Automated calculation of N-jet soft functions

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In collaboration with
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Introduction

- Computation of jet cross sections beyond LO complicated by IR-divergences
- Subtraction techniques: subtract IR soft and collinear behaviors from the real emission, then add them back to virtual contributions to cancel the IR poles
- q_T subtraction: e.g. top quark production at hadron colliders

Catani, Grazzini (2007)
Bonciani,Catani, Grazzini, Sargsyan, Torre (2015)

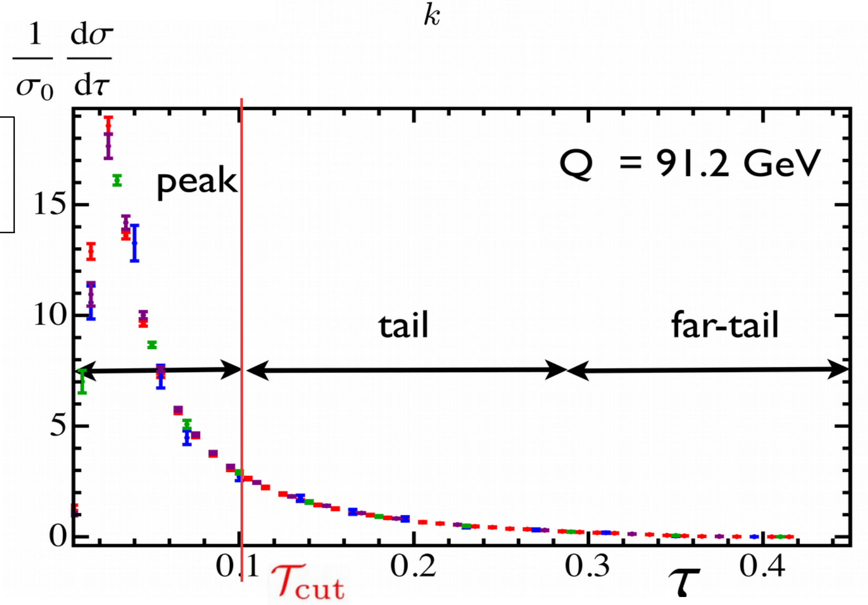
- N-jettiness slicing: e.g. H+jet, W+jet and Z+jet

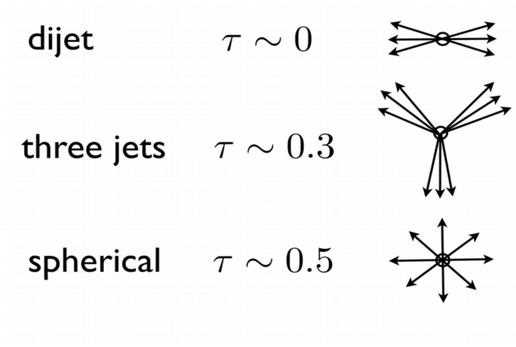
Boughezal, Focke, Liu, Petriello(2015)
Gaunt, Stahlhofen, Tackmann, Walsh (2015)
Boughezal et. al. (2015)

N-jettiness variable:

$$\mathcal{T}_N = \sum_k \min_i \{n_i \cdot p_k\}$$

e^+e^- 2-jettiness (thrust)





Introduction

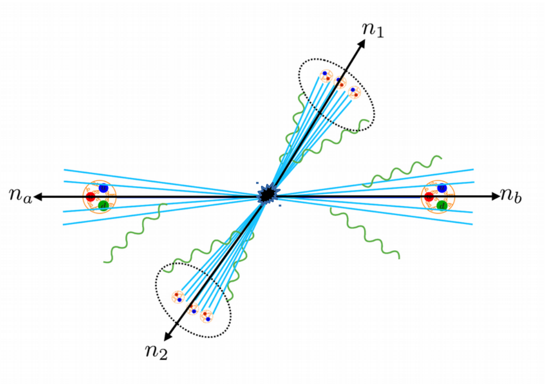
N-jet cross section:

Boughezal, Focke, Liu, Petriello(2015)
Gaunt, Stahlhofen, Tackmann, Walsh (2015)

$$\sigma(X) = \int_0 \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} = \int_0^{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} + \int_{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$

singular
N-jet final state

non-singualr
N+1-jet / multi-jets



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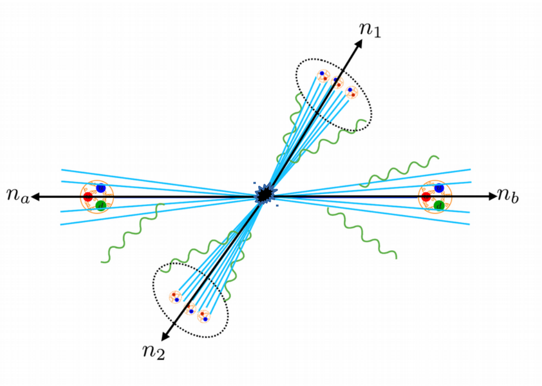


Compute using factorization theorems in soft/collinear limits

$$\frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} = \mathbf{H} \times \underbrace{B_a \otimes B_b} \otimes \underbrace{J_1 \otimes \ldots \otimes J_N} \otimes S_N + \mathcal{O}(\mathcal{T}_N)$$

Hard function | Beam functions | Jet functions | N-jet soft function

Hard function: for many processes known to NNLO (e.g. W+jet) Gehrmann et al, (2012)

Beam function: known to NNLO Gaunt et al, (2014)

Jet function: known to **NNNLO** → **Rubin's Talk** Brüser et al. (2018), Banerjee et al. (2018)

Soft function: for **N=0,1,2** known to NNLO (heavy to light decay known at **NNNLO**)

Gaunt et al (2015), Boughezal et al (2015),
Compbell et al (2017), Jen et al (2019),
Brüser et al. (2019)

This Talk

Calculate the N-jet soft function for arbitrary N to NNLO

Automate soft function calculations

Idea: Automation

- Find generic strategy to evaluate soft functions (to NNLO)
- Set up a numerical method based on universal structure of divergences
  - ✓ Isolate singularities with universal phase-space parametrization
  - ✓ Compute observable dependent integrations numerically
  - ✓ SoftSERVE

    Bell, Rahn, Talbert (2019)
    - Dijet soft functions (two light-like directions)
    - Explicit NNLO results for O(15) observables (e.g. jet grooming, jet vetoes, threshold and transverese momentum ressumation, e^+e^- event shapes)

