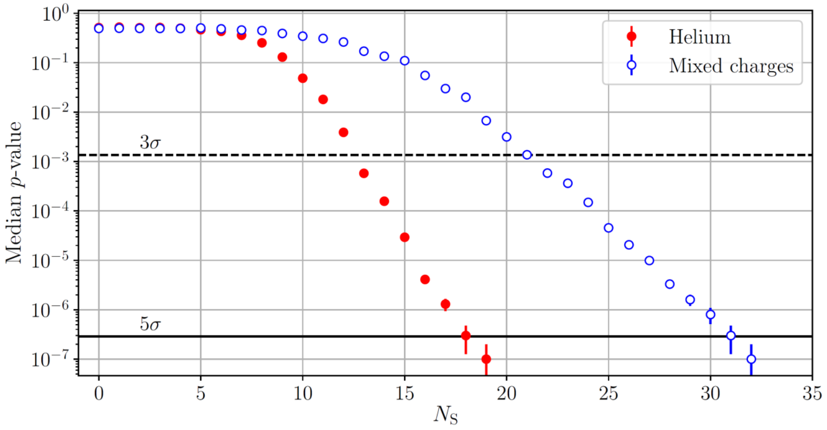
At $\boldsymbol{\rho_S^{95} = 1.2 \cdot 10^{-4}/\text{Mpc}^3}$: $\text{p} \geq 0.63$ for 5%

$\rightarrow$ Lower limit of ρ_S at 95% confidence

Conclusion



- (Dynamic) GCNN well-suited for **sparse cosmic-ray data**, efficiently using all available information
- Network automatically performs **clustering**



- High **sensitivity for a single pattern** of one strong source (5σ at $f_{\mathrm{sig}} = 1.8\%$)
  - ➞ Possibility to identify **individual source cosmic rays**
- Identify the **global structure** of simulated universe
  - ➞ **Constrain source density** ($\rho_S \geq 1.2 \cdot 10^{-4}/\mathrm{Mpc}^3$ from AGASA data)

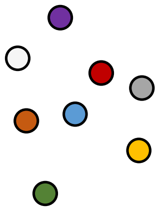
➞ **Great potential** for data from current observatories ($N_{\mathrm{CRs}} \sim 1000$)

Backup

Dynamic Graph Convolutional Neural Network

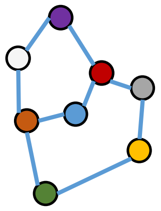
Input: Point Cloud of N points of dimension M

Our task: 4D points (x, y, z, E)



k=2

Build graph by connecting points x_i to k nearest neighbors x_{i_j}



Perform 'EdgeConv' [1] operation

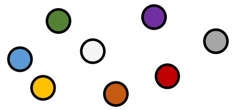
$a = 1 \ldots K$ (number of filters)

$$h_\Theta^a\left(x_{i,c}, x_{i_j,c}\right) = \sum_{c=1}^{M} \theta_c^a\, x_{i,c} + \sum_{c=1}^{M} \theta_c'^a\left(x_{i_j,c} - x_{i,c}\right)$$

