Zero-jettiness beam functions at NNLO to higher orders in epsilon

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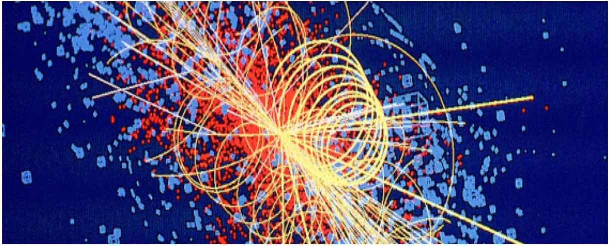

Outline

1. Motivation/background
2. Calculational set up
3. Beam functions at NNLO

Motivation/background

- Goal: use zero-jettiness τ as a slicing variable for $pp \to V$ at N3LO
- zero-jettiness variable is defined as

$$\tau = \sum_m \min_{i\in 1,2} \left[\frac{2p_i \cdot k_m}{Q_i} \right]$$

  and we slice at $\tau \ll 1$
- Simplification due to factorization theorem derived in SCET

$$\lim_{\tau\to 0} \mathrm{d}\sigma_{pp\to V}(\tau) = B_\tau \otimes B_\tau \otimes S_\tau \otimes H_\tau \otimes \mathrm{d}\sigma_V^{LO}$$

[Stewart, Tackmann, Waalewijn '10]

- beam function B, describes collinear radiation of incoming particles
- soft function S, describes soft radiation
- hard function H, describes corrections to the Born process

Beam function

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- beam function is a non-perturbative quantity defines as

$$B_i(t,x,\mu) = \int \mathrm{d}x_1 \mathrm{d}x_2 I_{ij}(t,x_1,\mu) f_j(x_2,\mu) \delta(x - x_1 x_2)$$

- f_j non-perturbative parton density function(PDF) for parton j
- I_{ij} perurbative matching coefficient, describing transition of parton j into parton i due to collinear emission

partonic beam function

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- Partonic beam function is a perturbative quantity defines as

$$B_{ij}(t,x,\mu) = \int \mathrm{d}x_1 \mathrm{d}x_2 I_{ik}(t,x_1,\mu) f_{kj}(x_2,\mu) \delta(x - x_1 x_2)$$

- f_{kj} perturbative partonic PDF
- I_{ij} same perurbative matching coefficient as before

In practice

- Calculate B_{ij}
- Solve above equations for I_{ij} in terms of B_{ij} (renormalization) e.g.

$$I_{qq}^{(3)} \sim B_{qq}^{(3)} + \frac{4C_F}{\epsilon^2} B_{qq}^{(2)} + \dots$$

$\Rightarrow$ require $B_{ij}^{(2)}$ up to ϵ^2, right now known up to ϵ^0

[Gaunt,Stahlhofen,Tackmann'14]

- Perform convolution

$$B_i = I_{ij} \otimes f_j$$

Calculational set up

- Two possible approaches
  - SCET
  - full QCD
- Focus on full QCD
  $\Rightarrow$ allows use of well-developed loop calculation techniques

Calculation of B_{ij}

- B_{ij} as an integral over the collinear QCD splitting function $P_{j\to i^*\{m\}}$

$$B_{ij} \sim \sum_{\{m\}} \int \mathrm{dPS}^{(m)} P_{j\to i^*\{m\}}$$

[Ritzmann,Waalewijn'14]

- Obtain splitting function through collinear projection operator $\mathcal{P}$

[Catani, Grazzini '00]

$$P_{j\to i^*\{m\}} \sim \mathcal{P}\left|M_{j\to i^*\{m\}}\right|^2$$

- Since QCD is charge-conjugation invariant we only need to consider the sets $(i,j) \in \{(q_i,q_j),(q_i,g),(q_i,\bar{q}_j),(g,g),(g,q_j)\}$

Projection operator

- Projection operator defined as
  - for $i \in \{q\}$

$$\mathcal{P}|M_{j\to i^*\{m\}}|^2 \sim \mathrm{Tr}\left[M_{j\to i^*\{m\}} \frac{\hat{\bar{p}}}{4\bar{p}\cdot(p-\sum_m k_m)} M^\dagger_{j\to i^*\{m\}}\right]$$