Aim: extend framework for calculating N-jet soft functions at NNLO

Outline

Automating generic N-jet soft function calculation

(a) NLO: Real emission

Boost invariant parametrization

(b) NNLO: Virtual-Real interference

Double-Real emissions

N-jettiness soft function

(a) Constraints from RGE

(b) 1-jettiness ***Preliminary Results***

(c) 2-jettiness ***Preliminary Results***

Summary and outlook

N-jet soft functions

N-jet soft functions at NLO

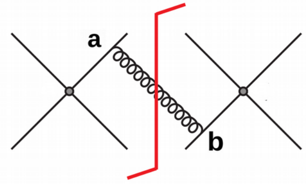
- N-jet Soft functions

$$S_N(\tau,\mu) = \sum_X \mathcal{M}(\tau,\{k_i\})\, \langle 0|\, \left(S_{n_1}\, S_{n_2}\, S_{n_3}\dots\right)^{\dagger} |X\rangle \langle X|\, \left(S_{n_1}\, S_{n_2}\, S_{n_3}\dots\right) |0\rangle$$

multiple soft Wilson lines $\boldsymbol{S}_i(x) = \mathcal{P}\exp\left(ig_s\int_{-\infty}^{0} ds\, n_i\cdot A^a(x+sn_i)\,\boldsymbol{T}_i^a\right)$

$\mathcal{M}(\tau,\{k_i\})$ **generic measurement function**

✓ One-loop: Virtual corrections scaleless, real emissions diagrams contribute



✓ N-jet soft function at NLO: $S_N = \sum_{a\neq b} T_a\cdot T_b\, S_{ab}$

Catani, Grazzini (2000)
Catani, Seymour (1996)

$$S_{ab} \sim \int \mathrm{d}^d k\, \delta(k^2)\, \theta(k^0)\, \mathcal{M}(\tau,\{k_i\})\, |\mathcal{A}_{ab}(k)|^2$$

dipole matrix element

$$|\mathcal{A}_{ab}(k)|^2 \sim \frac{n_a\cdot n_b}{2\, n_a\cdot k\, n_b\cdot k}$$

Dijet matrix element

$$|\mathcal{A}(k)|^2 \sim \frac{n_+\cdot n_-}{2\, k_-\, k_+}$$

Setup of calculation at NLO

Strategy

1. Parametrization: use the **transverse momentum** and **rapidity** measure

$$k_T = \sqrt{\frac{2\, k_a\, k_b}{n_{ab}}} \qquad y = \frac{k_a}{k_b} \qquad \begin{aligned} n_{ab} &\equiv n_a \cdot n_b \\ k_X &\equiv n_X \cdot k \end{aligned}$$

- **Parameterizing the solid angle:** Sudakov decomposition is a **Lorenz covariant relation**

$$k^\mu = k_b \frac{n_a^\mu}{n_{ab}} + k_a \frac{n_b^\mu}{n_{ab}} \underbrace{- k_{x_3} n^\mu_{x_3} - k_{x_4} n^\mu_{x_4} + \dots}_{k^\mu_\perp}$$

$$\begin{aligned} k_{x_3} &= -k_T \cos(\theta_1) \\ k_{x_4} &= -k_T \cos(\theta_2) \sin(\theta_1) \\ k_{x_d} &= -k_T \cos(\theta_{d-2}) \sin(\theta_{d-3}) \dots \sin(\theta_1) \end{aligned}$$

Setup of calculation at NLO

2. Generic measurement function (inspired by **Laplace space**)

$$\boxed{\mathcal{M}(\tau;k) = \exp\Big(-\tau\, k_T\, y^{n/2}\ \sqrt{n_{ab}/2}\ f(y,\theta_1,\theta_2)\Big)}$$

- k_T dependence fixed on dimensional grounds
- $f(y,\theta_1,\theta_2)$ finite and non-zero in collinear limit y → 0
- Factorized part of kinematic dependences on n_{ab} : improves numerical convergence
- External kinematics are limited to 4-dim $\longrightarrow$ **2 angles** for N-jet processes

Setup of calculation at NLO

3. Integrate k_T analytically

4. Derive a master formula

$$S_{ab}(\tau,\mu) \sim \frac{\overbrace{\Gamma(-2\epsilon)}^{\textbf{Soft divergence}}}{\Gamma(-\epsilon)} \left(\sqrt{n_{ab}/2}\,\tau e^{\gamma_E} \mu \right)^{2\epsilon}$$

$$\times \int_0^1 \mathrm{d}y \int_{-1}^1 \mathrm{d}\cos\theta_1\, \mathrm{d}\cos\theta_2\, \sin^{-1-2\,\epsilon}\theta_1\, \sin^{-2-2\,\epsilon}\theta_2\, \overbrace{y^{-1+n\epsilon}}^{\textbf{Collinear divergences}} \underbrace{\Big[f(y,\theta_1,\theta_2) \Big]^{2\epsilon}}_{\textbf{Measurement function}}$$

✓ Singularities from k_T → 0 and y → 0 are factorized

N-jettiness soft function

5. Isolate singularities with standard subtraction techniques:

$$\int_0^1 dx\ x^{-1+n\varepsilon}\ f(x) = \int_0^1 dx\ x^{-1+n\varepsilon}\ [\ \underbrace{f(x) - f(0)}_{\text{finite}} + \underbrace{f(0)}_{1/\varepsilon}\ ]$$

Numerical implementations:

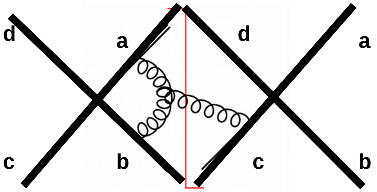
- Based on pySecDec Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk,Zirke (2017)

  general implementation of sector decomposition algorithm
  Cuba library (use Vegas routine) for numerical integrations

- Based on SoftSERVE Bell, Rahn, Talbert (2019)

  dedicated C++ implementation for N-jet soft function
  Cuba library (use Divonne routine) for numerical integrations

Setup of calculation at NNLO



- ✓ Two-Loop: Virtual corrections scaleless
- ✓ Real-Virtual contribution   Catani, Grazzini (2000)

$$S_{\rm RV} = \sum_{a\neq b} T_a \cdot T_b \, S^R_{ab} + \underbrace{\sum_{a\neq b\neq c} (\lambda_{ab} - \lambda_{ak} - \lambda_{bk}) \, f_{ABC} \, T^A_a \, T^B_b \, T^C_c \, S^{Im}_{abc}}_{\text{Three-parton correlation (process dependent)}}$$

$$\lambda_{XY} = \begin{cases} +1 & \text{if X and Y are both incoming/outgoing} \\ 0 & \text{otherwise} \end{cases}$$

- ➢ **dipole contribution :** follow the same strategy of NLO

$$|\mathcal{A}^R_{ab}(k)|^2 \sim \left(\frac{n_{ab}}{2\,k_a\,k_b}\right)^{1+\epsilon}$$

Dijet matrix element

$$|\mathcal{A}(k)|^2 \sim \left(\frac{n_+ \cdot n_-}{2\,k_+\,k_-}\right)^{1+\epsilon}$$

- ➢ **tripole contribution:**
  - ✓ only present in processes with four or more hard partons
  - ✓ choose dipole n_a- n_c and follow the same strategy of NLO

$$|\mathcal{A}^{Im}_{abc}(k)|^2 \sim \left(\frac{n_{ac}}{2\,k_a\,k_c}\right)\left(\frac{n_{ab}}{2\,k_a\,k_b}\right)^{\epsilon}$$

