Karlsruhe Institute of Technology

Physics with the NNLO QCD

Colour-singlet decay to massive partons
in the nested soft-collinear subtraction scheme

based on [1911.11524: A.Behring, WB]

Wojciech Bizoń

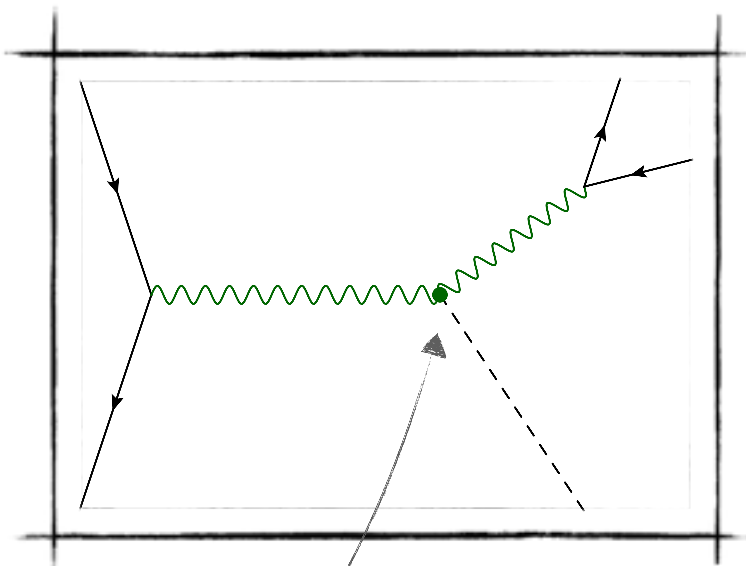
Institute for Theoretical Particle Physics - Karlsruhe Institute of Technology

Workshop on the Soft-Collinear QCD Dynamics,
Siegen, 23/01/2020

Introduction

- Precise QCD predictions
  $\Longrightarrow$ a lot of progress observed in recent years
  $\Longrightarrow$ a number of NNLO QCD subtraction schemes has appeared
  $\Longrightarrow$ this allows for fully-differential description of many processes

- How can we utilise this knowledge for physics at the LHC?
  $\Longrightarrow$ consider associated Higgs production as a case study



- Associated Higgs production (ZH/WH)
  - third channel of Higgs production
  - ideal trigger: leptons from the V decay
  - important role in the Higgs physics explorations at the LHC
  - direct access to the HVV couplings (V=W/Z)
    $\Longrightarrow$ they are completely fixed by the gauge symmetry of the SM
  - access to the b-quark Yukawa coupling (considering Hbb decay channel)

VH production case

- a fully-differential NNLO QCD calculation not available a decade ago
  - On one hand, complexity is similar to the DY process (colour-singlet production).
  - On the other hand, the phase space is much larger (more final-state particles)

- since then a number of independent results has appeared

  [1107.1164: Ferrera,Grazzini,Tramontano] ⟵ (slicing)
  [1601.00658: Campbell,Ellis,Williams] ⟵ (slicing)
  [1712.06954: Caola,Luisoni,Melnikov,Rontsch] ⟵ (nested soft-coll subtractions)
  [1907.05836: Gauld,Gehrmann-De Ridder,Glover,Huss,Majer] ⟵ (antenna subtractions)

- $H \to bb$ decay is <u>the decay channel to look for</u>
  $\Longrightarrow$ the largest branching fraction $\Longrightarrow$ save cross-section

  - Higgs has a very small width $\Longrightarrow$ narrow-width approx. justified
  - Higgs is a scalar $\Longrightarrow$ no spin-correlations

$\Longrightarrow$ as such NNLO QCD description can be independently implemented

$H \to bb$ decay

- **Q:** Why do we need radiative corrections?
  A: b-quarks are QCD partons which get confined inside hadrons
  $\Longrightarrow$ we will be looking at jets (the more we know about radiation the better)
  $\Longrightarrow$ higher-order corrections desirable!

- $m_b \ll M_H$: massless b-quarks are a good approximation
  - many NNLO QCD calculations available

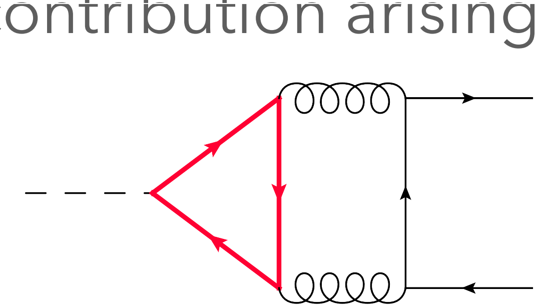
    [1110.2368: Anastasiou, Herzog, Lazapoulos]
    [1712.06954: Caola, Luisoni, Melnikov, Rontsch]
    [1501.07226: Del Duca, Duhr, Somogyi, Tramontano, Trocsanyi]
    [1907.05836: Gauld, Gehrmann-De Ridder, Glover, Huss, Majer]
  - first N3LO QCD results appearing