  - for $i \in \{g\}$

$$\mathcal{P}|M_{j\to i^*\{m\}}|^2 \sim -\frac{1}{2(1-\epsilon)} d^\rho_\mu \left(p - \sum_m k_m\right) d_{\nu\rho}\left(p - \sum_m k_m\right)$$
$$\times M^\mu_{j\to i^*\{m\}} M^{\nu\dagger}_{j\to i^*\{m\}}$$

where we have to use the gluon polarization tensor $d_{\mu\nu}$in axial gauge

$$d_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_\mu \bar{p}_\nu + \bar{p}_\mu k_\nu}{k\cdot\bar{p}}$$

Phase space measure

- Consider collinear emission only of p_1

$$\tau = \sum_m \min_{i \in 1,2} \left[\frac{2p_i \cdot k_m}{Q_i} \right] \Rightarrow \sum_m \frac{2p_1 \cdot k_m}{Q_1}$$

- m particle phase space now defined as

$$\int \mathrm{dPS}^{(m)} = \left(\prod_m \int \frac{\mathrm{d}^d k_m}{(2\pi)^{d-1}} \delta^+ \left(k_m^2 \right) \right) \times \delta \left(2 \sum_m k_m \cdot p - \frac{t}{z} \right) \delta \left(\frac{2 \sum_m k_m \cdot \bar{p}}{s} - (1-z) \right)$$

Calculation of B_{ij}

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- Altogether we write

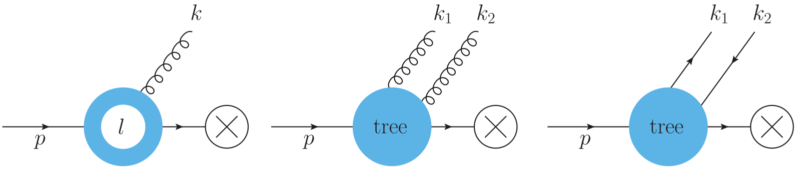
$$B_{ij} \sim \sum_{\{m\}} \int \mathrm{dPS}^{(m)} \mathcal{P} |M_{j \to i^* \{m\}}|^2$$

- for example consider $q_i \to q_j$ at NNLO

$$B_{q_i q_j} \sim \int \mathrm{dPS}^{(2)} \mathcal{P} |M_{q_j \to q_i^* \{g,g\}}|^2 + \int \mathrm{dPS}^{(2)} \mathcal{P} |M_{q_j \to q_i^* \{q,\bar{q}\}}|^2$$
$$+ \int \mathrm{dPS}^{(1)} \int \frac{\mathrm{d}^d l}{(2\pi)^d} \mathcal{P} |M^{\mathrm{loop}}_{q_j \to q_i^* \{g\}}|^2$$
$$B_{q_i q_j} \sim B^{\{g,g\}}_{q_i q_j} + B^{\{q,\bar{q}\}}_{q_i q_j} + B^{\{g\}}_{q_i q_j}$$

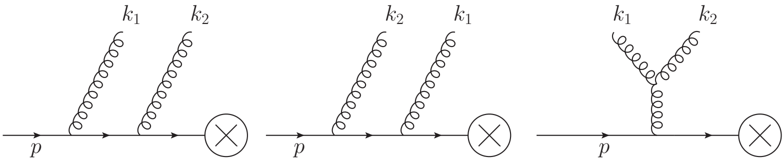
Calculation of B_{ij}

- Thus we have to consider the following diagrams



(a) Diagrams for $B^{\{g\}}_{q_i q_j}$ (b) Diagrams for $B^{\{g,g\}}_{q_i q_j}$. (c) Diagrams for $B^{\{q,\bar{q}\}}_{q_i q_j}$.