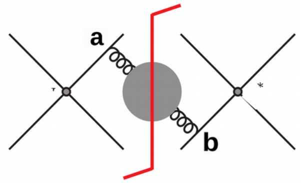
Setup of calculation at NNLO

✓ Double real corrections:

Catani, Grazzini (2000)

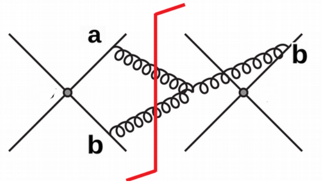
I) radiation of soft q$\bar{\text{q}}$ pair

$$S_N^{q\bar{q}} = T_F\, n_f \sum_{a\neq b} T_a \cdot T_b\, S_{ab}^{T_F n_f}$$



II) radiation of double-real gluons

$$S_N^{gg} = C_A \sum_{a\neq b} T_a \cdot T_b\, S_{ab}^{C_A}$$



III) tripole and quadrupole contributions are accounted for by non-abelian exponentiation

assume non-abelian exponentiation

➢ T_F n_f structure

$$S_{ab}^{T_F n_f} \sim \int \mathrm{d}^d k\, \delta(k^2)\, \theta(k^0) \int \mathrm{d}^d l\, \delta(l^2)\, \theta(l^0)\, \mathcal{M}(\tau; k, l) \left|\mathcal{A}_{ab}(k,l)\right|^2_{T_F n_f}$$

matrix element

$$\left|\mathcal{A}_{ab}(k,l)\right|^2_{T_F n_f} \sim \frac{2\,k\cdot l(k_i+l_i)(k_j+l_j) - (k_i\,l_j - l_i\,k_j)^2}{(k_i+l_i)^2\,(k_j+l_j)^2\,\boxed{(2\,k\cdot l)^2}} \longrightarrow \text{overlapping divergence}$$

$$\left|\mathcal{A}(k,l)\right|^2_{C_F T_F n f} \sim \frac{2\,k\cdot l(k_-+l_-)(k_++l_+) - (k_-\,l_+ - l_-\,k_+)^2}{(k_-+l_-)^2\,(k_++l_+)^2\,(2\,k\cdot l)^2}$$

Setup of calculation at NNLO

Strategy

1. Parametrization: collective and relative variables related to a two body system

$$p_T = \sqrt{\frac{2}{n_{ab}}(k_a + l_a)(k_b + l_b)} \qquad a = \frac{k_b\, l_a}{k_a\, l_b} = \sqrt{\frac{y_l}{y_k}}$$

$$y = \frac{k_a + l_a}{k_b + l_b} \qquad b = \sqrt{\frac{k_a\, k_b}{l_a\, l_b}} = \frac{k_T}{l_T}$$

2. Generic form of the measurement function: five angles in transverse plane

$$\boxed{\mathcal{M}(\tau; k, l) = \exp\Big(-\tau\, p_T\, y^{n/2}\ \sqrt{n_{ab}/2}\ F(a, b, y, \theta_{kl}, \theta_{nk_1}, \theta_{nk_2}, \theta_{nl_1}, \theta_{nl_2})\Big)}$$

- p_T dependence fixed on dimensional grounds
- $F(a, b, y, \theta_{kl}, \theta_{nk_1}, \theta_{nk_2}, \theta_{nl_1}, \theta_{nl_2})$ finite and non-zero for y → 0
- External kinematics are limited to 4-dim $\longrightarrow$ **5 angles** for N-jet processes

Setup of calculation at NNLO

3&4. Integrate P_T analytically and obtain the master formula

$$
\begin{aligned}
S_{C_F T_F n_f}(\tau,\mu) \sim{}& \frac{\overbrace{\Gamma(-4\epsilon)}^{\textbf{Soft divergence}}}{\Gamma(-\epsilon)\Gamma(1/2-\epsilon)} \left(\tau e^{\gamma_E}\mu\right)^{4\epsilon} \int_0^1 \mathrm{d}y\, \mathrm{d}a\, \mathrm{d}b \\
&\times \int_{-1}^{1} \mathrm{d}\cos\theta_{kl}\ \mathrm{d}\cos\theta_{nk_1}\ \mathrm{d}\cos\theta_{nk_2}\ \sin^{-1-2\,\epsilon}\theta_{kl}\ \sin^{-2-2\,\epsilon}\theta_{nk_1}\ \sin^{-3-2\,\epsilon}\theta_{nk_2} \\
&\times \int_{-1}^{1} \mathrm{d}\cos\theta_{nl_1}\ \mathrm{d}\cos\theta_{nl_2}\ \sin^{-1-2\,\epsilon}\theta_{nl_1}\ \sin^{-2-2\,\epsilon}\theta_{nl_2} \\
&\times \underbrace{\frac{y^{-1+2n\epsilon}}{\left(1+a^2-2\,a\,\cos\theta_{kl}\right)^2}}_{\textbf{Collinear divergences}} \underbrace{\Big[F(a,b,y,\theta_{kl},\theta_{nk_1},\theta_{nk_2},\theta_{nl_1},\theta_{nl_2})\Big]^{4\epsilon}}_{\textbf{Measurement function}} \underbrace{\mathcal{J}(a,b,y,\epsilon)}_{\textbf{Matrix element Jacobian}}
\end{aligned}
$$

Overlapping singularity:

I) Sector decomposition

II) Factorize the singularity with a simple change of variable

Applications:

N-jettiness soft function

Solving RGE

- RGE for the renormalized soft function and the counterterm

$$\mu \frac{\mathrm{d}\, S(\tau,\mu)}{\mathrm{d}\mu} = \frac{1}{2}\gamma_s\, S(\tau,\mu) + \frac{1}{2} S(\tau,\mu)\, \gamma_s^\dagger$$

$$\mu \frac{\mathrm{d}\, Z_S(\tau,\mu)}{\mathrm{d}\mu} = -\frac{1}{2}\gamma_s S(\tau,\mu)$$

$$\boxed{i\pi\alpha_s^2 \left[\sum_{a\neq b} T_a \cdot T_b \,\ln(\sqrt{2\, n_{ab}})\,,\, \sum_{c\neq d} T_c \cdot T_d\, \Delta_{cd} \right]}$$

- Soft anomalous dimension given by consistency relation(RG invariance)

$$\gamma_s = \Gamma_{\text{cusp}} \left[-2 \sum_{a\neq b} T_a \cdot T_b \,\ln\left(\sqrt{2\, n_{ab}}\, \mu\, \bar{\tau}\right) + i\pi \sum_{a\neq b} T_a \cdot T_b \Delta_{ab} \right] + \gamma_s^{\text{non-cusp}}$$

$$\Delta_{ab} = \begin{cases} +1 & \text{if a and b are both incoming/outgoing} \\ 0 & \text{otherwise} \end{cases}$$

related to the anomalous dimension of hard Wilson Coefficient from matching QCD to SCET