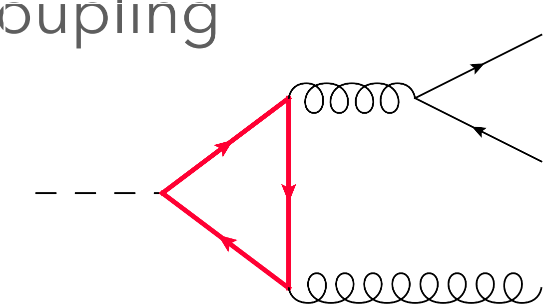
    [1904.08960: Mondini,Schiavi,Williams]

- Is there any reason to look into b-quark mass corrections?
  - **Yes!** Corners of phase-space where b-mass impact might play a role (no stone unturned)
  - **Yes!** Detection of b-jets based on a displaced vertex (can take a massive b-quark in its rest-frame and simulate the decay)
  - **Yes!** Peculiar contribution from the top-quark mediated Higgs decay

H→bb decay via top-loop

- An extra contribution arising from the Htt coupling



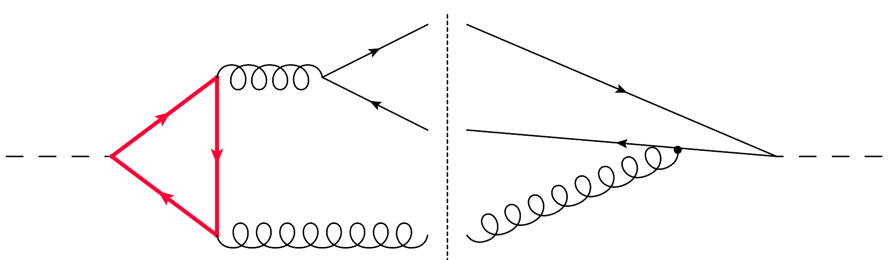


- Appears at $\mathcal{O}(\alpha_s^2)$
- Contribution to the partial decay width calculated in the '90s

  [hep-ph/9506465: Larin, van Ritbergen, Vermaseren] [hep-ph/9505358: Chetyrkin, Kwiatkowski]

- It is UV & IR finite $\Longrightarrow$ it can be considered separately

- Fully-differential calculation [1712.06954: Caola,Luisoni,Melnikov,Rontsch]



  - vanishes in the limit $m_b \rightarrow 0$ $\Longrightarrow$ a need for mass suppressed terms
  - then peculiar divergences appear which are hard to regulate by jet-algorithms
  - massive calculation preferable $\Longrightarrow$ this is what we do…

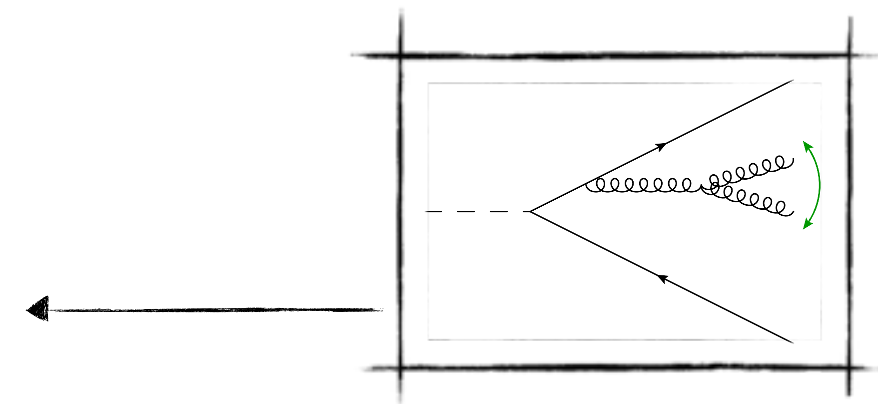
[1911.11524, A.Behring, WB]

H→bb decay with massive b-quarks

- Project in collaboration with Arnd Behring
- A pedestrian guide to an NNLO calculation (what can we do nowadays)

- An opportunity to extend the nested soft-collinear subtraction scheme to the case of colour-singlet decay into massive fermions

- Main differences wrt. the massless calculation
  - Advantage:
    simple and transparent singularity structure
    - NLO: only single-soft limit
    - NNLO: soft limits, but only one collinear limit

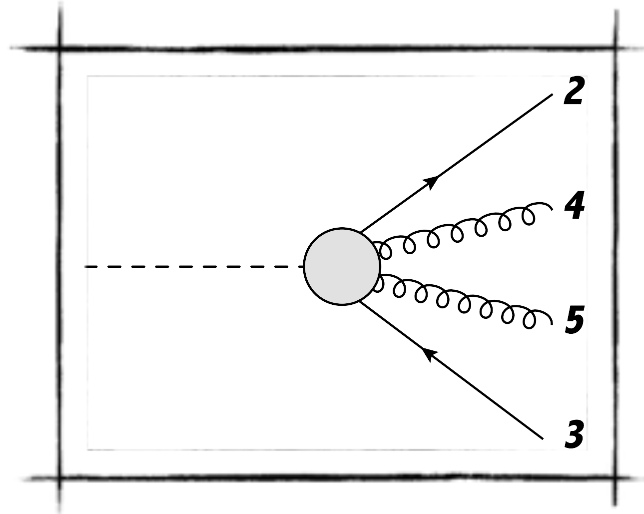


  - Possible problem:
    b-mass regulates the collinear divergences of the b-quarks, but is small
    ⟹ large logarithms appear $L = log(m_b/M_H)$
    ⟹ some amplitudes (RV: one-loop needs quadruple precision for some points)

