Multi-emission Kernels for Parton Branching Algorithms

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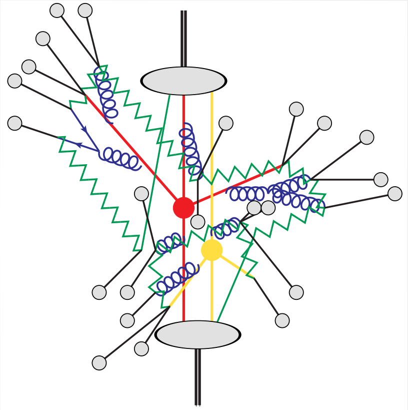
24 January 2020

[a]in collaboration with Simon Plätzer and Emma Simpson Dore



Motivation

General Idea: investigate novel routes in understanding soft and collinear dynamics in multi-parton final states.

- Going beyond iterated $1 \to 2$ splittings in parton showers
- Combine with global recoil scheme to capture

} higher logarithmic accuracy

- Include colour and spin correlations
- Refine ad hoc models of MC-programs, *e.g.* azimuthal correlations
- Define language for connecting FO to parton showers
- Systematic expansion to handle uncertainties
- Work in a diagrammatic approach
- Address non-global observables

Relation to fixed order

A variety of *e.g.* NNLO QCD subtraction methods are available (on the cross-section level).

Relation to fixed order

Well established subtraction schemes at NLO

- Frixione-Kunst-Signer (FKS) subtraction — Frixione, Kunszt, Signer
- Catani-Seymour (CS) Dipole subtraction — Catani, Seymour
- Nagy-Soper subtraction — Nagy, Soper

Many methods available at NNLO

- Antenna subtraction — Gehrmann De Ridder, Gehrmann, Glover, Heinrich, et al.
- CoLoRFul subtraction — Del Duca, Duhr, Kardos, Somogyi, Troscanyi, et al.
- Sector-improved residue subtraction — Czakon et al.
- Nested soft-collinear subtraction — Melnikov et al.
- Local analytic sector subtraction — Magnea, Maina, Torrielli, U. et al.
- qT-slicing — Catani, Grazzini, et al.
- N-jettiness slicing — Boughezal, Petriello, et al.
- Projection to Born — Cacciari, Salam, Zanderighi, et al.
- Sector decomposition — Anastasiou, Binoth, et al.
- $\mathcal{E}$-prescription — Frixione, Grazzini
- Unsubtraction — Rodrigo et al.
- Geometric — Herzog

S. Uccirati @ Vienna CES 2019

Relation to fixed order

A variety of *e.g.* NNLO QCD subtraction methods are available (on the cross-section level).

Instead of adding to the list, want to:

- Combine real and virtual contributions differentially
- Smooth phase-space coverage
- Let a MonteCarlo do the integrals
- Still keep a bridge to the fixed order and EFT side

Project overview

Goal: study leading singular behavior for multiple emissions

1. Purely virtual corrections (Ines Ruffa, SP)$_{\text{2-loop}}$
2. Virtual-real interference (Ines Ruffa, SP)$_{\text{1-loop, 1-emission}}$

   $\rightarrow$ As phase-space integrals using Feynman-Tree-Theorem
3. Purely real corrections (ML, SP, Emma Simpson Dore)
   $\uparrow$ **this talk**

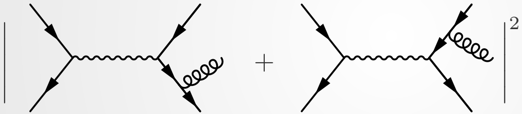
- Derive generalized splitting functions (input for parton showers)
- Only take collinear/soft limits at a late stage
- Keep interpolation over whole phase space
- Include overlapping singular regions
- Understand IR cancellations on a diagrammatic level

One emission example

One emission example
Setup

- Leading IR singular behavior for gluon emission:

$$\frac{\mathrm{d}\sigma_{e^+e^-\to q\bar{q}g}}{\mathrm{d}z\,\mathrm{dcos}\,\theta} \approx \sigma_{q\bar{q}} \times C_F \frac{\alpha_S}{\pi} \frac{1}{\sin^2\theta} \frac{1+(1-z)^2}{z}$$