Solve iteratively for the bare soft function (provides a cross check for the poles)

$$S^{\text{bare}}(\tau) = Z_S(\tau,\mu)\, S(\tau,\mu)\, Z_S^\dagger(\tau,\mu)$$

N-jettiness soft function

The soft function in Laplace space

$$Z_\alpha = 1 - \left(\frac{\alpha_s}{4\,\pi}\right)\frac{\beta_0}{\epsilon}$$

$$\bar{\tau} = \tau\, e^{\gamma_E}$$

$$S(\tau,\mu) = 1 + \left(\frac{Z_\alpha\,\alpha_s}{4\,\pi}\right)\sum_{a\neq b}\mathbf{T_a}\cdot\mathbf{T_b}\left(\sqrt{2\,n_{ab}}\,\mu\,\bar{\tau}\right)^{2\epsilon} S^{(1)}_{ab}(\epsilon)$$

$$+ \left(\frac{Z_\alpha\,\alpha_s}{4\,\pi}\right)^2\Bigg[\sum_{a\neq b}\mathbf{T_a}\cdot\mathbf{T_b}\left(\sqrt{2\,n_{ab}}\,\mu\,\bar{\tau}\right)^{4\epsilon} S^{(2)}_{ab}(\epsilon) + \sum_{a\neq b\neq c}\mathbf{f_{ABC}}\,\mathbf{T^A_a}\,\mathbf{T^B_b}\,\mathbf{T^C_c}\left(\mu\,\bar{\tau}\right)^{4\epsilon}\, S^{(2,Im)}_{ab}(\epsilon)$$

$$+ \frac{1}{2}\sum_{a\neq b, c\neq d}\mathbf{T_a}\cdot\mathbf{T_b}\,\mathbf{T_c}\cdot\mathbf{T_d}\left(2\,\sqrt{n_{ab}\,n_{cd}}\,\mu^2\,\bar{\tau}^2\right)^{2\epsilon} S^{(1)}_{ab}(\epsilon) S^{(1)}_{cd}(\epsilon)\ \Bigg] + \mathcal{O}(\alpha_s^3)$$

Jouttenus, Stewart, Tackmann, Waalewijn (2011)

This work: Preliminary Results

$$S^{(1)}_{ab}(\epsilon) = \overbrace{\frac{2}{\epsilon^2} + \frac{0}{\epsilon} + \mathbf{I^1_{ab}}}^{\textbf{known results for any number of jets}} + \epsilon\, K^1_{ab}$$

$$S^{(2)}_{ab}(\epsilon) = \Bigg(T_F n_f\left[-\frac{2}{3\,\epsilon^3} - \frac{10}{9\,\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{56}{7} + \frac{\pi^2}{9} - \frac{4}{3}\mathbf{I^1_{ab}}\right) + I^{T_F n_f}_{ab}\right]$$

$$+ C_A\left[\frac{0}{\epsilon^4} + \frac{11}{6\,\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{67}{18} - \frac{\pi^2}{6}\right) + \frac{1}{\epsilon}\left(\frac{202}{27} - \frac{11\,\pi^2}{36} - 7\zeta_3 + \frac{11}{3}\mathbf{I^1_{ab,c}}\right) + I^{C_A}_{ab}\right]\Bigg)$$

poles are known from RGE

Numerical checks

One-jettiness in pp collision

$$S^{(1)}_{ab}(\epsilon) = \frac{\mathbf{C^1_{-2}}}{\epsilon^2} + \frac{\mathbf{C^1_{-1}}}{\epsilon} + \mathbf{I^1_{ab}} + \epsilon\,\mathbf{K^1_{ab}}$$

$$S^{(2)}_{ab}(\epsilon) = \left(\mathbf{T_F n_f}\left[\frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{T_F n_f}_{ab}}\right] + \mathbf{C_A}\left[\frac{\mathbf{C^2_{-4}}}{\epsilon^4} + \frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{C_A}_{ab}}\right]\right)$$

