SOFT-COLLINEAR EFFECTIVE THEORY

[GUIDO BELL]

UNIVERSITÄT SIEGEN

OUTLINE

SCET OVERVIEW

SCALES, MODES, FACTORISATION

SCET-1 AND SCET-2

APPLICATIONS

ANGULARITIES

NNLL$'$+NLO PREDICTIONS

NON-PERTURBATIVE EFFECTS

COMPARISON TO DATA

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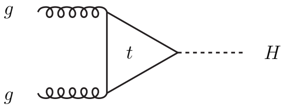
Momentum scales

Inclusive Higgs production



$m_t \simeq 175$ GeV

$m_H \simeq 125$ GeV

$\Lambda_{\rm QCD} \simeq 0.5$ GeV

Momentum scales

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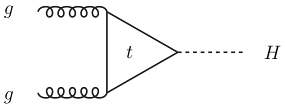

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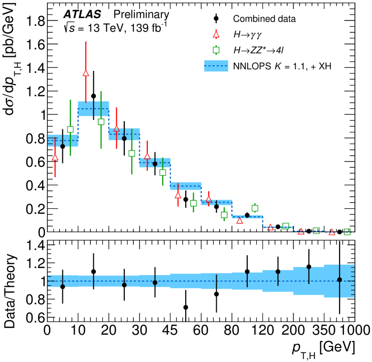
Higgs p_T spectrum



$p_T \simeq 0 - 500$ GeV

Momentum scales

Peak region: $p_T \simeq 0 - 30$ GeV

- fixed-order expansion breaks down
- resummation of $\ln^2 \frac{p_T}{m_H}$ necessary
- (non-perturbative effects for $p_T \lesssim 10$ GeV)

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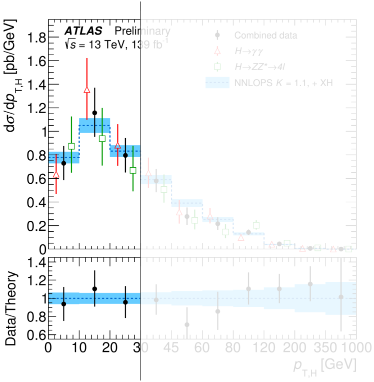

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Tail region: $p_T \simeq 60 - 500$ GeV

- fixed-order expansion applicable
- (resummation for $p_T \gg m_H$)

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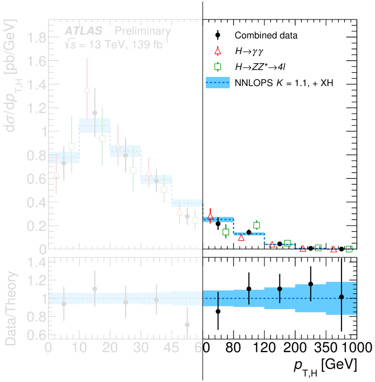

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- combine fixed-order and resummation

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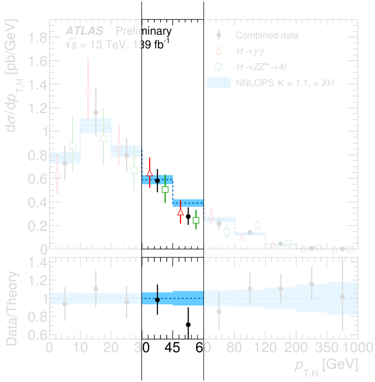

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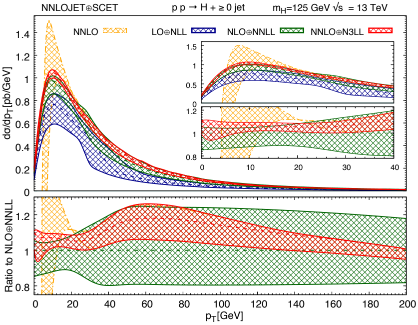


[Chen et al 18]