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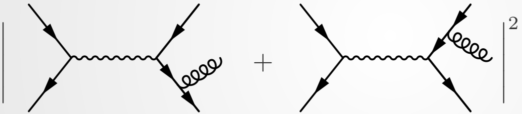

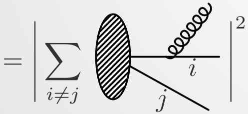

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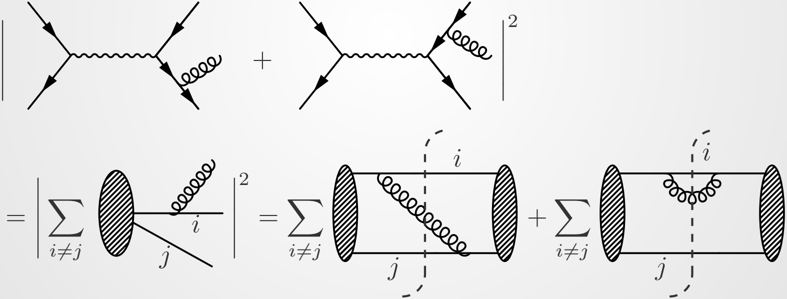
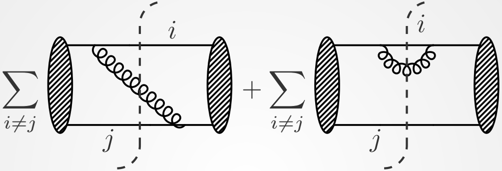


One emission example
Partitioning

- Extract IR singular behavior:



$$= \sum_{i \neq j} \tilde{\mathcal{M}}^* \Big[\frac{\mathcal{N}^{\rm int}(i,j)}{(2q_i \cdot k)(2q_j \cdot k)} + \frac{\mathcal{N}^{\rm self}(i,j)}{(2q_i \cdot k)^2} \Big] \tilde{\mathcal{M}}$$

- Introduce **partitioning** (or **weighting factors**) using $S_{ik} \equiv 2q_i \cdot k$:

$$1 = \frac{S_{ik}}{S_{ik} + S_{jk}} + \frac{S_{jk}}{S_{ik} + S_{jk}}$$

One emission example

Partitioning

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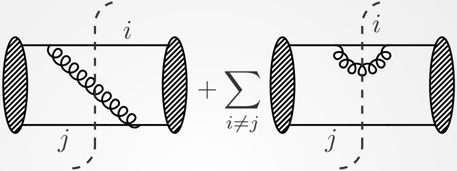

$$\sum_{i\neq j} \quad + \sum_{i\neq j}$$

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$$1 = \frac{S_{ik}}{S_{ik}+S_{jk}} + \frac{S_{jk}}{S_{ik}+S_{jk}}$$

$$\frac{\mathcal{N}^{\text{int}}(i,j)}{S_{ik}S_{jk}} \simeq \frac{1}{S_{ik}} \frac{\mathcal{N}^{\text{int}}(S_{ik}=0)}{S_{ik}+S_{jk}} + \frac{1}{S_{jk}} \frac{\mathcal{N}^{\text{int}}(S_{jk}=0)}{S_{ik}+S_{jk}}$$

One emission example
Splitting kernels

- Collect singular structures in **splitting kernels**

$$U_{(ik)j} \equiv \frac{1}{S_{ik}} \underbrace{\frac{\mathcal{N}^{\text{int}}(S_{ik}=0)}{S_{ik}+S_{jk}}}_{\text{non-singular in } \substack{S_{ik}\to 0 \\ \text{or} \\ S_{jk}\to 0}}$$

- Each kernel smoothly approaches soft singular behavior

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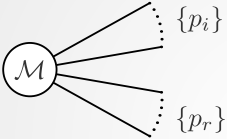
- Each kernel smoothly approaches soft singular behavior