$$n_{12} = 2$$
$$n_{13} = 1 - \cos(\theta)$$
$$n_{23} = 2 - n_{13}$$

1-Loop →

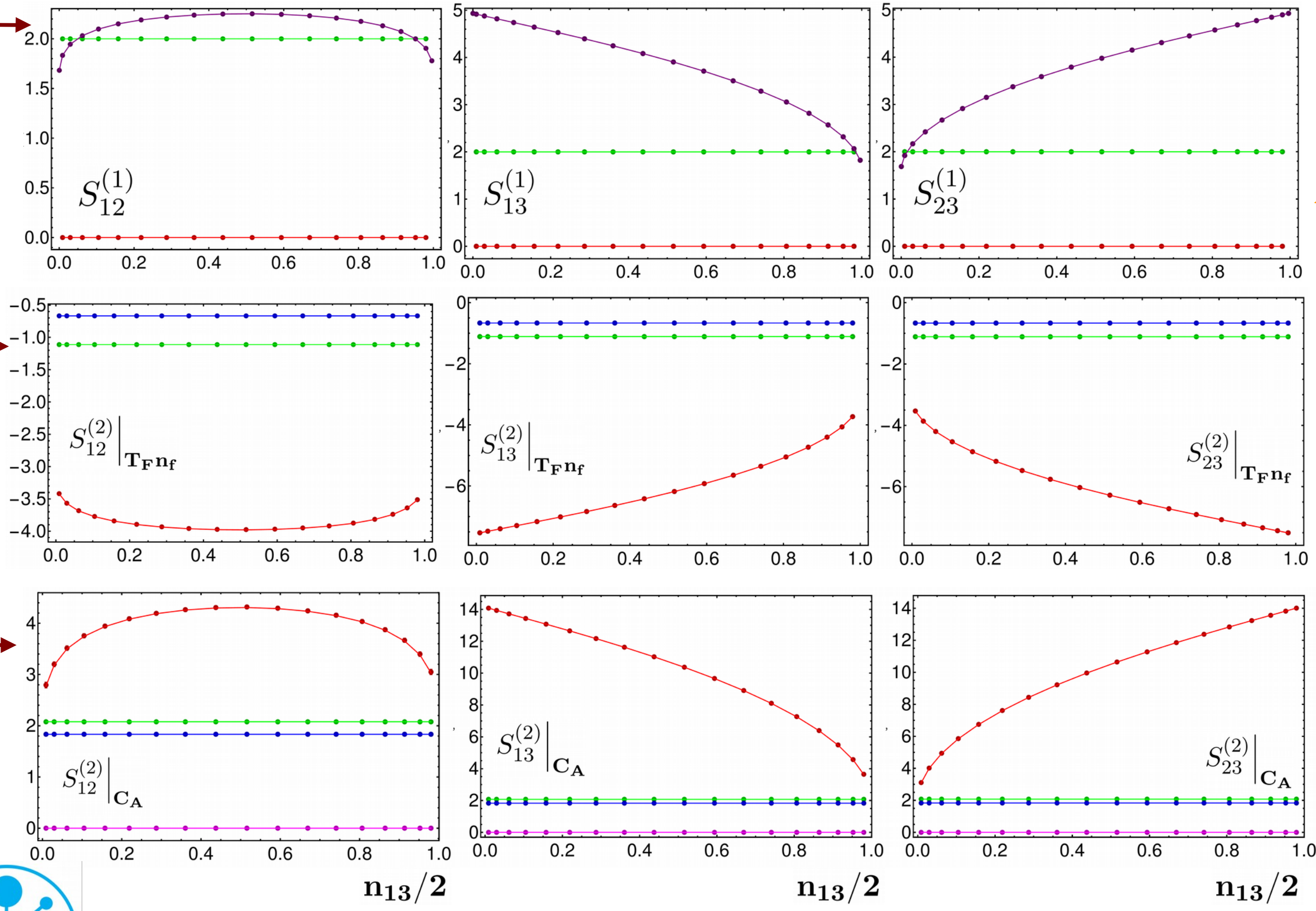
Relative Uncertainty < 0.01%

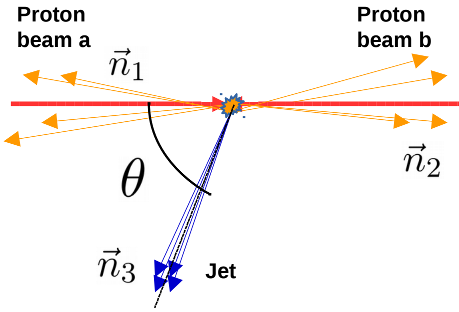
2-Loop $T_F\, n_f$ →

Relative Uncertainty < 0.1%

2-Loop C_A →

Relative Uncertainty < 0.1%





Our numerical results using ***pySecDec (VEGAS)*** *and* ***SoftSERVE (Divonne)*** *routines agree with each other. They* ***(dots)*** *also agree within the uncertainty with the known* ***results at NLO*** *and* ***divergent terms at NNLO (lines)****.*

Preliminary Results

Our predictions

One-jettiness in pp collision

$$S^{(1)}_{ab}(\epsilon) = \frac{\mathbf{C^1_{-2}}}{\epsilon^2} + \frac{\mathbf{C^1_{-1}}}{\epsilon} + \mathbf{I^1_{ab}} + \epsilon\, \mathbf{K^1_{ab}}$$

$$S^{(2)}_{ab}(\epsilon) = \left(\mathbf{T_F n_f}\left[\frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{T_F n_f}_{ab}}\right] + \mathbf{C_A}\left[\frac{\mathbf{C^2_{-4}}}{\epsilon^4} + \frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{C_A}_{ab}}\right]\right)$$

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Relative Uncertainty < 0.01%

2-Loop $T_F n_f$

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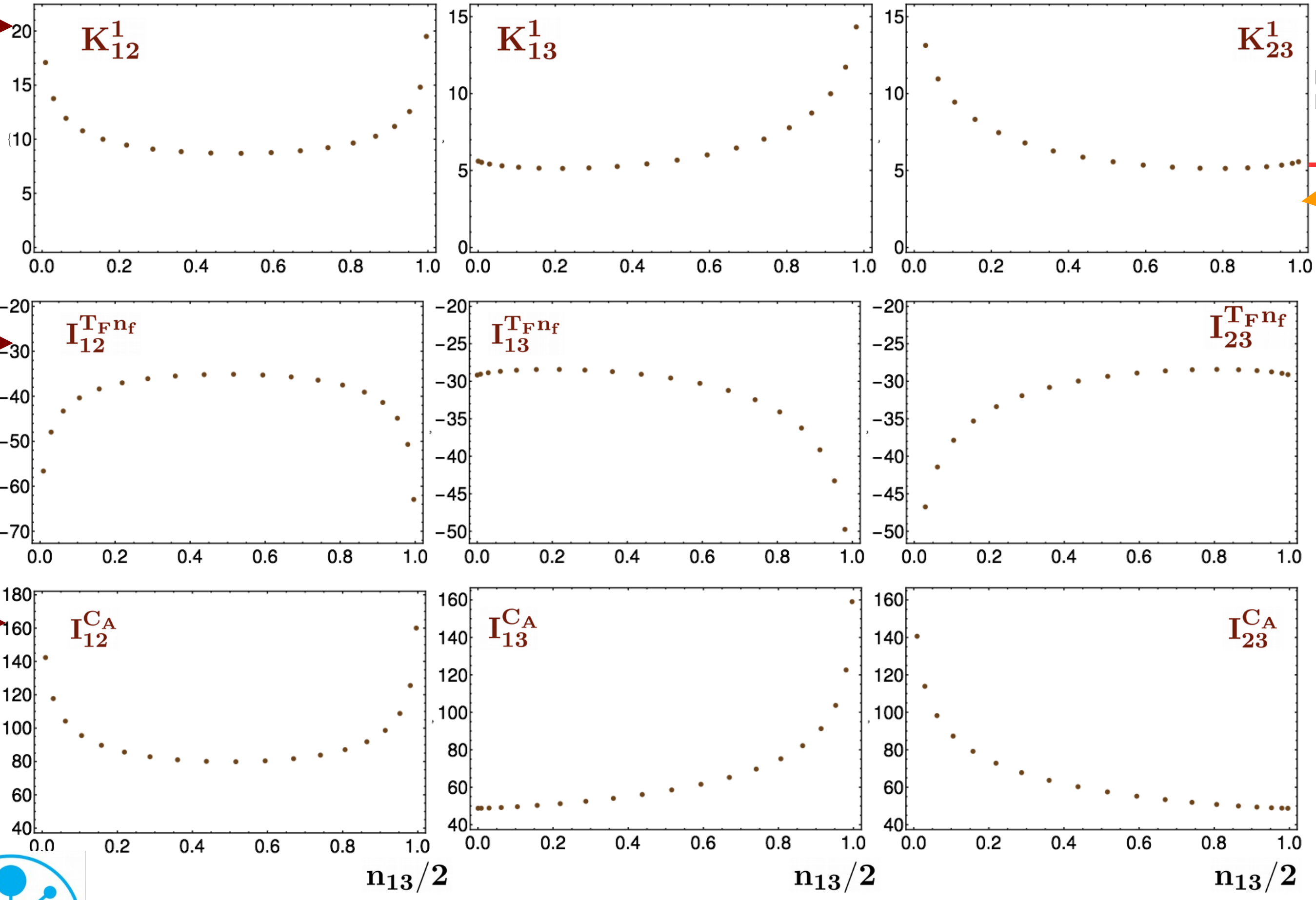
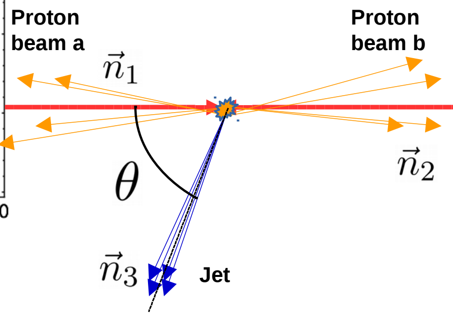