  - Disadvantage:
    slightly more complicated factorisation formulae (and integrated counter-terms)

[1911.11524, A.Behring, WB]

H→bb: Phase-space parametrisation



- Double-real contribution
  ⟹ only one collinear divergence appears
  ⟹ no reason to partition the phase space

- The only partitioning is the energy-ordering, i.e. $1 = \Theta(E_4-E_5) + \Theta(E_5-E_4)$

- Global phase-space parametrisation possible

$$\int \mathrm{d}\Phi_{bbgg}^{E_4<E_5}(q_H) = \int \frac{\mathrm{d}\Omega_4^{(3-2\epsilon)}}{2(2\pi)^{3-2\epsilon}} \int \frac{\mathrm{d}\Omega_5^{(2-2\epsilon)}}{2(2\pi)^{3-2\epsilon}} \int_0^1 \frac{\mathrm{d}\eta}{\eta^\epsilon(1-\eta)^\epsilon}$$
$$\times \int_0^1 \mathrm{d}\xi_1 \int_0^1 \mathrm{d}\xi_2 \, (E_{45,\max})^{4-4\epsilon} \xi_1^{3-4\epsilon} \xi_2^{1-2\epsilon} (2-\xi_2)^{1-2\epsilon} \int \mathrm{d}\Phi_{bb}(q_H - q_{45})$$

⇝ collinear divergence captured by: $\eta = \frac{1}{2}(1-\cos\theta_{45})$
⇝ soft divergences captured by: $\xi_1 \, , \xi_2$
with energies parametrised as

$$E_4 = \tfrac{1}{2} E_{45,\max}\, \xi_1 (2-\xi_2)$$
$$E_5 = \tfrac{1}{2} E_{45,\max}\, \xi_1 \xi_2$$

$$E_{45,\max} = \frac{M_H \beta^2}{1+\sqrt{1-\beta^2\xi_2(2-\xi_2)\eta}}$$

[1911.11524, A.Behring, WB]

H→bb: Amplitudes

- Tree-level amplitudes
  ⇝ using spinor-helicity formalism extended to treat massive fermions (simple)
  ⇝ four-dimensional amplitudes suffice
  ⇝ two- / three- / four-parton amplitudes

- One-loop amplitudes
  ⇝ using spinor-helicity formalism and Passarino-Veltman reduction (simple)
  ⇝ only two- / three-parton amplitudes

- Two-loop amplitude
  ⇝ available as a heavy-quark form factor (independent results in the literature)

$$|\mathcal{M}_{bb}\rangle = F_{\mathrm{s}}(M_H^2, m_b^2, \mu_R^2)|\mathcal{M}_{bb}^{(0)}\rangle$$

$$F_{\mathrm{s}}(M_H^2, m_b^2, \mu_R^2) = 1 + \left(\frac{\alpha_s}{4\pi}\right) F_{\mathrm{s}}^{(1)}(M_H^2, m_b^2, \mu_R^2) + \left(\frac{\alpha_s}{4\pi}\right)^2 F_{\mathrm{s}}^{(2)}(M_H^2, m_b^2, \mu_R^2) + \mathcal{O}(\alpha_s^3)$$

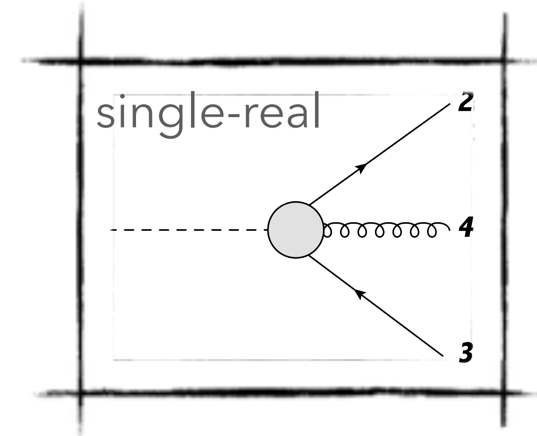
[hep-ph/0508254: Bernreuther et al.]
[0905.1137: Gluza et al.]
[1712.09889: Ablinger et al.]

H→bb: Soft/collinear limits

- QCD amplitudes with massless partons feature soft/collinear divergences [independent of a hard process]
- Divergences can be regulated using end-point subtraction
- At NLO
  - ⇝ soft limit

zero in massless case

$$S_4|\mathcal{M}^{(0)}_{bbg}|^2 \approx -g_s^2 C_F \left(\boxed{S^{(0)}_{22,4}} - 2S^{(0)}_{23,4} + \boxed{S^{(0)}_{33,4}}\right) |\mathcal{M}^{(0)}_{bb}|^2$$



eikonal factor $S_{ij,k} = \dfrac{(q_i \cdot q_j)}{(q_i \cdot q_k)(q_j \cdot q_k)}$

- At NNLO
  - ⇝ single-soft limit

$$S_5|\mathcal{M}^{(0)}_{bbgg}|^2 \approx -g_s^2 \left[C_F \left(S^{(0)}_{22,5} - 2S^{(0)}_{23,5} + S^{(0)}_{33,5}\right) - C_A \left(S^{(0)}_{24,5} + 2S^{(0)}_{34,5} - S^{(0)}_{23,5}\right)\right] |\mathcal{M}^{(0)}_{bbg}|^2$$