- **Keep colour (& potentially spin) information in kernels**
- Yields potential for tracking **colour correlations** in PS
- **Algorithmic generalization** for higher emissions possible
- Recover **splitting functions** and **Eikonal factors** in collinear/soft limits

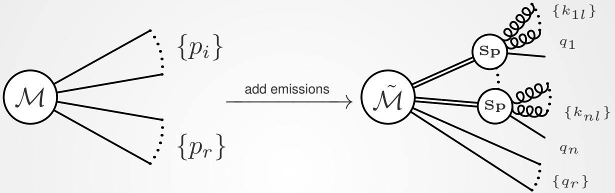
Momentum mapping

Momentum mapping
Adding emissions



- Start with **on-shell** (OS) momenta p_i (to be **emitters**) and p_r (to be **recoilers**) with overall momentum transfer $Q \equiv \sum_i p_i + \sum_r p_r$

Momentum mapping
Adding emissions



- Start with **on-shell** (OS) momenta p_i (to be **emitters**) and p_r (to be **recoilers**) with overall momentum transfer $Q \equiv \sum_i p_i + \sum_r p_r$
- Add emissions to the process with:
  1. Momentum conservation: $\sum_i q_i + \sum_{i,l} k_{il} + \sum_r q_r = Q$
  2. On-shellness of all partons
  3. Parametrization of soft & collinear behavior for any # of emissions

Momentum mapping

$$q_r = \frac{\Lambda}{\alpha_L} p_r$$

$$k_{il} = \frac{\Lambda}{\alpha_L} \left[\alpha_{il}\, p_i + \beta_{il}(1 - A_i)\, n_i + \sqrt{\alpha_{il}\beta_{il}(1 - A_i)}\, n_{il}^{\perp} \right], \quad A_i \equiv \sum_l \alpha_{il}$$

$$q_i = \frac{\Lambda}{\alpha_L} \left[(1 - A_i) p_i + \Big(y_i - \sum_l \beta_{il}(1 - A_i) \Big) n_i - \sum_l \sqrt{\alpha_{il}\beta_{il}(1 - A_i)}\, n_{il}^{\perp} \right]$$

- Lorentz transformation $\Lambda \Rightarrow$ non-trivial **global recoil**
- Decomposition w/ light-like momentum n_i and $n_{il}^{\perp} \cdot p_i = n_{il}^{\perp} \cdot n_i = 0$
- $k_{il}^2 = 0$ implies $(n_{il}^{\perp})^2 = -2 p_i \cdot n_i$
- $q_i^2 = 0$ fixes y_i in terms of the α_{il} and β_{il}
- Need $\alpha_L^2 = (Q + N)^2 / Q^2$ for momentum conservation

$$Q = \sum_r q_r + \sum_i q_i + \sum_{i,l} k_{il} = \frac{\Lambda}{\alpha_L} \Big[\underbrace{\sum_r p_r + \sum_i p_i}_{Q} + \underbrace{\sum_i y_i n_i}_{N} \Big]$$

Momentum mapping
Soft and collinear scaling

- Soft and collinear behavior studied via scaling and $\lambda \to 0$:

k_{il}	$(p_i, n_i, n_{il}^{\perp})$	$(\alpha_{il}, \beta_{il}, y_i)$
(forward) collinear	$Q(1, \lambda^2, \lambda)$	$(1, \lambda^2, \lambda^2)$
soft	$Q(\lambda, \lambda, \lambda)$	$(\lambda, \lambda, \lambda)$

- Induces collinear scaling of propagators

$$\frac{1}{(q_i + \sum_l k_{il})^2} = \frac{1}{y_i\, 2p_i \cdot n_i} \to \frac{1}{\lambda^2} \frac{1}{y_i\, 2p_i \cdot n_i}$$

- Take limits in splitting kernels to find leading singular behavior
- Note: y_i non-singular in α_{il}, β_{il}:

$$y_i = \sum_l \beta_{il}(1 - A_i) - \frac{\left(\sum_l \sqrt{\alpha_{il}\beta_{il}(1 - A_i)}\, n_{il}^{\perp}\right)^2}{2p_i \cdot n_i}$$