⋮ Additional fully-connected layers

Mean

N points in K dimensions

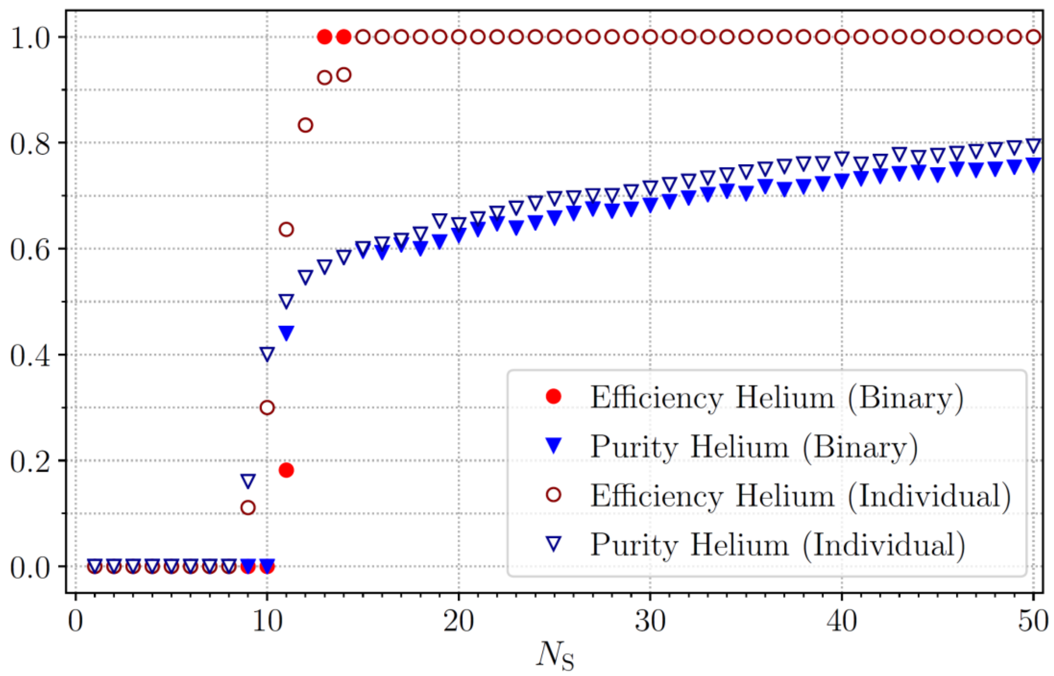


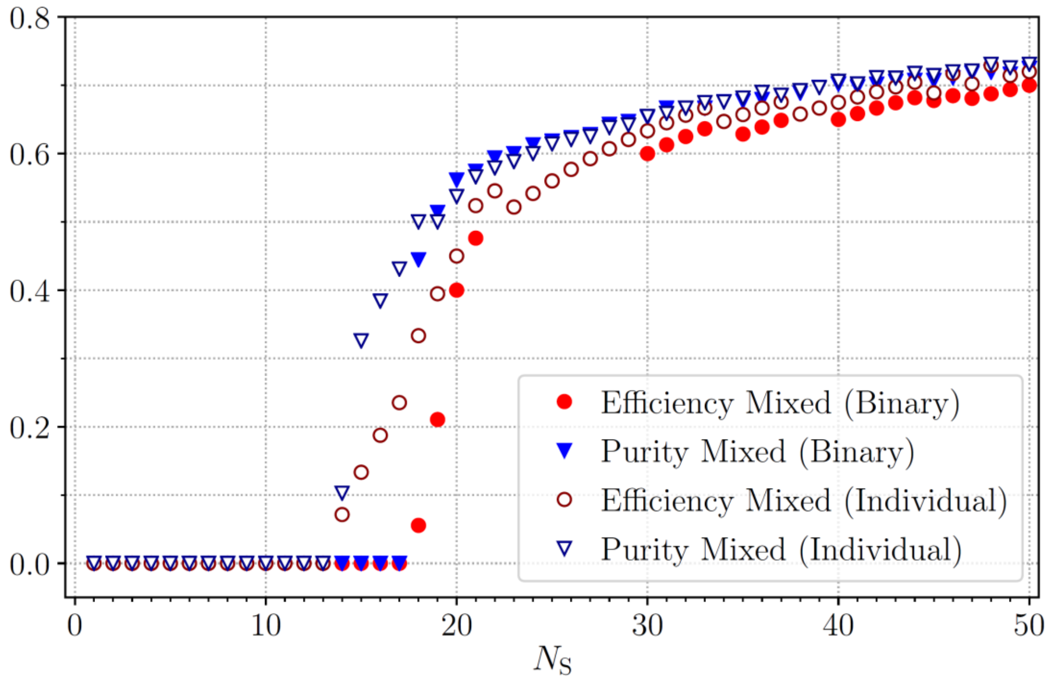
[1] https://arxiv.org/abs/1801.07829

Individual Classifier

Comparison of strategies to achieve classification of individual cosmic rays

1. 'Binary': Binary cosmic-ray sky classifier with threshold
2. 'Individual': Network explicitly trained to classify individual CRs (1000 outputs)