  - ⇝ collinear-limit

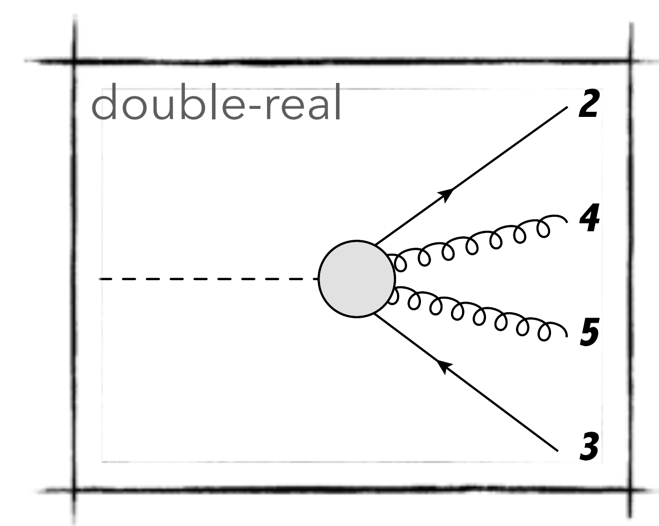
$$C_{45}|\mathcal{M}^{(0)}_{bbij}|^2 \approx \frac{g_s^2}{(q_4 \cdot q_5)} \langle \mathcal{M}^{(0)}_{bbg,\mu}| P^{\mu\nu}_{ij}(z, k_T, \epsilon) |\mathcal{M}^{(0)}_{bbg,\nu}\rangle \longleftarrow P^{\mu\nu}_{ij}(z, k_T, \epsilon)$$

spin-correlated splitting function

  - ⇝ no triple-collinear limit
  - ⇝ double-soft limit

$$S_{45}|\mathcal{M}^{(0)}_{bbii}|^2 \approx g_s^4 \, \mathrm{DSoft}^{(0)}_{ii}(q_2, q_3; q_4, q_5) \, |\mathcal{M}^{(0)}_{bb}|^2$$

[Catani,Grazzini '99; Czakon '10]



  - ⇝ single-soft limit at one-loop [later]

[1911.11524, A.Behring, WB]

H→bb: Integrated subtraction terms

- Subtraction terms are added back after integrating them over the unresolved phase space (eg. soft-gluon, pair of soft gluons/quarks)
- Soft singularities
  - we need integrated eikonal factors

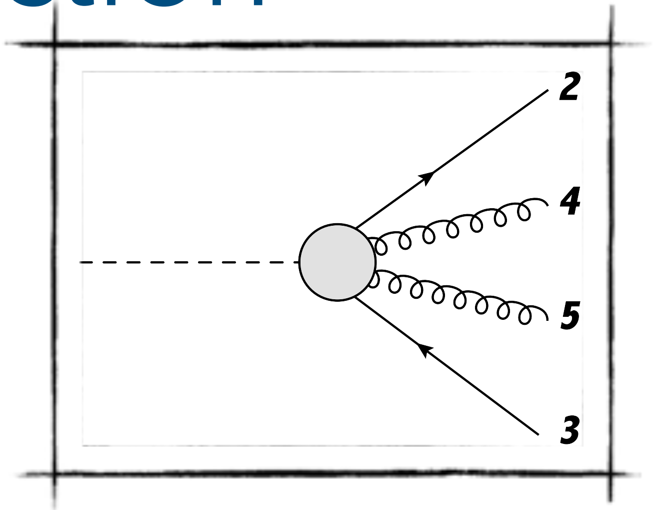
angular integrals solved for the NLO FKS subtraction scheme, see eg. [1002.2581: POWHEG]

$$S^{(0)}_{ij,\mathrm{int}} = \int [dg_k] \frac{(q_i \cdot q_j)}{(q_i \cdot q_k)(q_j \cdot q_k)} \sim \frac{1}{\epsilon} \int \frac{\mathrm{d}\Omega_k^{(3-2\epsilon}}{2(2\pi)^{(3-2\epsilon)}} \frac{(q_i \cdot q_j)}{(q_i \cdot \hat{q}_k)(q_j \cdot \hat{q}_k)}$$

- Collinear singularities
  - very simple since global phase-space parametrisation
  - take collinear factorisation formula and integrate using phase-space parametrisation
  - no complications related to the b-mass since only massless partons take part in collinear splittings
- Additionally:
  - double-soft function
  - one-loop single soft function

we need to tackle these two parts on our own

H→bb: Integrated double-soft function



- Double-soft limit ($E_4 \to 0$, $E_5 \to 0$)

$$S_{45}|\mathcal{M}^{(0)}_{bbgg}|^2 \approx g_s^4\,\mathrm{DSoft}^{(0)}_{gg}(q_2,q_3;q_4,q_5)\,|\mathcal{M}^{(0)}_{bb}|^2$$

[Catani,Grazzini '99; Czakon '10]

- Born kinematics in the double-soft limit
  $\Longrightarrow$ a back-to-back configuration $\Longrightarrow$ result becomes just a set of 4 numbers

- Slight deviation from the original nested soft-collinear subtraction scheme

$$[dg_i] = \frac{d^3k_i}{(2\pi)^3 2E_i}\cancel{\Theta(E_{\max}-E_i)} \longrightarrow$$