Phase space

- Can write down factorized phase space using momentum mapping

$$\mathrm{d}\phi\left(\{q_i\}_{\mathbf{S}}, \{q_r\}_{\mathbf{R}}, \{k_{il}\}_{\mathbf{E}_i}|Q\right) = \mathrm{d}\phi\left(\{p\}_{\mathbf{R}}|P_R\right) \alpha^{d-n_R(d-2)} (2\pi)^d \delta\left(P_S + P_R - Q\right)$$
$$\times \frac{\mathrm{d}m^2}{2\pi} [\mathrm{d}P_R] \frac{\omega(\vec{P}_R, \alpha m)}{\omega(\vec{Q}_R, m)} \Theta(Q_R^0) \prod_{i\in\mathbf{S}} [\mathrm{d}p_i] \frac{\omega(\vec{p}_i)}{\omega(\vec{q}_i)} \Theta(q_i^0) \left| \frac{\partial(\{\vec{q}\}_{\mathbf{S}}, \vec{Q}_R)}{\partial(\{\vec{p}\}_{\mathbf{S}}, \vec{P}_R)} \right| \prod_{l\in\mathbf{E}_i} [\mathrm{d}k_{il}] \Theta(k_{il}^0).$$

- Emission phase space:

$$\mathrm{d}^{d-1}k_{il} = |\mathcal{J}(\alpha_{il}, \beta_{il}, \Omega)|\ \mathrm{d}\alpha_{il}\, \mathrm{d}\beta_{il}\, \mathrm{d}^{d-3}\Omega,$$

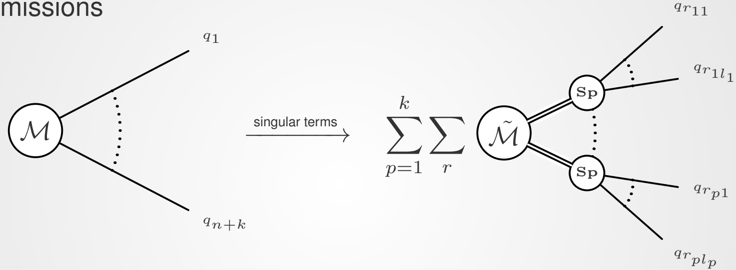
- Computable in d dimensions (where $\tilde{\beta}_{il} = \beta_{il}(1 - A_i)$):

$$[\mathrm{d}k_{il}] = \frac{1}{(2\pi)^{d-1}} \Theta(\alpha_{il}) \Theta(\tilde{\beta}_{il}) \frac{\alpha^{2-d}}{4} \frac{(2p_i \cdot n_i)^{\frac{d-2}{2}}}{(\alpha_{il}\tilde{\beta}_{il})^{\frac{d-4}{2}}} \mathrm{d}\alpha_{il}\, \mathrm{d}\tilde{\beta}_{il}\, \mathrm{d}\Omega^{d-3}.$$

General Algorithm and two emissions

General Algorithm
Amplitude

- Devise general setup for extracting singular behavior for k emissions

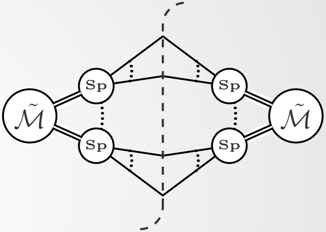


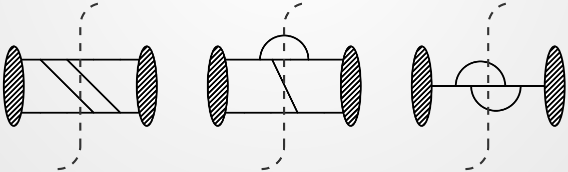
- Write amplitude in terms of splitting operators and factorized matrix element

$$|\mathcal{M}_{n+k}(q_1, ..., q_{n+k})\rangle = \sum_{p=1}^{k}\sum_{\{r\}} \mathbf{Sp}_{(r_{11}|...|r_{1\ell_1})} ... \mathbf{Sp}_{(r_{p1}|...|r_{p\ell_p})}$$
$$|\mathcal{M}_n(q_1, ..., q_{(r_{11}|...|r_{1\ell_1})}, ..., q_{(r_{p1}|...|r_{p\ell_p})}, ..., q_{n+k})\rangle$$