Scale separation

For $\Lambda_{\text{QCD}} \ll p_T, m_H, m_t$ the cross section factorises

$$d\sigma \simeq \sum_{i,j} f_{i/p}(\Lambda_{\text{QCD}}, \mu) \otimes f_{j/p}(\Lambda_{\text{QCD}}, \mu) \otimes d\hat{\sigma}_{ij\to HX}(p_T, m_H, m_t, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{p_T}\right)$$

- universal parton-distribution functions $f_{i/p}$
- perturbative partonic cross section $d\hat{\sigma}_{ij\to HX}$

Factorisation scale μ separates short- and long-distance dynamics

- single-logarithmic evolution controlled by DGLAP equations

$$\frac{df_{i/p}(\mu)}{d\ln\mu} = \sum_j P_{ij}(\alpha_s) \otimes f_{j/p}(\mu)$$

Scale separation

For $\Lambda_{\rm QCD} \ll p_T, m_H, m_t$ the cross section factorises

$$d\sigma \simeq \sum_{i,j} f_{i/p}(\Lambda_{\rm QCD},\mu) \otimes f_{j/p}(\Lambda_{\rm QCD},\mu) \otimes d\hat{\sigma}_{ij\to HX}(p_T, m_H, m_t, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{p_T}\right)$$

- universal parton-distribution functions $f_{i/p}$
- perturbative partonic cross section $d\hat{\sigma}_{ij\to HX}$

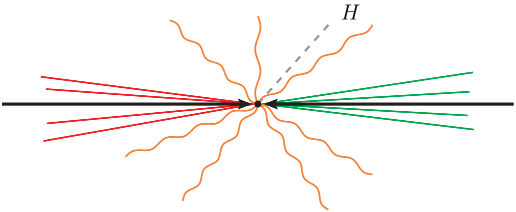
Factorisation can be derived using methods from SCET

- PDFs are matrix elements of (non-local) SCET operators
- partonic cross sections are Wilson coefficients of these operators

Small p_T

For $p_T \ll m_H, m_t$ the partonic cross section factorises further

$$\frac{d\hat{\sigma}}{dp_T} \simeq H(m_H, m_t, \mu)\; J_1(p_T, \mu) \otimes J_2(p_T, \mu) \otimes S(p_T, \mu) \;+\; \mathcal{O}\left(\frac{p_T}{m_H}\right)$$



- hard function H
- jet (beam) functions J_i
- soft function S

double-logarithmic RG evolution

$\Rightarrow$ Sudakov logarithms $\alpha_s^n \ln^{2n} \frac{p_T}{m_H}$

Soft-Collinear Effective Theory

[Bauer, Fleming, Pirjol, Stewart 00; Beneke, Chapovsky, Diehl, Feldmann 02]

Expand QCD in powers of $\frac{p_T}{m_H} \ll 1$

$$\mathcal{L}_{\text{QCD}} = \underbrace{\mathcal{L}^{(0)}_{\text{SCET}}}_{\text{leading power}} + \underbrace{\mathcal{L}^{(1)}_{\text{SCET}}}_{\text{next-to-leading power}} + \mathcal{O}\left(\frac{p_T^2}{m_H^2}\right)$$

- various IR modes: soft, 1-collinear, 2-collinear, ...
- various IR scales: $\mu_s, \mu_{c1}, \mu_{c2}, \ldots$

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- various IR modes: soft, 1-collinear, 2-collinear, ...
- various IR scales: $\mu_s,\ \mu_{c1},\ \mu_{c2},\ \dots$

Factorisation at leading power

- interactions of different collinear modes

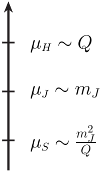
  QCD-SCET matching $\quad\Rightarrow\quad \chi(x) = W_c^\dagger(x)\,\psi(x)$

- interactions of soft and collinear modes

$$\chi(x) = S_n(x_-)\,\chi^{(0)}(x) \quad\Rightarrow\quad \mathcal{L}_{\text{SCET}}^{(0)} = \mathcal{L}_s + \mathcal{L}_{c1} + \mathcal{L}_{c2} + \dots$$