Keep ξ_1, ξ_2, η of the $\mathrm{d}\Phi_{bbgg}$ in the $[0,1]$ interval

- Integration performed numerically
  - the double-soft function still divergent in the soft/collinear limits
    $\Longrightarrow$ but easy to regulate using end-point subtraction

$$\mathrm{DSoft}^{(0)}_{gg,\mathrm{int}}(q_2,q_3) = \int [dg_4][dg_5]\,\mathrm{DSoft}^{(0)}_{gg}(q_2,q_3;q_4,q_5)$$

$$= \frac{1}{(4\pi)^2}\left(\frac{\mu_R}{E_{\max}}\right)^{4\epsilon}\left[\frac{C^{(-3)}_{gg}}{\epsilon^3}+\frac{C^{(-2)}_{gg}}{\epsilon^2}+\frac{C^{(-1)}_{gg}}{\epsilon}+C^{(0)}_{gg}\right]$$

$$E_{\max} = (M_H^2 - 4m_b^2)/2$$

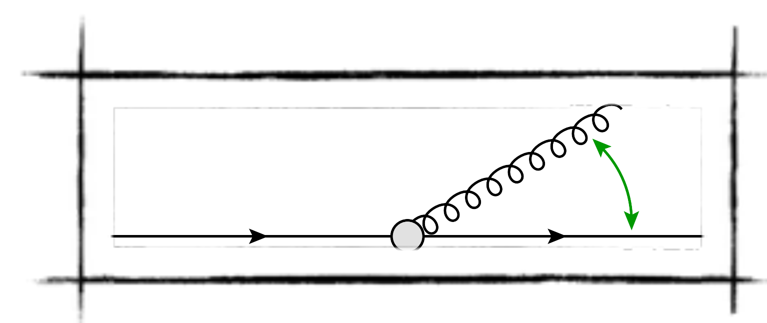
H→bb: Integrated one-loop soft function

- Soft divergences of one-loop amplitudes with massive partons studied in the past
  $\Longrightarrow$ factorisation formula ready-to-use

$$S_4 2\mathrm{Re}\langle \mathcal{M}^{(0)}_{bbg} | \mathcal{M}^{(1)}_{bbg} \rangle = -\, g_s^2 C_F \left(S^{(0)}_{22,4} = 2S^{(0)}_{23,4} + S^{(0)}_{33,4} \right) \longleftarrow S_{ij,k} = \frac{(q_i \cdot q_j)}{(q_i \cdot q_k)(q_j \cdot q_k)}$$

$$\times \left[2\mathrm{Re}\langle \mathcal{M}^{(0)}_{bb} | \mathcal{M}^{(1)}_{bb} \rangle + \left(\underbrace{\mathcal{R}^{(1)}_{23,4}}_{\text{one-loop soft function}} + \underbrace{Z^{(1)}_{\alpha_s} + Z^{(1)}_{A}}_{\text{renormalisation constants}} \right) |\mathcal{M}^{(0)}_{bb}|^2 \right]$$

- Integrated one-loop soft function can be partially assembled using the NLO integrated subtraction terms
- Z_A piece appears explicitly
- Born kinematics in the soft-limit reduces to a back-to-back configuration
- Integration over the soft-gluon phase space reduces to
  - energy integral trivial (an overall factor)
  - angular integral (no divergences!)
    ⇝ performed numerically (quick and easy, only 1-dim)
    ⇝ performed analytically (5-letters, HPLs up to weight 4)



$$\frac{\mathrm{d}(\cos\theta)}{\cos\theta}\,, \qquad \frac{\mathrm{d}(\cos\theta)}{1+\cos\theta}\,, \qquad \frac{\mathrm{d}(\cos\theta)}{1-\cos\theta}\,, \qquad \frac{\mathrm{d}(\cos\theta)}{1+\beta\cos\theta}\,, \qquad \frac{\mathrm{d}(\cos\theta)}{1-\beta\cos\theta}$$

H→bb: Pole cancellation

- All integrated subtraction terms are known
  ⟹ trade phase-space divergences into 1/ε singularities
- Reduced matrix elements are factored out in the factorisation formulas
  ⟹ demonstrate pole cancellation at each phase-space point
- Single-unresolved term [H→bbg] (RR+RV)

$$F_{LM}(bbg) = \mathrm{d}\Phi_{bbg}\,|\mathcal{M}^{(0)}_{bbg}|^2\,\mathcal{F}_{\mathrm{obs}}(bbg)$$

$$\mathrm{d}\Gamma^{\mathrm{SU}}_{\mathrm{RR}}(bbgg+bbqq) + \mathrm{d}\Gamma^{\mathrm{SU}}_{\mathrm{RV}}(bbg) = \left[\frac{\#}{\epsilon^2} + \frac{\#}{\epsilon} + \cdots\right] \underbrace{(I - S_4)}_{\text{soft-regulated term}} F_{LM}(bbg)$$

☞ cancellation shown analytically

- Fully-unresolved term [H→bb](RR+RV+VV)