- Actual diagrams for $B^{\{g,g\}}_{q_i q_j}$



Reverse unitarity, IBPs

- Use reverse unitarity to rewrite phase-space delta functions

[Anastasiou,Melnikov'02]

$$\delta\left(p^2-m^2\right)=\frac{\mathrm{i}}{2\pi}\left[\frac{1}{p^2-m^2+\mathrm{i}\epsilon}-\frac{1}{p^2-m^2-\mathrm{i}\epsilon}\right],$$

- Use IBPs to reduce beam function to sum of master integrals
- Solve master integrals (MIs)

Beam functions at NNLO

- We find that all five beam functions B_{q_i,q_j}, $B_{q_i,g}$, $B_{q_i,\bar{q}_j}$, $B_{g,g}$, B_{g,q_j} can be expressed through 13 master integrals
- Integrals easy enough to be straightforwardly integrated
- Many integrals can be computed in closed form in ϵ

Real-real master integral

- Consider the following integral

$$I = \int \frac{\mathrm{d}^d k_1}{(2\pi)^{d-1}} \int \frac{\mathrm{d}^d k_2}{(2\pi)^{d-1}} \, \delta^+\left(k_1^2\right) \; \delta^+\left(k_2^2\right) \delta\left(2k_{12}\cdot p - \tfrac{1}{z}\right)$$
$$\times \frac{\delta\left(2k_{12}\cdot\bar{p} - (1-z)\right)}{(p-k_1)^2 \; k_{12}^2 \; \bar{p}\cdot k_2}.$$

- We insert $1 = \int \mathrm{d}^d Q \delta^d(k_1 + k_2 - Q)$ and change the order of integration.

$$I = \int \mathrm{d}^d Q \; \delta\left(2Q\cdot p - \tfrac{1}{z}\right) \delta\left(2Q\cdot\bar{p} - (1-z)\right) \frac{\mathrm{F}(Q^2, p\cdot Q, \bar{p}\cdot Q)}{Q^2}$$

$$\mathrm{F} = \int \frac{\mathrm{d}^d k_1}{(2\pi)^{d-1}} \int \frac{\mathrm{d}^d k_2}{(2\pi)^{d-1}} \; \frac{\delta^+\left(k_1^2\right) \delta^+\left(k_2^2\right)}{(p-k_1)^2 \; \bar{p}\cdot k_2} \delta^d(Q - k_1 - k_2)$$

Real-real master integral

- We exploit Lorentz-invariance and set $Q = (Q_0, 0, 0, 0)$.

$$F = -\frac{1}{2}\int \frac{\mathrm{d}^{d-1}\vec{k}_1}{(2\pi)^{2d-2}4|\vec{k}_1|^2}\frac{\delta(Q_0 - 2|\vec{k}_1|)}{\bar{p}_0|\vec{k}_1|+\vec{\bar{p}}\vec{k}_1}\frac{1}{p_0|\vec{k}_1|-\vec{p}\,\vec{k}_1}$$

- Introduce spherical coordinates for $\vec{k}_1$

$$F = -\frac{1}{(2p_0Q_0)(2\bar{p}_0Q_0)}\left(\frac{Q_0}{2}\right)^{d-4}\int \frac{\mathrm{d}^{d-1}\Omega_k}{(2\pi)^{2d-2}}\frac{1}{(k_n\cdot p_1)\,(k_n\cdot p_2)}$$

and we abbreviated $p_1 = (1, \vec{n}_p)$, $p_2 = (1, -\vec{n}_{\bar{p}})$, $k_n = (1, \vec{n}_k)$

- Calculate the angular integral [Somogyi'14]

$$\int \frac{\mathrm{d}^{d-1}\Omega_k}{(k_n\cdot p_1)\,(k_n\cdot p_2)} = -\Omega_{d-2}\frac{2^{-2\epsilon}}{\epsilon}\frac{\Gamma(1-\epsilon)^2}{\Gamma(1-2\epsilon)}\,{}_2F_1\left(1,1,1-\epsilon,1-\frac{\rho_{12}}{2}\right)$$

with $\rho_{12} = (1 - \vec{n}_{p_1}\cdot\vec{n}_{p_2})$

Real-real master integral

- Restore Lorentz-invariance

$$F(Q^2, p\cdot Q, \bar{p}\cdot Q) = -\frac{\Omega_{d-2}}{(2\pi)^{2d-2}} \frac{(Q^2)^{-\epsilon}}{(2p\cdot Q)(2\bar{p}\cdot Q)} \frac{\Gamma(1-\epsilon)\Gamma(-\epsilon)}{\Gamma(1-2\epsilon)} \times \ {}_2F_1\left(1,1,1-\epsilon, \tfrac{Q^2}{(2Q\cdot p)(2Q\cdot\bar{p})}\right)$$