SCET-1

Three-scale problem: $\mu_S \ll \mu_J \ll \mu_H$



$$\boxed{d\hat{\sigma} \;\simeq\; H(Q,\mu)\;\; J(m_J,\mu) \;\otimes\; S(m_J^2/Q,\mu)}$$

$$\ln^2 \frac{Q^2}{m_J^2} = \frac{1}{2}\ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{m_J^2}{\mu^2} + \frac{1}{2}\ln^2 \frac{m_J^4/Q^2}{\mu^2}$$

SCET-1

Three-scale problem: $\mu_S \ll \mu_J \ll \mu_H$

$\mu_H \sim Q$

$\mu_J \sim m_J$

$\mu_S \sim \frac{m_J^2}{Q}$

$$\boxed{d\hat{\sigma} \;\simeq\; H(Q,\mu)\;\; J(m_J,\mu) \;\otimes\; S(m_J^2/Q,\mu)}$$

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Sudakov resummation with standard RG techniques

$$\frac{dH(Q,\mu)}{d\ln\mu} = \left[2\,\Gamma_{\rm cusp}(\alpha_s)\;\ln\frac{Q^2}{\mu^2} + 4\gamma_H(\alpha_s)\right]\; H(Q,\mu)$$

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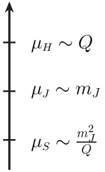

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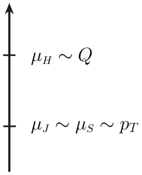
LL solution

$$H(Q,\mu) = H(Q,\mu_H)\,\exp\left\{\frac{4\pi\Gamma_0}{\beta_0^2}\,\frac{1}{\alpha_s(\mu_H)}\left(1-\frac{1}{r}-\ln r\right)\right\} \qquad r = \frac{\alpha_s(\mu)}{\alpha_s(\mu_H)}$$

SCET-2

Two-scale problem: $\mu_S \sim \mu_J \ll \mu_H$



$$\boxed{d\hat{\sigma} \simeq H(Q,\mu)\; J(p_T,\mu) \otimes S(p_T,\mu)}$$

$$\ln^2 \frac{Q^2}{p_T^2} = \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{p_T^2}{\mu^2} + \quad ?$$

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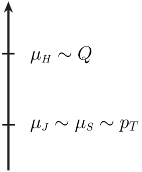

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Jet and soft functions are ill-defined in dimensional regularisation

$$\int d^4k \;\delta(k^2)\,\theta(k^0) \quad \Rightarrow \quad \int d^dk \left(\frac{\nu}{k_+}\right)^{\alpha} \delta(k^2)\,\theta(k^0)$$

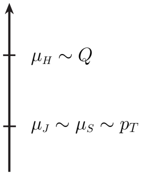
[Becher, GB 11]

$$\left.\begin{aligned} \Rightarrow\; S &\sim +\frac{1}{\alpha} + \ln\frac{\nu}{p_T} \\ J &\sim -\frac{1}{\alpha} - \ln\frac{\nu}{Q} \end{aligned}\right\} \ln\frac{Q}{p_T}$$

induces rapidity logarithms that cannot be resummed with standard RG techniques

SCET-2

Two-scale problem: $\mu_S \sim \mu_J \ll \mu_H$



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Rapidity logarithms exponentiate (in position space)

[Becher, Neubert 10; Chiu, Jain, Neill, Rothstein 11]

$$\mathcal{J}(x_T,\mu,Q,\nu)\,\mathcal{S}(x_T,\mu,x_T,\nu) = \left(Q^2 x_T^2\right)^{-F(x_T,\mu)}\; W(x_T,\mu)$$