(a) part proportional to one-loop H→bb matrix element

$$F^{\mathrm{fin}}_{LV}(bb) = \mathrm{d}\Phi_{bb}\;2\mathrm{Re}\langle\mathcal{M}^{(0)}_{bb}|\mathcal{F}^{(1)}_{bb}\rangle\,\mathcal{F}_{\mathrm{obs}}(bb)$$

$$\mathrm{d}\Gamma^{\mathrm{FR}}_{\mathrm{RV}}(bbg) + \mathrm{d}\Gamma^{\mathrm{FR}}_{\mathrm{VV}}(bb) = \left[\frac{\#}{\epsilon} + \cdots\right] F^{\mathrm{fin}}_{LV}(bb)$$

☞ cancellation shown analytically

(b) part proportional to tree-level H→bb matrix element

$$\mathrm{d}\Gamma^{\mathrm{DU}}_{\mathrm{RR}} + \mathrm{d}\Gamma^{\mathrm{DU}}_{\mathrm{RV}} + \mathrm{d}\Gamma^{\mathrm{DU}}_{\mathrm{VV}} = \left[\frac{\#}{\epsilon^3} + \frac{\#}{\epsilon^2} + \frac{\#}{\epsilon} + \cdots\right] F_{LM}(bb)$$

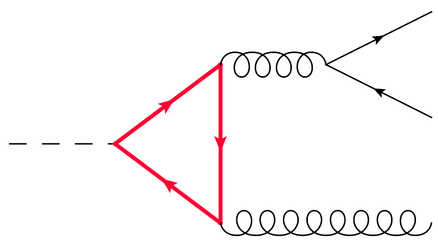
☞ cancellation shown numerically

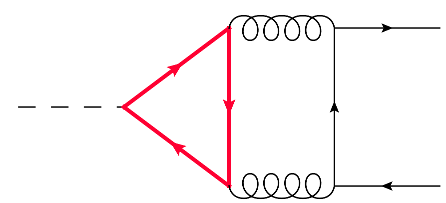
⇝ since double-soft function integrated numerically
⇝ at least 7digits of cancellation

[1911.11524, A.Behring, WB]

H→bb: top-induced contribution

- An additional contribution related to diagrams with a top-quark loop
- Two parts: real-virtual and double-virtual





- real-virtual: one-loop H→bbg diagram
  - standard techniques (easy to obtain)
  - UV & IR finite $\Longrightarrow$ just integrate over the phase space

- double-virtual: two-loop H→bb diagram
  - UV & IR finite $\Longrightarrow$ just integrate over the phase space
  - exact master integrals computed recently ← [1812.07811: Primo, Sasso, Somogyi, Tramontano]
  - in case this two-loop amplitude not available $\Longrightarrow$ a simpler way (our choice)
    - ⇝ start with result for the total contribution ← [hep-ph/9506465: Larin, van Ritbergen, Vermaseren]
    - ⇝ subtract the integrated real-virtual piece

[1911.11524, A.Behring, WB]

H→bb: Results

[hep-ph/9704436: Harlander, Steinhauser]

- Validation: comparison with approximate analytical result

$$\Gamma^{\text{NNLO}}_{H\to b\bar{b}} = \Gamma^{\text{LO}}_{H\to b\bar{b}} \cdot \left[1 + \left(\frac{\alpha_s}{\pi}\right)\gamma_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \gamma_2\right]$$

$$\gamma_1 = C_F \cdot \gamma_1^{C_F}$$
$$\gamma_2 = C_F^2 \cdot \gamma_2^{C_F^2} + C_F C_A \cdot \gamma_2^{C_F C_A} + C_F T_F n_l \cdot \gamma_2^{C_F T_F n_l} + C_F T_F \cdot \gamma_2^{C_F T_F}$$

compare our result to analytical prediction (M_H=125.09 GeV and m_b=4.78 GeV)

	$\gamma_1^{C_F}$	$\gamma_2^{C_F^2}$	$\gamma_2^{C_F C_A}$	$\gamma_2^{C_F T_F n_l}$	$\gamma_2^{C_F T_F}$
HS '97	−7.446648	+19.4192	−53.5558	+18.6286	+14.7946
Our result	−7.446648(7)	+19.4199(10)	−53.5557(20)	+18.6283(2)	+14.7945(1)