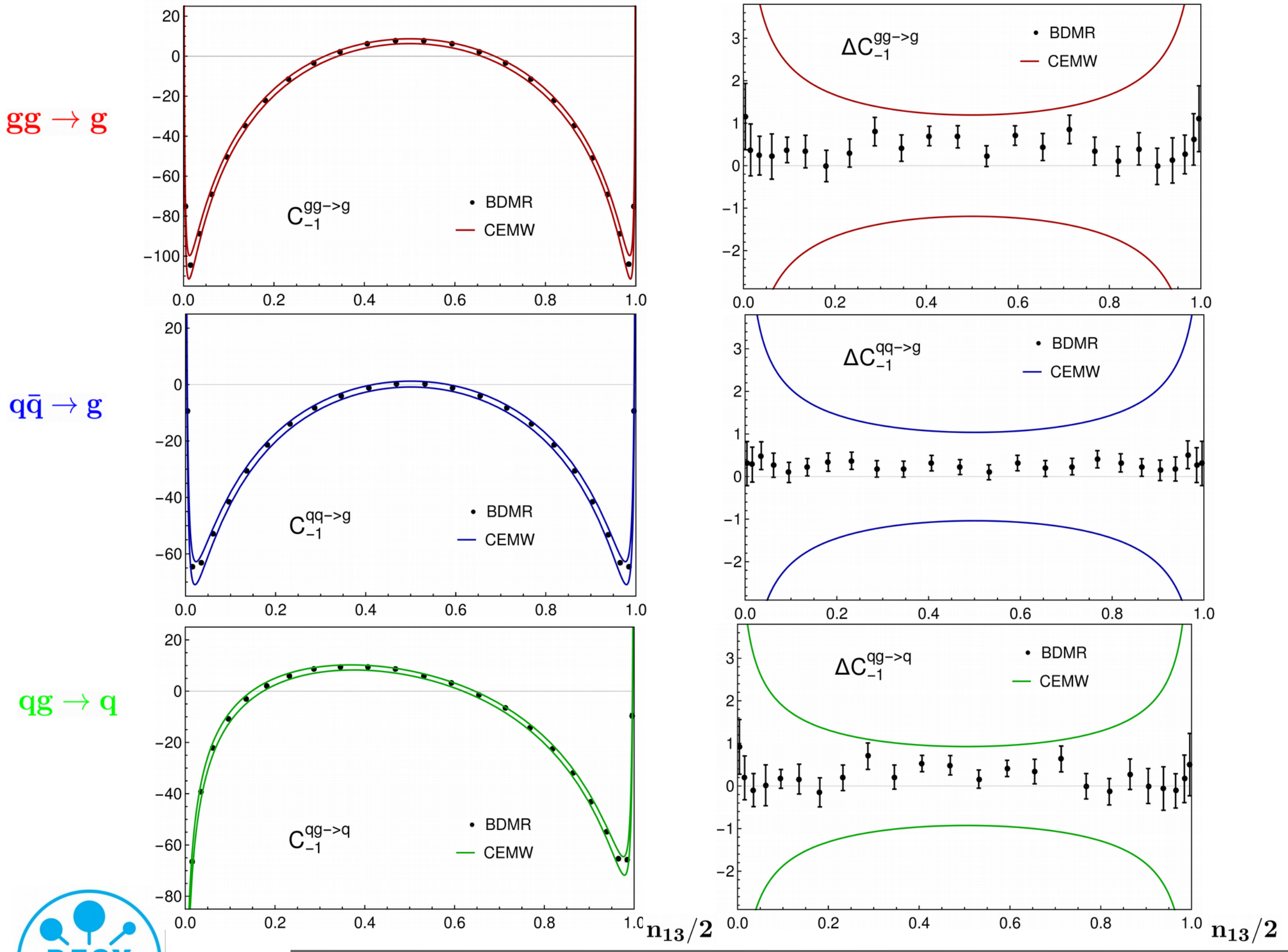
Preliminary Results

Numerical checks

One-jettiness in pp collision

Sum of the dipole contributions and **color factors** at NNLO for different partonic channels $gg \to g$, $q\bar{q} \to g$, $qg \to q$ in the distribution space (coefficients of $\delta(\mathcal{T}_1)$). **Our results (black dots)** vs. fit result in **Ref.[2] (lines)**

$$n_{12} = 2$$
$$n_{13} = 1 - \cos(\theta)$$
$$n_{23} = 2 - n_{13}$$



Ref. [2]: Campbell, Ellis, Mondini, Williams (2017)

Ref. [2] provides useful fits to their numerical results. However we could not reconstruct their uncertainties!

*Our numerical results from SoftSERVE with **Divonne** integrator*

Preliminary Results

Numerical checks

Two-jettiness in pp collision

$$S^{(1)}_{ab}(\epsilon) = \frac{\mathbf{C^1_{-2}}}{\epsilon^2} + \frac{\mathbf{C^1_{-1}}}{\epsilon} + \mathbf{I^1_{ab}} + \epsilon\,\mathbf{K^1_{ab}}$$

$$S^{(2)}_{ab}(\epsilon) = \left(\mathbf{T_F n_f}\left[\frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{T_F n_f}_{ab}}\right] + \mathbf{C_A}\left[\frac{\mathbf{C^2_{-4}}}{\epsilon^4} + \frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{C_A}_{ab}}\right]\right)$$