- collinear anomaly exponent $F(x_T,\mu)$
- remainder function $W(x_T,\mu)$

Counting logs

Perturbative expansion

$$1 + \frac{\alpha_s}{4\pi} \left\{ \# \, L^2 + \# \, L + \# \right\} + \left(\frac{\alpha_s}{4\pi} \right)^2 \left\{ \# \, L^4 + \# \, L^3 + \# \, L^2 + \# \, L + \# \right\} + \dots$$

Counting logs

Perturbative expansion

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$$= \exp \left\{ \frac{1}{\alpha_s} \, g_1(\alpha_s L) + \, g_2(\alpha_s L) + \alpha_s \, g_3(\alpha_s L) + \dots \right\}$$

LL NLL NNLL

Counting logs

Perturbative expansion

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LL NLL NNLL

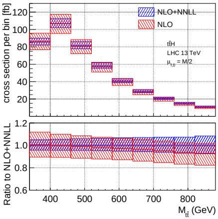
Accuracy	Γ_{cusp}	$\gamma_H, \begin{cases} \gamma_J, \gamma_S \\ F \end{cases}$	$c_H, \begin{cases} c_J, c_S \\ W \end{cases}$
LL	1-loop	–	–
NLL	2-loop	1-loop	tree
NNLL	3-loop	2-loop	1-loop
N^3LL	4-loop	3-loop	2-loop

SCET-1
SCET-2

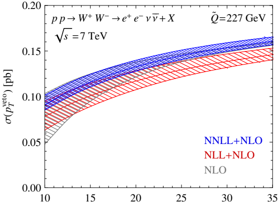
Applications

1) High-precision resummations

- e^+e^- event shapes
- threshold resummation
- p_T resummation
- jet veto resummation



[Broggio et al 16]



[Becher et al 16]

Applications

1) High-precision resummations

2) Resummation for jet observables

- complicated soft radiation pattern generates non-global logarithms [Dasgupta, Salam 01]
- resummation of non-global logarithms has similarities to parton shower evolution
- effect can be reduced by applying grooming techniques

Applications

1) High-precision resummations

2) Resummation for jet observables

3) Resummation at subleading power

- SCET provides the tools to systematically study power corrections
- first LL results for thrust and threshold resummation [Moult et al 18; Beneke et al 18]
- key problem: endpoint-divergent convolutions

Applications

1) High-precision resummations

2) Resummation for jet observables

3) Resummation at subleading power

4) Counterterms for fixed-order calculations

- N-jettiness slicing [Boughezal et al 15; Gaunt et al 15]

$$d\sigma = \int_0^\delta d\mathcal{T}_N \, \frac{d\sigma_{\text{SCET}}}{d\mathcal{T}_N} + \int_\delta^\infty d\mathcal{T}_N \, \frac{d\sigma}{d\mathcal{T}_N}$$

- soft and collinear counterterms for any other subtraction method

Applications

1) High-precision resummations

2) Resummation for jet observables

3) Resummation at subleading power

4) Counterterms for fixed-order calculations

5) Non-QCD applications

- resummation of electroweak Sudakov logarithms
- dark matter annihilation

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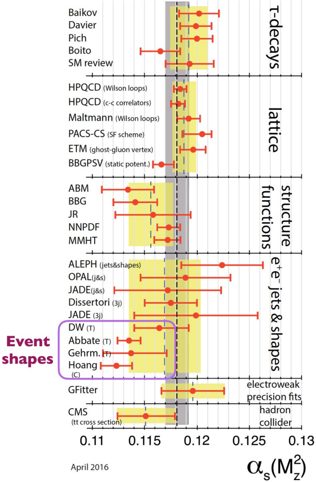
e^+e^- event shapes

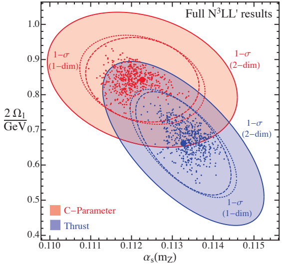
Extraction of the strong coupling $\alpha_s(M_Z)$



precision thrust and C-parameter analyses

give significantly lower values for $\alpha_s(M_Z)$

[Hoang et al 15]



are the non-perturbative corrections under control?