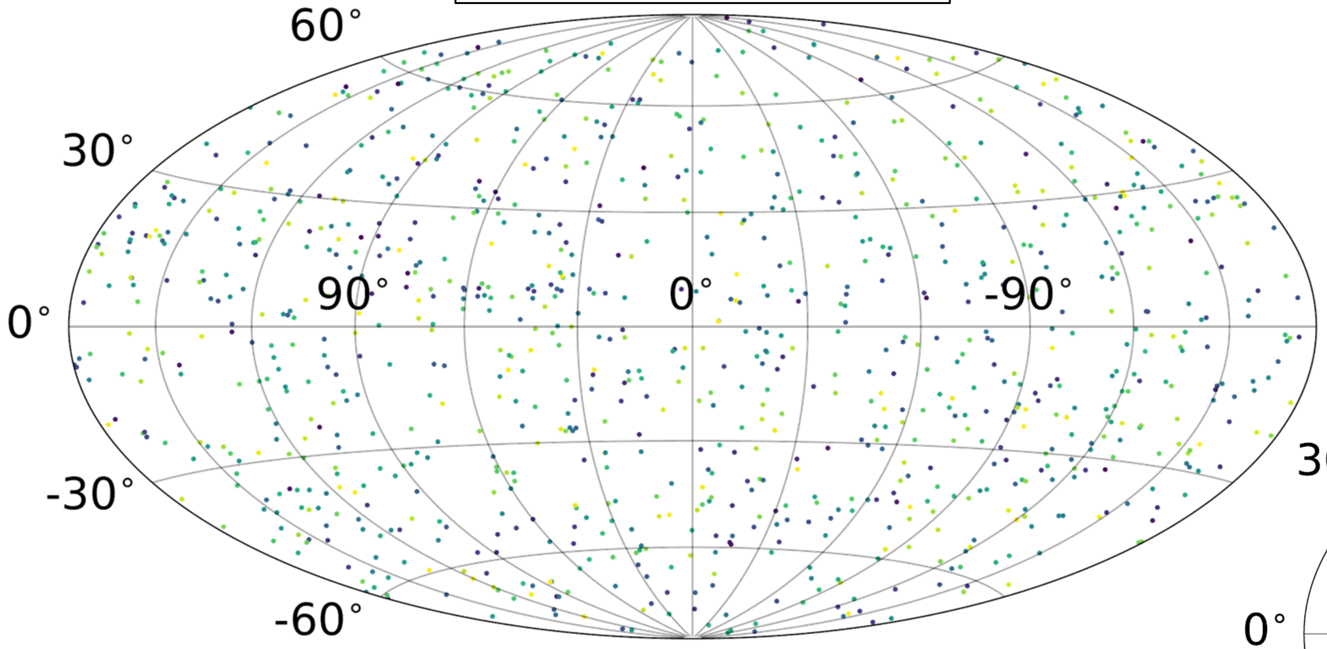




Representative skymaps

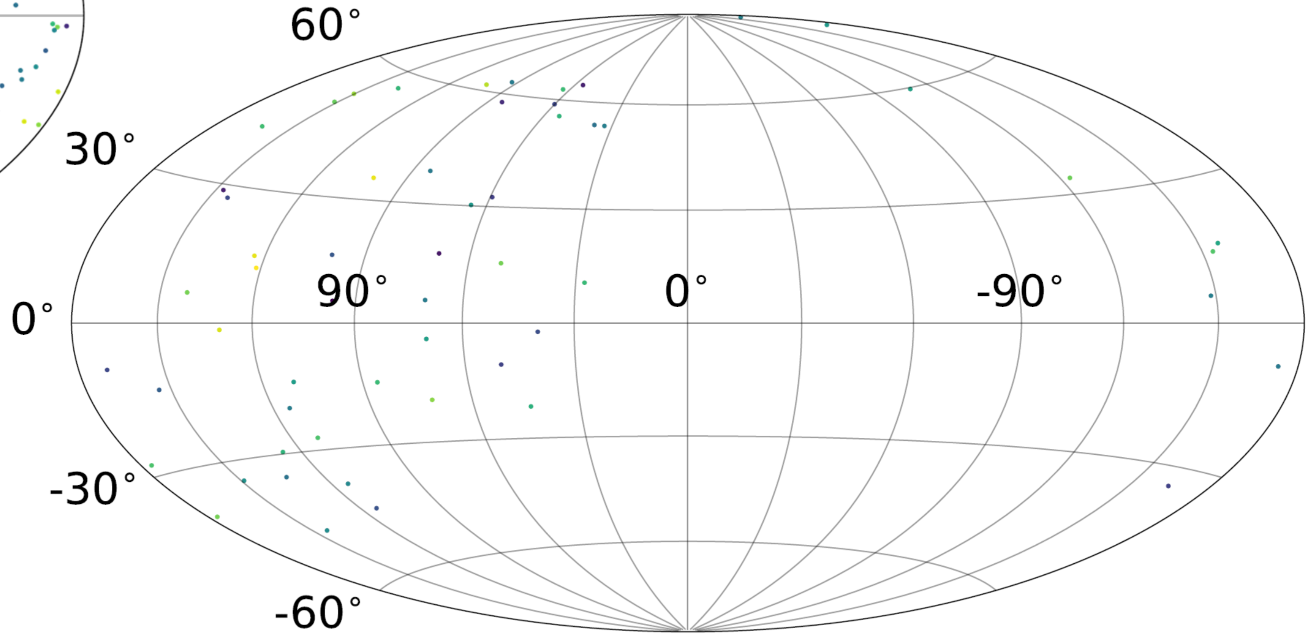
(Cosmic-ray skies that result in the median network-response of a given source density.)