- Substitute F back into I

$$I = \int \mathrm{d}^dQ\, \delta\left(2Q\cdot p - \tfrac{1}{z}\right)\delta\left(2Q\cdot\bar{p} - (1-z)\right) \frac{F(Q^2, p\cdot Q, \bar{p}\cdot Q)}{Q^2}$$

- Introduce a Sudakov decomposition $Q^\mu = \alpha p^\mu + \beta\bar{p}^\mu + Q_\perp^\mu$

$$I = -\frac{z}{(1-z)} \frac{(\Omega_{d-2})^2}{4(2\pi)^{2d-2}} \frac{\Gamma(1-\epsilon)\Gamma(-\epsilon)}{\Gamma(1-2\epsilon)} \int_0^{\frac{1-z}{z}} \mathrm{d}Q_\perp^2\, (Q_\perp^2)^{-\epsilon} \times \left(\tfrac{1-z}{z} - Q_\perp^2\right)^{-(1+\epsilon)} {}_2F_1\left(1,1,1-\epsilon, 1-\tfrac{Q_\perp^2\, z}{1-z}\right)$$

Real-real master integral

- We finally find

$$I = -\frac{(\Omega_{d-2})^2}{4(2\pi)^{2d-2}}\left(\frac{1-z}{z}\right)^{-1-2\epsilon}\frac{\Gamma(1-\epsilon)^2\Gamma(-\epsilon)^2}{\Gamma(1-2\epsilon)^2}$$
$$\times\ {}_3\mathrm{F}_2\left(1,1,-\epsilon;1-2\epsilon,1-\epsilon,1\right)$$

- If the last parametric integral is unsolvable we expand in ϵ and integrate order by order

Real-virtual master integral

- Consider the following integral

$$I = \int \frac{\mathrm{d}^d k}{(2\pi)^{d-1}} \int \frac{\mathrm{d}^d l}{(2\pi)^d}\, \delta^+\left(k^2\right)\, \delta\left(2k\cdot p - \tfrac{1}{z}\right) \times \frac{\delta\left(2k\cdot\bar{p} - (1-z)\right)}{l^2\ (l\cdot\bar{p})\ (p-l)^2\ (p-k-l)^2}$$

- We begin with the loop integration

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{1}{l^2\ (l\cdot\bar{p})\ (p-l)^2\ (p-k-l)^2}$$

- Combine the two propagators $1/l^2$ and $1/l\cdot\bar{p}$ using a special choice of Feynman parameters [Becher'14]

$$\frac{1}{AB} = \int_0^1 \frac{\mathrm{d}x}{[x\,A + (1-x)\,B]^2} \overset{y=\frac{x}{1-x}}{=} \int_0^\infty \mathrm{d}y \frac{1}{[A+yB]^2}.$$

Real-virtual master integral

- The propagators are now written in a standard form

$$\frac{1}{l^2}\frac{1}{(2l\cdot\bar{p})} = \int_0^\infty \frac{\mathrm{d}y}{(l^2+2\ \ l\cdot\bar{p}\ \ y)^2} = \int_0^\infty \frac{\mathrm{d}y}{[(l+y\ \ \bar{p})^2]^2}$$

  ...and we obtain the standard loop integral

$$\tilde{I} = \int_0^\infty \mathrm{d}y \int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{1}{[(l+y\ \ \bar{p})^2]^2\ \ (p-l)^2\ \ (p-k-l)^2}$$

- The integration is straightforward. We find

$$\tilde{I} = -i\,2^{-3+2\epsilon}\pi^{-2+\epsilon}\mathrm{B}(-\epsilon,1)\,\mathrm{B}(-\epsilon,-\epsilon)\,\Gamma(1+\epsilon) \times (2p\cdot k)^{-1-\epsilon}\,{}_2\mathrm{F}_1\left(1,-\epsilon,1-\epsilon,2\bar{p}\cdot k\right).$$

Real-virtual master

- Substitute into the remaining k integration

$$I = \int \frac{\mathrm{d}^d k}{(2\pi)^{d-1}}\, \delta^+\left(k^2\right)\; \delta\left(2k\cdot p - \tfrac{1}{z}\right) \delta\left(2k\cdot \bar{p} - (1-z)\right) \tilde{I}$$