Angularities

e^+e^- event shape that depends on a continous parameter a [Berger, Kucs, Sterman 03]

$$e_a(\{k_i\}) = \sum_i |k_\perp^i|\, e^{-|\eta_i|\,(1-a)}$$

- transverse momentum and rapidity refer to thrust axis
- interpolates between thrust ($a = 0$) and total broadening ($a = 1$)

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Factorisation theorem for $e_a \to 0$

$$\frac{1}{\sigma_0}\frac{d\sigma}{de_a} \simeq H(Q,\mu) \int de_n \, de_{\bar{n}} \, de_s \;\; J_n(e_n,\mu) \; J_{\bar{n}}(e_{\bar{n}},\mu) \; S(e_s,\mu) \; \delta(e_a - e_n - e_{\bar{n}} - e_s)$$

- relevant scales: $\mu_H^2 \sim Q^2 \quad \gg \quad \mu_J^2 \sim Q^2 e_a^{\frac{2}{2-a}} \quad \gg \quad \mu_S^2 \sim Q^2 e_a^2$
- thrust: $\mu_H^2 \sim Q^2 \quad \gg \quad \mu_J^2 \sim Q^2 e_0 \quad \gg \quad \mu_S^2 \sim Q^2 e_0^2$ (SCET-1)
- broadening: $\mu_H^2 \sim Q^2 \quad \gg \quad \mu_J^2 \sim Q^2 e_1^2 \quad \sim \quad \mu_S^2 \sim Q^2 e_1^2$ (SCET-2)

State of the art

Accuracy	$\Gamma_{\rm cusp}$	$\gamma_H, \gamma_J, \gamma_S$	c_H, c_J, c_S
NLL	2-loop	1-loop	tree
NLL'	2-loop	1-loop	1-loop
NNLL	3-loop	2-loop	1-loop
NNLL'	3-loop	2-loop	2-loop

⇐ [Hornig, Lee, Ovanesyan 09] (NLL')

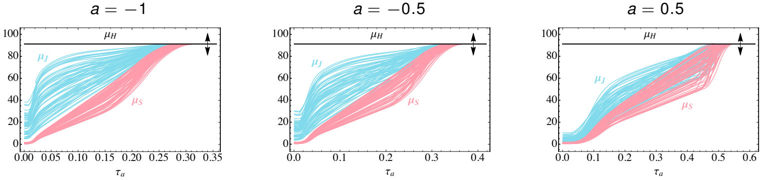
- γ_H and c_H are known to 2-loop
- 2-loop γ_S and c_S from `SoftSERVE` [GB, Rahn, Talbert 18]
- $\gamma_J = \frac{1}{2}\,\gamma_H + \frac{1}{2(1-a)}\,\gamma_S$ fixed by RG invariance
- extract 2-loop c_J from `EVENT2`-fit [GB, Hornig, Lee, Talbert 18]

⇒ extend resummation to NNLL' accuracy

Theory input

Further refinements

- matching to fixed-order α_s^2 calculation $\Rightarrow$ NNLL' + NLO accuracy
- angularity-dependent profile scales