- Total H→bb decay width at LO / NLO / NNLO

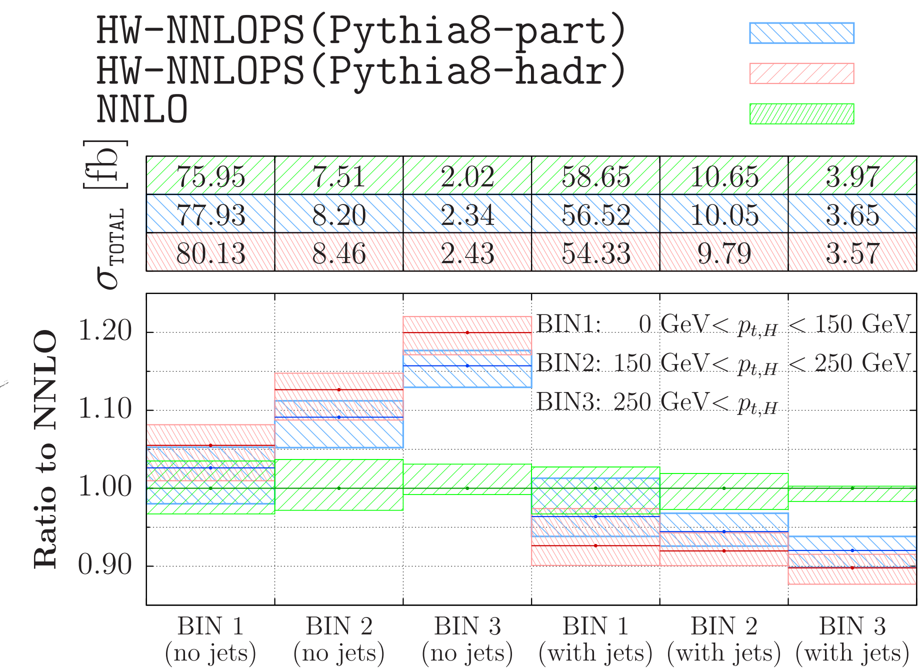
$\Gamma^{b\bar{b}}_{\text{LO}}$ [MeV]	+2.17005	+1.92702	+1.73274
$\Gamma^{b\bar{b}}_{\text{NLO}}$ [MeV]	+2.43161	+2.32781	+2.21731
$\Gamma^{b\bar{b}}_{\text{NNLO}}$ [MeV] (w/o $y_b y_t$)	+2.42041(1)	+2.40333(1)	+2.36344(1)
$\Gamma^{b\bar{b}}_{\text{NNLO}}$ [MeV] (with $y_b y_t$)	+2.44441(1)	+2.42059(1)	+2.37628(1)

⇝ About 20% of the NNLO correction comes from the top-induced contribution
⇝ About 85% of this top-induced contribution comes from real-virtual part (distributions?)

V(→ll)H(→bb) process

- Phenomenological studies of the VH process with massive H→bb decay under way
  - check impact of the mass corrections & top-induced contribution on important distributions (eg. M_{bb}, ΔR_{bb}, $p_{T,bb}$, etc.)

- Is this everything we can do for the VH process?
  - a study of HW production @NNLO+PS
  - cross-section binned according to Higgs p_T and presence of jets…



[1603.01620: Astill, WB, Re, Zanderighi]

⇝ large corrections in bins with boosted Higgs
⇝ high transverse-momentum correlated with hard jets
⇝ note that VH@NNLO is just NLO for jet observables

⇝ VH+1jet @ NNLO desirable?
⇝ This is a different story: colour-singlet + 1jet production

Conclusions

- The situation of the precision QCD has changed dramatically in recent years
  ⇝ a number of independent fully-differential schemes available

  ⇝ many ingredients towards an local & analytical subtraction scheme available

  ⇝ this allows for many interesting NNLO QCD calculations relevant for the LHC physics programme

  ⇝ nevertheless, work still to be done… (eg. colour-singlet+1jet, …)

Backup: pole cancellation (1)

- single-unresolved

$$2M_H \langle \mathrm{d}\Gamma^{\mathrm{FR}}_{\mathrm{RV}}(b\bar{b}g)\rangle = \frac{1}{\epsilon}\left[\left(\frac{\alpha_s}{4\pi}\right)4C_F\left[1+\frac{1+\beta^2}{2\beta}\log\left(\frac{1-\beta}{1+\beta}\right)\right]\langle F^{\mathrm{fin}}_{LV}(b\bar{b})\rangle\right] + \mathcal{O}\left(\epsilon^0\right)\,, \tag{4.22}$$

and the explicit expansion of the double-virtual contribution, Eq. (4.20), yields

$$2M_H \langle \mathrm{d}\Gamma^{\mathrm{FR}}_{\mathrm{VV}}(b\bar{b})\rangle = -\frac{1}{\epsilon}\left[\left(\frac{\alpha_s}{4\pi}\right)4C_F\left[1+\frac{1+\beta^2}{2\beta}\log\left(\frac{1-\beta}{1+\beta}\right)\right]\langle F^{\mathrm{fin}}_{LV}(b\bar{b})\rangle\right]\,, \tag{4.23}$$

- fully-unresolved (proportional to one-loop H→bb matrix element)

$$\begin{aligned}&2M_H \langle \mathrm{d}\Gamma^{\mathrm{SU}}_{\mathrm{RR}}(b\bar{b}gg + b\bar{b}q\bar{q})\rangle \\ &= \Bigg\langle \left(\frac{\alpha_s}{4\pi}\right)\Bigg[\frac{2C_A}{\epsilon^2}+\frac{1}{\epsilon}\Bigg[4C_F+\beta_0(n_l)+\frac{C_A-2C_F}{v_{23}}\log\left(\frac{1+v_{23}}{1-v_{23}}\right) \\ &\quad + 2C_A\log\left(\frac{m_b\mu_R}{2(q_2\cdot q_4)}\right)+2C_A\log\left(\frac{m_b\mu_R}{2(q_3\cdot q_4)}\right)\Bigg]\Bigg](I-S_4)F_{LM}(b\bar{b}g)\Bigg\rangle + \mathcal{O}\left(\epsilon^0\right)\,.\end{aligned} \tag{4.24}$$