- Introduce Sudakov decomposition $k^\mu = \alpha p^\mu + \beta \bar{p}^\mu + k_\perp^\mu$ and immediately obtain

$$I = \frac{(\Omega_{d-2})^2}{8(2\pi)^{2d-2}} \frac{(1-z)^{-\epsilon} z^{1+2\epsilon}}{(1+\epsilon)} \mathrm{B}(-\epsilon,1)\mathrm{B}(-\epsilon,-\epsilon)$$
$$\times\; \Gamma(1-\epsilon)\,\Gamma(2+\epsilon)\; {}_2\mathrm{F}_1(1,-\epsilon,1-\epsilon,1-z).$$

Conclusion and Outlook

- Beam functions can be calculated as phase space integrasl over collinear enhanced cross sections with $\mathcal{P}$ as a special Feynman rule
- At NNLO all five partonic beam functions B_{q_i,q_j}, $B_{q_i,g}$, $B_{q_i,\bar{q}_j}$, $B_{g,g}$, B_{g,q_j} in terms of 13 master integrals
- All five matching coefficient calculated to order ϵ^2, and checked against literature
- Outlook: N3LO calculation....

Backup

- Consider the Integral

$$I = \int_0^1 \mathrm{d}r \int_0^1 \mathrm{d}\alpha_1 (1-\alpha_1)^{-2\epsilon} \alpha_1^{-1-2\epsilon} (1-r)^{-1-2\epsilon} r^{-1-\epsilon}$$
$$\frac{(1-\alpha_1+\alpha_1 r)^{2\epsilon}}{-1+(1-z)\alpha_1}\, {}_2\mathrm{F}_1\left(-\epsilon, -2\epsilon, 1-\epsilon, r\right),$$

  ...which diverges at $\alpha_1 = 0$ and $r = 0, 1$.

- We cannot solve this integral in a closed form. For this reason we would like to expand the integrand in a Laurent series in ϵ solving the integral order by order. To this end, the integrand needs to be finite in the whole integration interval. We achieve this by performing end-point subtractions.

Backup

- For example, we write

$$\frac{(1-\alpha_1+\alpha_1 r)^{2\epsilon}}{-1+(1-z)\alpha_1} = \left(\frac{(1-\alpha_1+\alpha_1 r)^{2\epsilon}}{-1+(1-z)\alpha_1} - \frac{(1-\alpha_1+\alpha_1 r)^{2\epsilon}}{-1+(1-z)\alpha_1}\bigg|_{\alpha_1=0} \right)$$
$$+ \frac{(1-\alpha_1+\alpha_1 r)^{2\epsilon}}{-1+(1-z)\alpha_1}\bigg|_{\alpha_1=0}$$
$$= \left(\frac{(1-\alpha_1+\alpha_1 r)^{2\epsilon}}{-1+(1-z)\alpha_1} + 1 \right) - 1.$$

- Substituting this back into the above equation we obtain the sum of two integrals $I = I_1 + I_2$

Backup

- We find

$$I_1 = \int_0^1 \mathrm{d}r \int_0^1 \mathrm{d}\alpha_1 (1-\alpha_1)^{-2\epsilon} \alpha_1^{-1-2\epsilon} (1-r)^{-1-2\epsilon} r^{-1-\epsilon} \times \left(\frac{(1-\alpha_1+\alpha_1 r)^{2\epsilon}}{-1+(1-z)\alpha_1} + 1 \right) {}_2\mathrm{F}_1\left(-\epsilon, -2\epsilon, 1-\epsilon, r\right),$$

$$I_2 = (-1) \int_0^1 \mathrm{d}r \int_0^1 \mathrm{d}\alpha_1 (1-\alpha_1)^{-2\epsilon} \alpha_1^{-1-2\epsilon} (1-r)^{-1-2\epsilon} r^{-1-\epsilon} \times {}_2\mathrm{F}_1\left(-\epsilon, -2\epsilon, 1-\epsilon, r\right).$$

- The $\alpha_1 = 0$ singularity in the integral inI_1 is now regulated by the term in parenthesis, while the second integral I_2 can be easily integrated if we identify the r integration with the integral representation of the generalized hypergeometric function ${}_3F_2$. The I_1 still diverges in $r = 0$ and $r = 1$.