- non-perturbative corrections

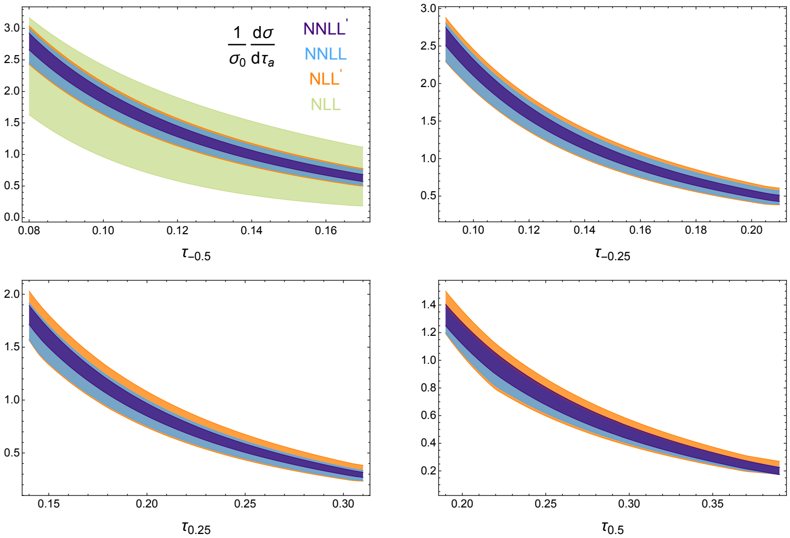
[Lee, Sterman 06]

$$\frac{d\sigma}{de_a}(e_a) \xrightarrow{\text{transition region}} \frac{d\sigma}{de_a}\left(e_a - \frac{2}{1-a}\frac{\Omega_1}{Q}\right)$$

same NP parameter Ω_1 that enters thrust and C-parameter analyses

a-dependent shift $\Rightarrow$ break correlation in two-dimensional fits?

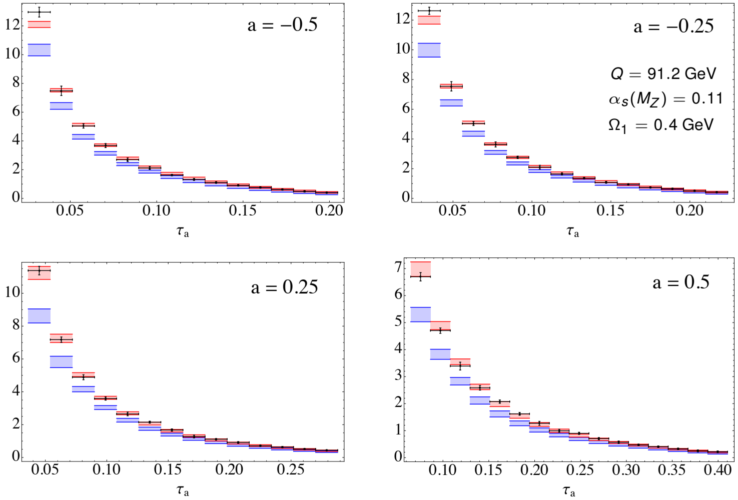
Convergence



Comparison to L3 data

[GB, Hornig, Lee, Talbert 18]

Binned distributions without / with non-perturbative effects



Conclusions

SCET

- high-precision resummations for global observables
- automation will become more and more important
- conceptual questions: non-global logs, factorisation violation, power corrections
- impact on NNLO subtractions + parton shower development?

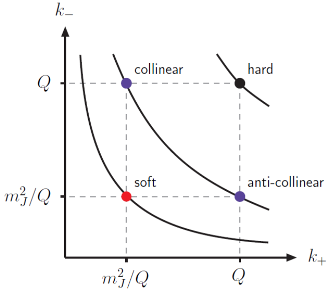
Angularities

- precision predictions at NNLL′ + NLO + NP accuracy
- extraction of $\alpha_s(M_Z)$ from a fit to LEP data is on-going

Backup slides

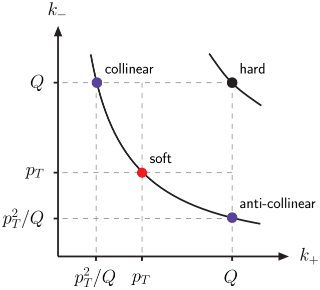
Momentum modes

SCET-1



$\mu_S \ll \mu_J$

SCET-2



$\mu_S \sim \mu_J$

In SCET-2 one cannot distinguish soft from collinear modes when radiated into jet direction

$\Rightarrow$ need additional regulator that distinguishes modes by their rapidities