$n_{12} = 2$

$n_{13} = 1 - \cos(\theta)$

$n_{23} = 2 - n_{13}$

1-Loop

Relative Uncertainty < 0.01%

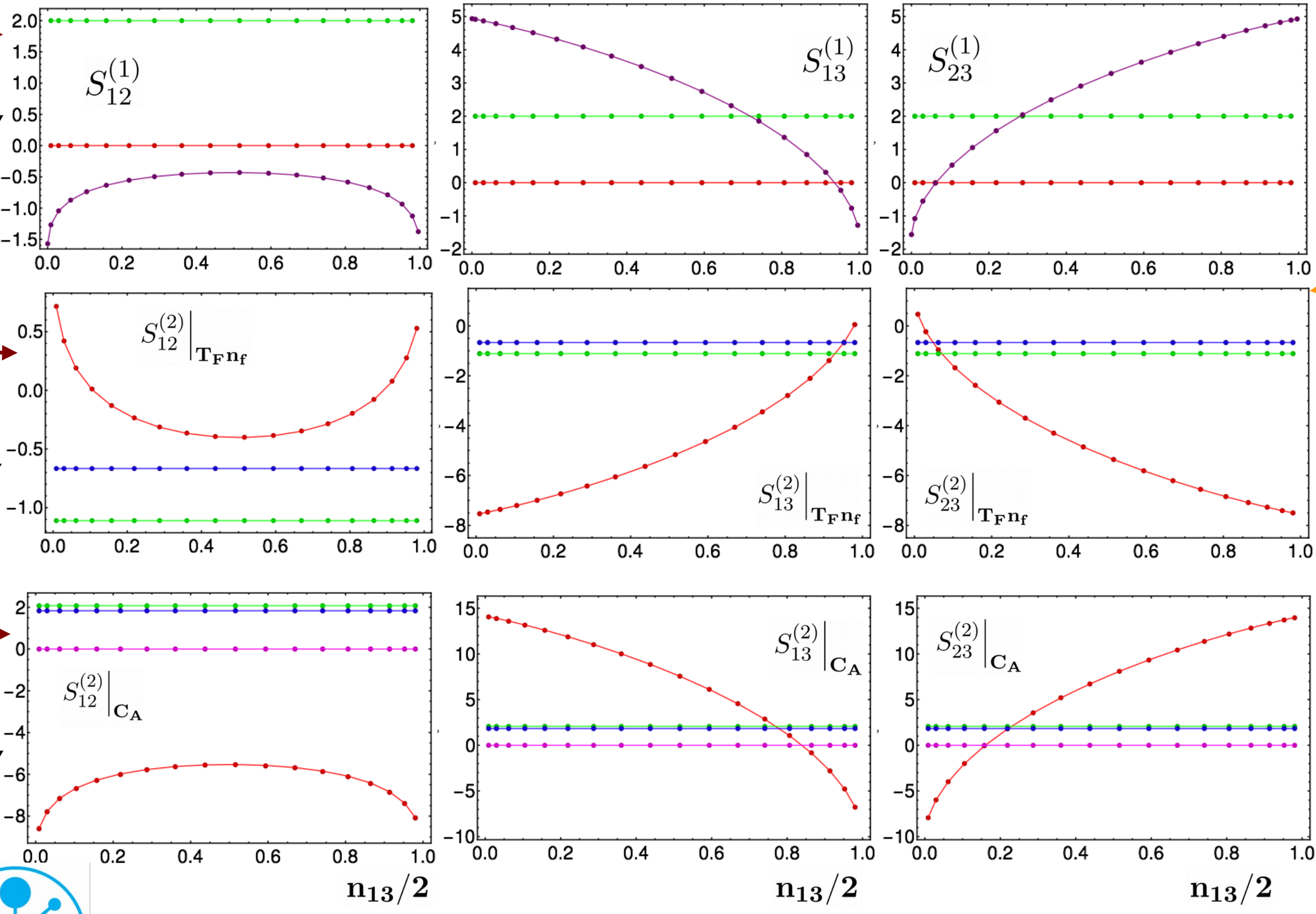
The first Purple line < 0.1%

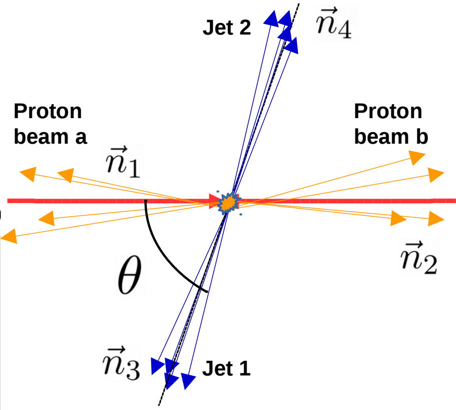
2-Loop $T_F n_f$

Relative Uncertainty < 0.1%

2-Loop C_A

Relative Uncertainty < 0.1%





*Our numerical results **(dots)** agree within the uncertainty with the known **results at NLO** and the **divergent terms at NNLO (lines)**.*

Preliminary Results

Our predictions

Two-jettiness in pp collision

$$S_{ab}^{(1)}(\epsilon) = \frac{\mathbf{C^1_{-2}}}{\epsilon^2} + \frac{\mathbf{C^1_{-1}}}{\epsilon} + \mathbf{I^1_{ab}} + \epsilon\,\mathbf{K^1_{ab}}$$

$$S_{ab}^{(2)}(\epsilon) = \left(\mathbf{T_F n_f}\left[\frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{T_F n_f}_{ab}}\right] + \mathbf{C_A}\left[\frac{\mathbf{C^2_{-4}}}{\epsilon^4} + \frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{C_A}_{ab}}\right]\right)$$