1000 CRs, no exposure



$10^{-1}/\mathrm{Mpc}^3$

57 CRs, exposure



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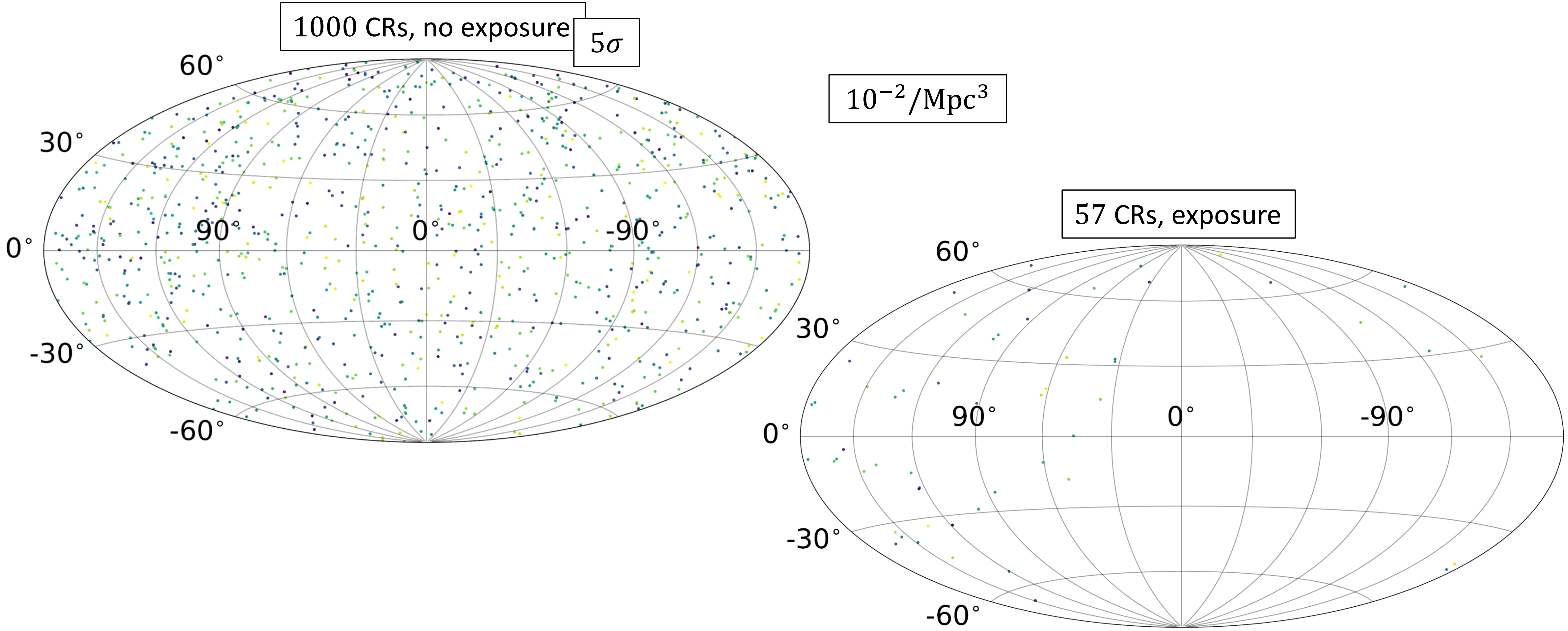