A similar expansion holds for the real-virtual single-unresolved contribution, Eq. (4.14),

$$\begin{aligned}&2M_H \langle \mathrm{d}\Gamma^{\mathrm{SU}}_{\mathrm{RV}}(b\bar{b}g)\rangle \\ &= \Bigg\langle \left(\frac{\alpha_s}{4\pi}\right)\Bigg[-\frac{2C_A}{\epsilon^2}-\frac{1}{\epsilon}\Bigg[4C_F+\beta_0(n_l)+\frac{C_A-2C_F}{v_{23}}\log\left(\frac{1+v_{23}}{1-v_{23}}\right) \\ &\quad + 2C_A\log\left(\frac{m_b\mu_R}{2(q_2\cdot q_4)}\right)+2C_A\log\left(\frac{m_b\mu_R}{2(q_3\cdot q_4)}\right)\Bigg]\Bigg](I-S_4)F_{LM}(b\bar{b}g)\Bigg\rangle\,.\end{aligned} \tag{4.25}$$

Backup: pole cancellation (2)

- fully-unresolved (proportional to tree-level H→bb matrix element)
  - cancellation of at least 7 digits

	$\mathcal{C}^{\mathrm{DU},(-3)}_{C_F C_A}$	$\mathcal{C}^{\mathrm{DU},(-2)}_{C_F C_A}$	$\mathcal{C}^{\mathrm{DU},(-2)}_{C_F^2}$	$\mathcal{C}^{\mathrm{DU},(-2)}_{C_F T_F n_l}$	$\mathcal{C}^{\mathrm{DU},(-1)}_{C_F C_A}$	$\mathcal{C}^{\mathrm{DU},(-1)}_{C_F^2}$	$\mathcal{C}^{\mathrm{DU},(-1)}_{C_F T_F n_l}$	$\mathcal{C}^{\mathrm{DU},(-1)}_{C_F T_F}$
RR	-22.11	-279.75	$+244.32$	$+14.74$	-1777.55	$+2672.58$	$+185.77$	0
RV	$+22.11$	$+320.28$	-488.64	-29.47	$+1732.44$	-2672.58	-161.20	$+257.20$
VV	0	-40.53	$+244.32$	$+14.74$	$+45.11$	0	-24.56	-257.20
Sum	10^{-13}	10^{-10}	10^{-8}	10^{-11}	10^{-6}	10^{-6}	10^{-5}	0
Rel. canc.	10^{-14}	10^{-13}	10^{-11}	10^{-13}	10^{-10}	10^{-9}	10^{-7}	0

Table 1: Numerical values of the pole coefficients of the double-unresovled term as defined in Eq. (4.27). The numerical values correspond to $m_b = 4.78$ GeV, $M_H = 125.09$ GeV and the renormalisation scale is $\mu_R = 3M_H$. Each column corresponds to a particular colour structure of a given ϵ pole. The three rows correspond to the double-real, real-virtual, and double-virtual contributions. In the last two rows, we report the absolute and relative level of cancellation after adding up RR + RV + VV contributions. The last row is normalised to the largest value of each column.

Backup: NNLO results

- Expansion coefficients and the H→bb decay width in the MS-bar scheme

μ_R	$\frac{1}{2}M_H$	M_H	$2M_H$
$\overline{\gamma}_1^{b\bar{b}}$ (our res.)	$+3.023597(10)$	$+5.796203(15)$	$+8.568783(11)$
$\overline{\gamma}_1^{b\bar{b}}$ (Ref. [20])	$+3.024$	$+5.798$	$+8.569$
$\overline{\gamma}_1^{b\bar{b}}$ (Ref. [71], $m_b = 0$)	$+2.8941$	$+5.6667$	$+8.4393$
$\overline{\gamma}_2^{b\bar{b}}$ (our res., w/o $y_b y_t$)	$-3.2466(31)$	$+30.4376(33)$	$+79.1755(38)$
$\overline{\gamma}_2^{b\bar{b}}$ (our res., with $y_b y_t$)	$+3.7123(31)$	$+37.3965(33)$	$+86.1345(38)$
$\overline{\gamma}_2^{b\bar{b}}$ (Ref. [20], with $y_b y_t$)	$+3.685$	$+37.371$	$+86.112$
$\overline{\gamma}_2^{b\bar{b}}$ (Ref. [71], $m_b = 0$)	-3.8368	$+29.1467$	$+77.1844$
$\overline{\Gamma}_{\mathrm{LO}}^{b\bar{b}}$ [MeV]	$+2.17005$	$+1.92702$	$+1.73274$
$\overline{\Gamma}_{\mathrm{NLO}}^{b\bar{b}}$ [MeV]	$+2.43161$	$+2.32781$	$+2.21731$
$\overline{\Gamma}_{\mathrm{NNLO}}^{b\bar{b}}$ [MeV] (w/o $y_b y_t$)	$+2.42041(1)$	$+2.40333(1)$	$+2.36344(1)$
$\overline{\Gamma}_{\mathrm{NNLO}}^{b\bar{b}}$ [MeV] (with $y_b y_t$)	$+2.44441(1)$	$+2.42059(1)$	$+2.37628(1)$

Table 3: The results for the LO, NLO and NNLO total decay width. The total width is calculated using our results for the expansion coefficients, $\overline{\gamma}_1^{b\bar{b}}$ and $\overline{\gamma}_2^{b\bar{b}}$. For comparison we include corresponding results from Ref. [20]. We also provide results in the limit of massless b-quarks from Ref. [71], which do not contain the $y_b y_t$ contribution. The uncertainties quoted for our results correspond to errors from numerical integration.