$n_{12} = 2$

$n_{13} = 1 - \cos(\theta)$

$n_{23} = 2 - n_{13}$

1-Loop →

Relative Uncertainty < 0.1%

2-Loop

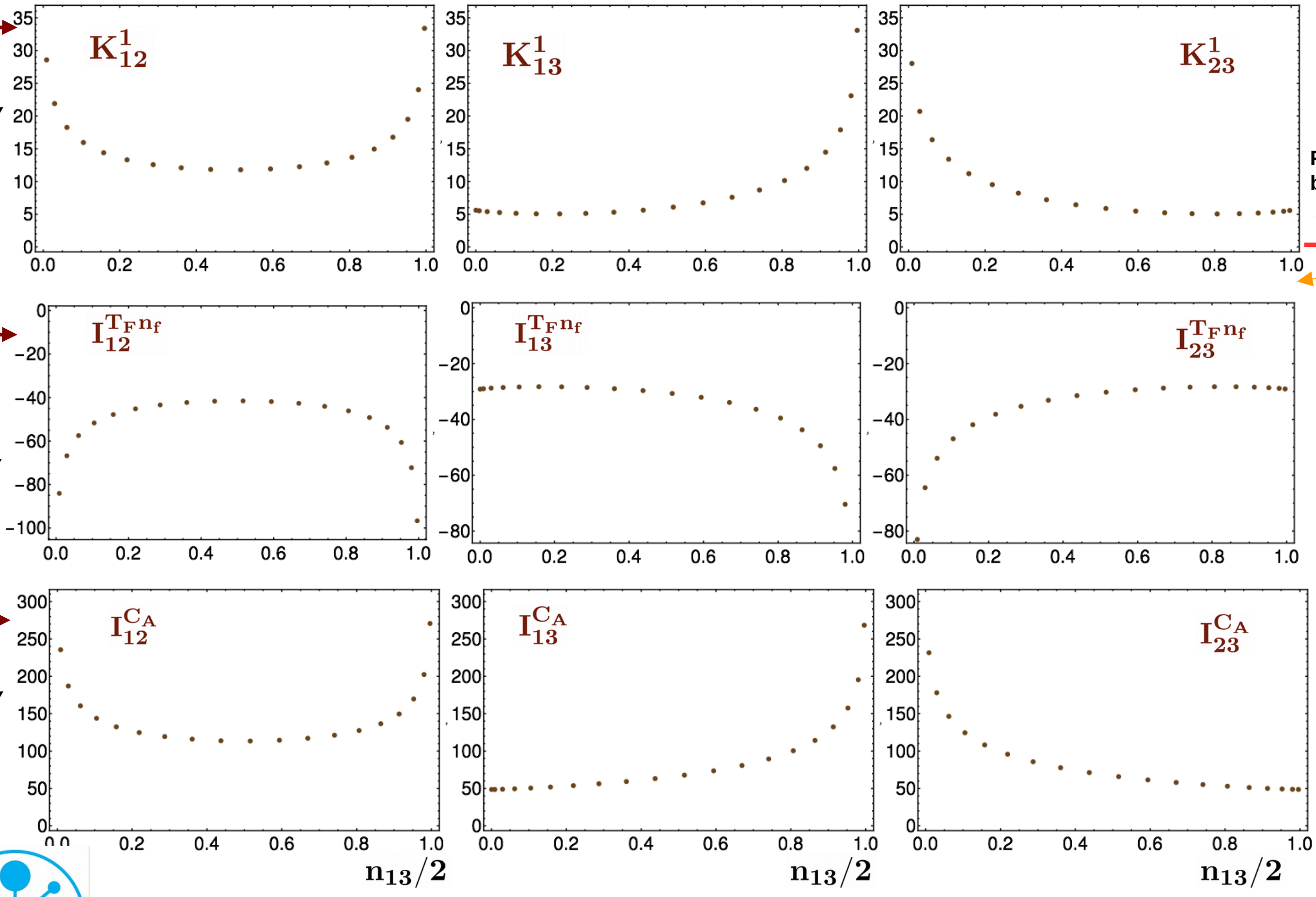
$T_F n_f$ →

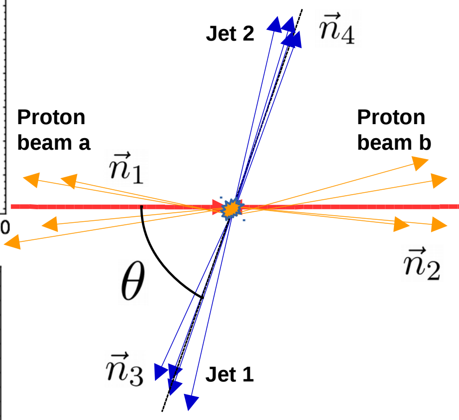
Relative Uncertainty < 0.1%

2-Loop

C_A →

Relative Uncertainty < 0.1%





Preliminary Results

Two-jettiness in pp collision

$$\sum_{a\neq b\neq c} f_{ABC}\, T_a^A\, T_b^B\, T_c^C\, S_{ab}^{(2,Im)}(\epsilon) = f_{ABC}\, T_1^A\, T_2^B\, T_3^C\, I^{(2,Im)}(\epsilon)$$

$$I^{(2,Im)}(\epsilon) = \frac{C_{-3}^{(2,Im)}}{\epsilon^3} + \frac{C_{-2}^{(2,Im)}}{\epsilon^2} + \frac{C_{-1}^{(2,Im)}}{\epsilon} + I_{ab}^{(2,Im)}$$

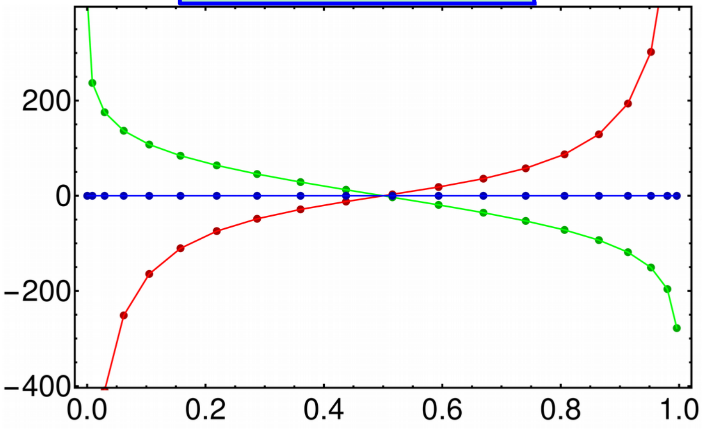
$n_{12} = 2$

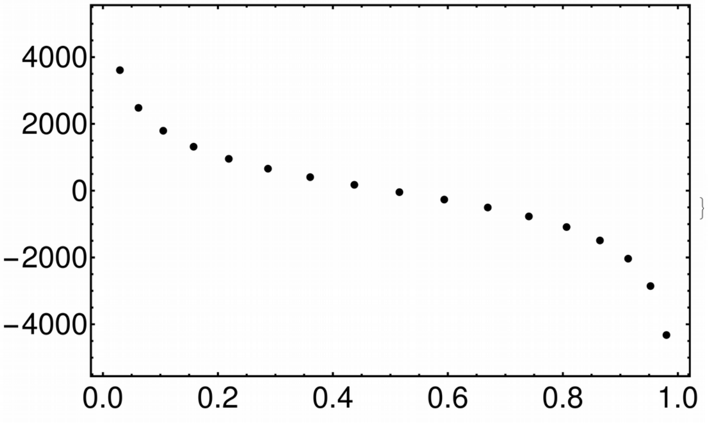
$n_{13} = 1 - \cos(\theta)$

$n_{23} = 2 - n_{13}$

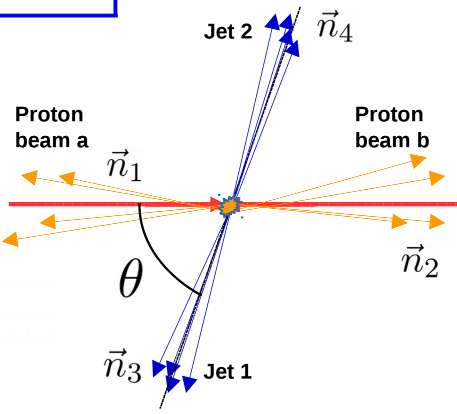
Numerical checks

Our predictions





Relative Uncertainty < 0.1%



- ✓ *Our numerical results* ***(dots)*** *agree within the uncertainty with the known* ***results at NLO*** *and the* ***divergent terms at NNLO (lines)****.*

Preliminary Results

Conclusions and outlook

Conclusions

✓ Systematic extension of our framework for automated calculations of N-jet soft functions

- First step assumes non-abelian exponentiation and SCET-1 type observable

✓ NNLO results

- Numerical results for 1-jettiness and 2-jettiness soft function
- N-jet implementation in SoftSERVE
- **Our calculation allows to extend the N-jettiness technique to processes with N-jets**

Outlook

➢ Other observables on the horizon (angularities, boosted-tops, hadronic event shapes, etc) (w.i.p)

- may trigger new ideas for subtraction techniques

Thank you for your attention!