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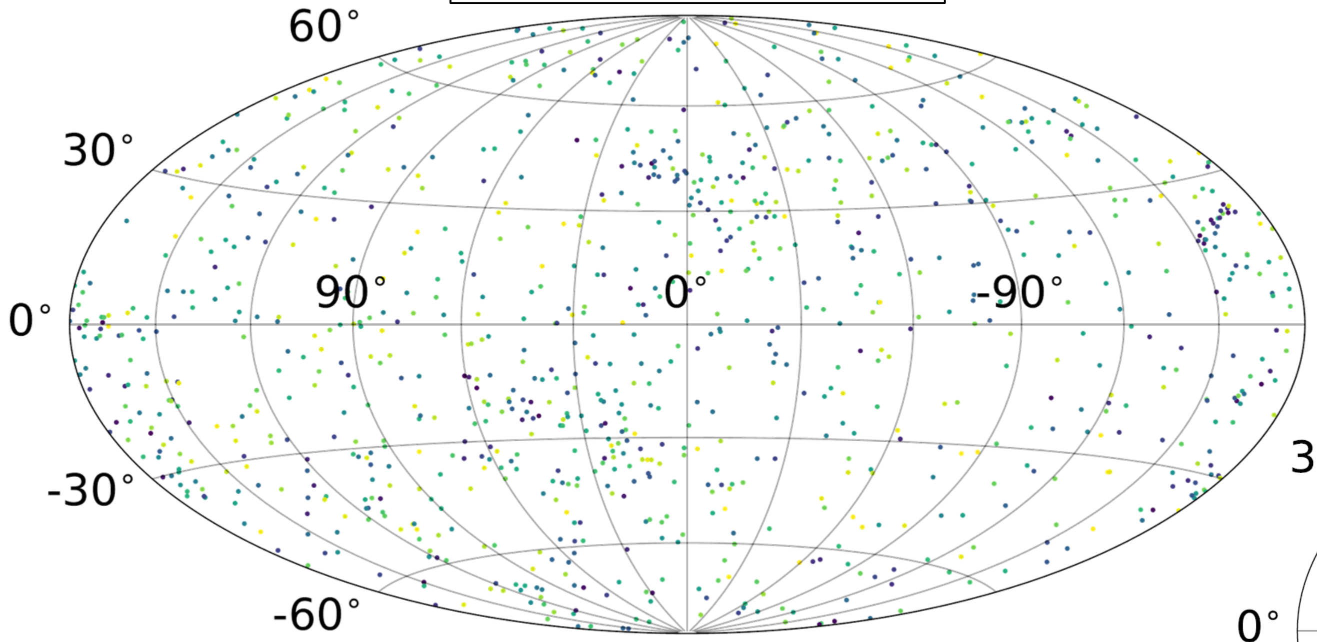
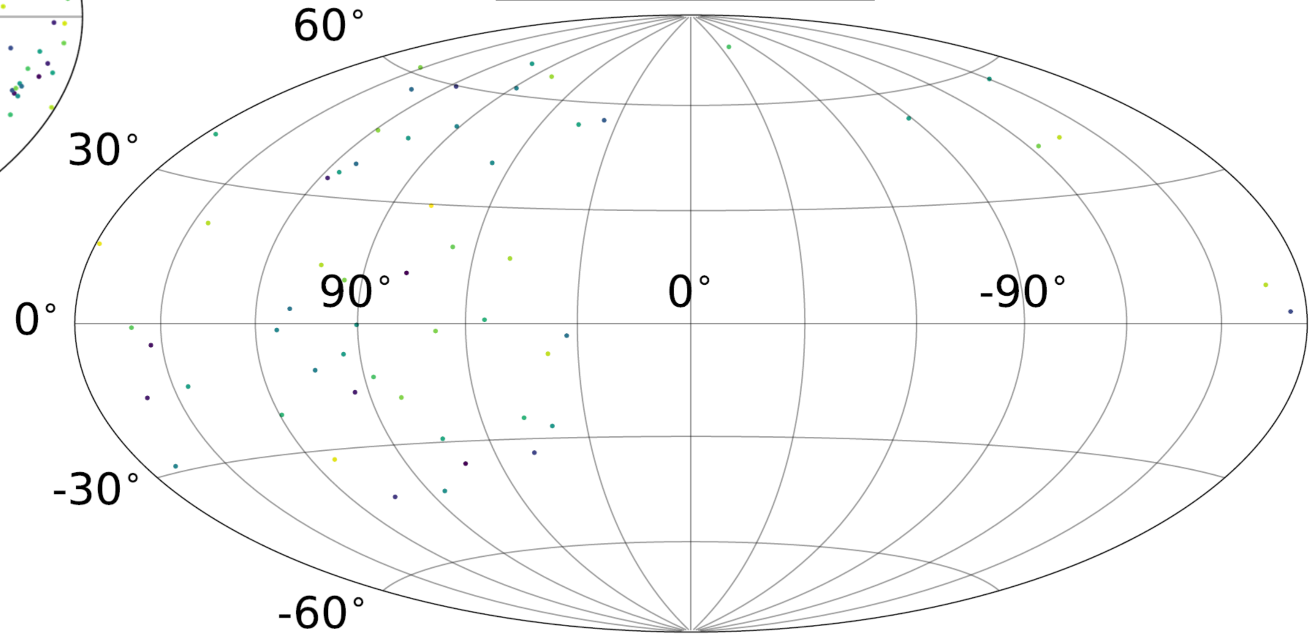