General Algorithm
Amplitude squared

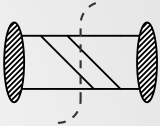
- Read amplitude squared in terms of **cut-diagrams** (reverse unitarity)
- Single out **topologies with leading singular behavior** (via # of unresolved partons)
- Examples for two emissions:





Two emissions

- For a given number of partons, find categorization of singular configs
- Read: $(i \parallel j \parallel k) \simeq S_{ijk} = (q_i + q_j + q_k)^2 \to 0$
- Triple collinear and double-soft contributions



	$\propto \frac{1}{S_{ij}S_{kl}S_{ijk}S_{jkl}}$
$i \parallel j \parallel k$	$S_{ij}S_{ijk}$
$i \parallel j \parallel l$	S_{ij}
$i \parallel k \parallel l$	S_{kl}
$j \parallel k \parallel l$	$S_{kl}S_{jkl}$
$(i \parallel j), (k \parallel l)$	$S_{ij}S_{kl}$
$(i \parallel k), (j \parallel l)$	$\times$
$(i \parallel l), (j \parallel k)$	$\times$

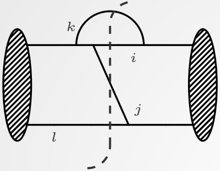
Construct partitioning factors from

$$1 = \frac{M^2(S_{kl}S_{jkl} + S_{ij}S_{ijk} + S_{ijk}S_{jkl}) + (S_{kl} + S_{ij})S_{ijk}S_{jkl}}{M^2(S_{kl}S_{jkl} + S_{ij}S_{ijk} + S_{ijk}S_{jkl}) + (S_{kl} + S_{ij})S_{ijk}S_{jkl}}$$

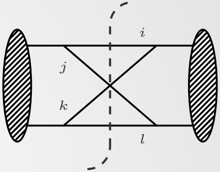
$\Rightarrow$ **non-singular in any configuration**

Two emissions

$B^{(2)}_{ijkl} \equiv$



$\propto \dfrac{1}{S_{ij}S_{ik}S_{jl}S_{ijk}}, \quad X_{ijkl} \equiv$



- Splitting kernels from partitioning factor $\times$ diagram

$$
\begin{aligned}
&U_{(i|j|k)}(B^{(2)}) \\
&= B^{(2)}_{ijkl} \frac{M^4 S_{jl}}{M^4 S_{jl} + M^2(S_{ij}S_{ijk} + S_{ik}S_{ijk}) + S_{ij}S_{ik}S_{ijk} + S_{ij}S_{jl}S_{ijk} + S_{ik}S_{jl}S_{ijk}} \\
&= \frac{1}{S_{ij}S_{ik}S_{ijk}} \frac{M^4 \mathcal{N}^{(2)}_{ijkl}(S_{ijk}=0)}{M^4 S_{jl} + S_{ijk}(M^2 S_{ij} + M^2 S_{ik} + S_{ij}S_{ik} + S_{ij}S_{jl} + S_{ik}S_{jl})}
\end{aligned}
$$

- Only leading-collinear in the $(i|j|k)$ configuration
- **Approaches singular behavior of full diagram** in any permutation of partons (triple-col, col-col, col-soft, soft-soft)

Conclusions

Goal: study leading singular behavior for multiple emissions
(for applications in parton showers and beyond)

- Momentum mapping for exposing collinear and soft factorization
- Global recoil via Lorentz transformation
- Partitioning algorithm to separate overlapping singularities
- Comprehensive framework for organising singular behavior

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- Check implementation for two emissions in QED and QCD, *e.g.* in double soft limits or via triple collinear splitting functions

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