$10^{-3}/\mathrm{Mpc}^3$

57 CRs, exposure

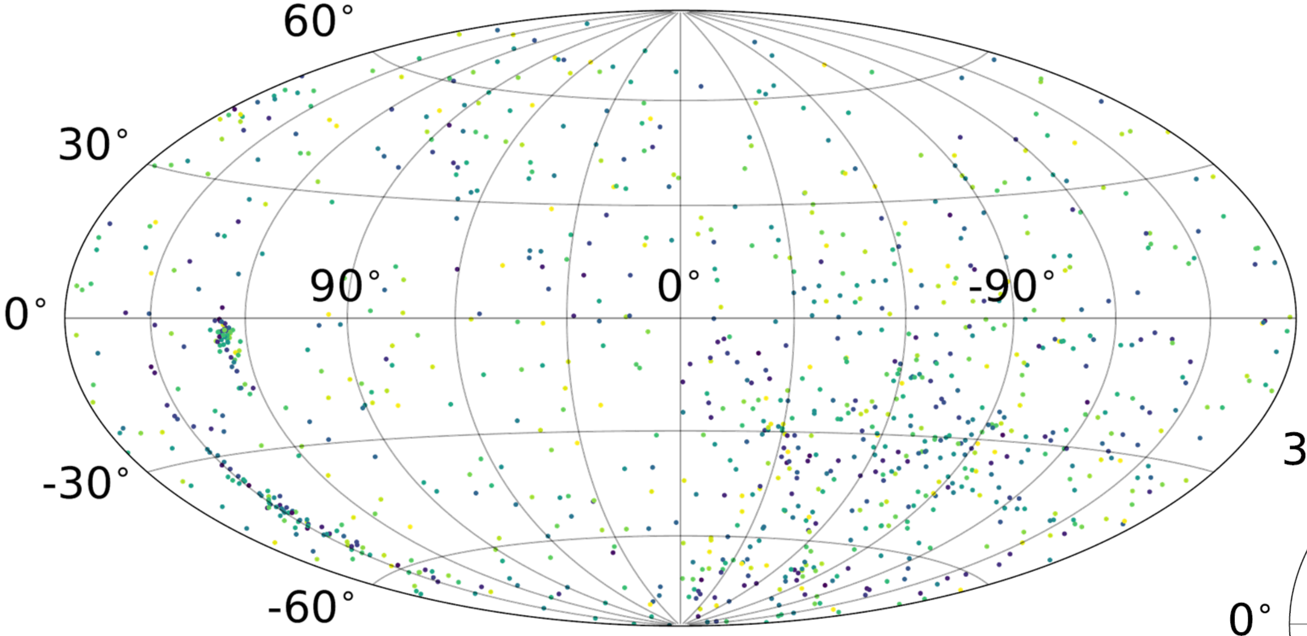


Representative skymaps

(Cosmic-ray skies that result in the median network-response of a given source density.)

$10^{-4}/\mathrm{Mpc}^3$

1000 CRs, no exposure



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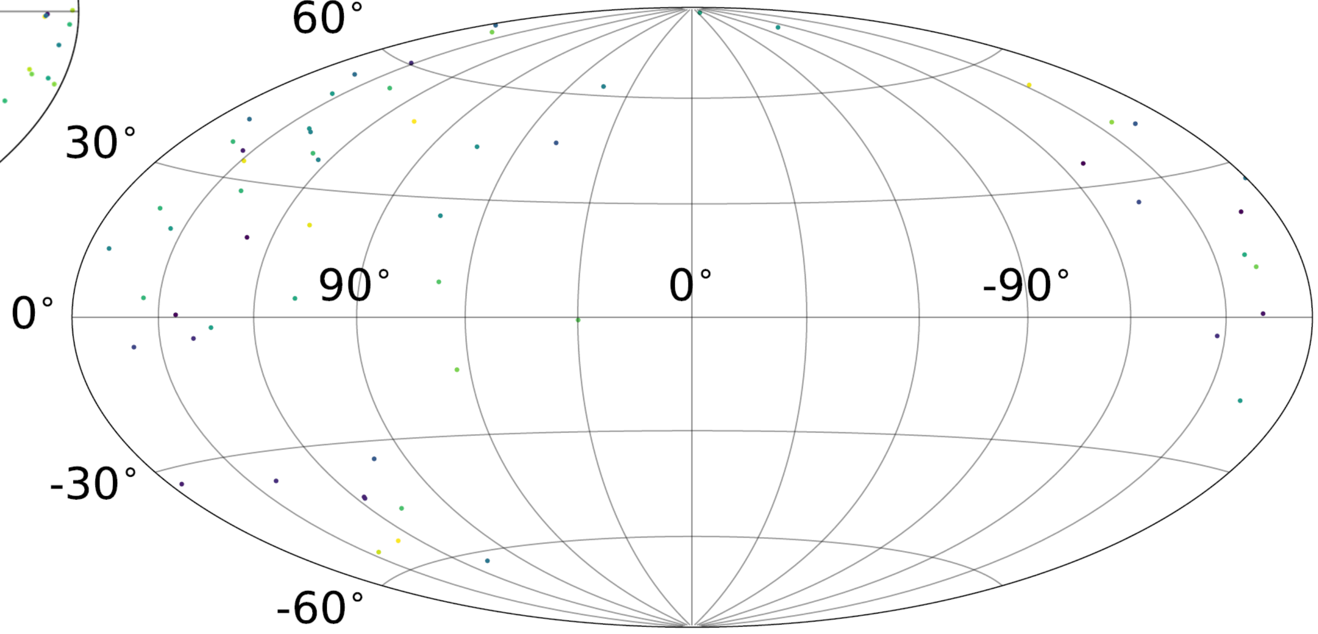

Training on simulation of multiple sources

- Based on Auger (parameters from Combined Fit): 1000 cosmic rays with $E > 40$ EeV,
- Simulate on the fly during training → no overfitting
- Train on $\rho_S = 10^{-3}/\mathrm{Mpc}^3$
  or $\rho_S = 10^{-2}/\mathrm{Mpc}^3$ (with exposure)
- Deflection strengths and directions maps from JF12 randomly rotated and used for all cosmic rays
- Turbulent deflection: 50% of maximum during training

EdgeConv dims 64/128/256	Loss Categorical cross entropy
Optimizer Adam	Concatenation Yes