NNLL Resummation for Transverse Thrust

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in collaboration with

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PRD 93 (2016) 054038 [079905(E)], arXiv:1512.00022

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- mostly used in lepton collisions
- precise extraction of α_s

- definitions for hadron collision exists as well
- use only momenta transverse to beam
- useful to study jet substructure and underlying event
- NLL resummation is available in automated framework `CAESAR`

[Banfi, Salam, Zanderighi]

Transverse Thrust

defined in analogy to thrust:

$$T_\perp = \max_{\vec{n}_\perp} \frac{\sum |\vec{p}_{m\perp} \cdot \vec{n}_\perp|}{\sum |\vec{p}_{m\perp}|} \qquad \tau_\perp = 1 - T_\perp$$

Goal: resum singular terms in dijet limit $\tau_\perp \to 0$ at NNLL using SCET

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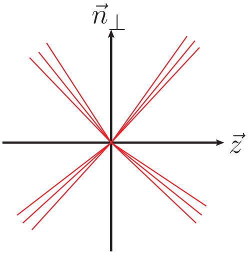
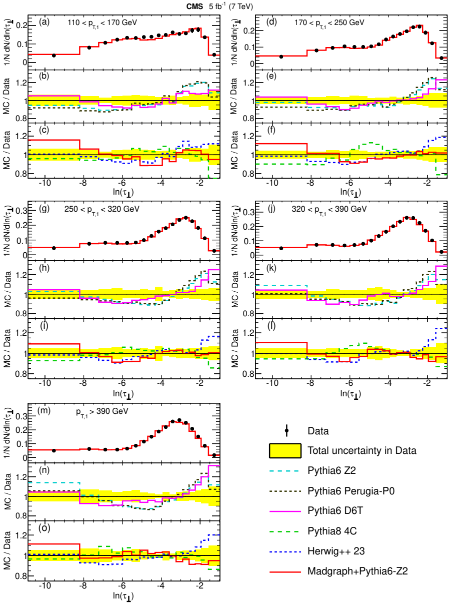
$\tau_\perp \to 0$ limit also contains non-singular configurations with all particles in the same plane

$\rightsquigarrow$ larger corrections when matching to fixed order

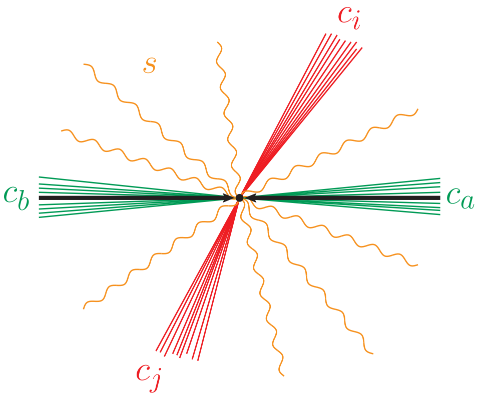


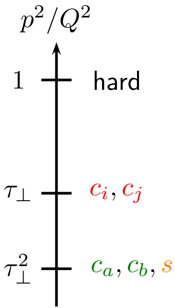
Transverse Thrust at LHC



[CMS]

consider transverse thrust in $pp \to 2$jets:

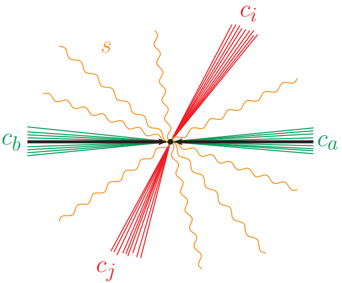




Factorisation Formula

partonic cross section $ab \to ij$ for recoil-free transverse event shape:

$$\tilde{t}(\kappa) \sim H_{IJ}^{ab\to ij} \left(\frac{Q^2}{\kappa^2}\right)^{-F^{ab\to ij}(\kappa)} \tilde{S}_{JI}^{ab\to ij}(\kappa)\tilde{B}_a(\kappa)\tilde{B}_b(\kappa)\tilde{J}_i(\kappa)\tilde{J}_j(\kappa)$$



(for transverse thrust: $F^{ab\to ij} = F^{ab}$)

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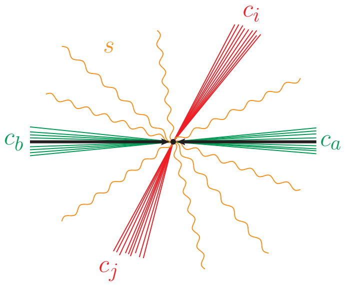

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needed for NNLL:

- 1-loop hard, soft, jet, and beam function
- 3-loop cusp anomalous dimension
- 2-loop anomalous dimensions
- 2-loop anomaly exponent

RG and Factorisation Constraints

$$\tilde{t}(\kappa) \sim H_{IJ}^{ab\to ij} \left(\frac{Q^2}{\kappa^2}\right)^{-F^{ab\to ij}(\kappa)} \tilde{S}_{JI}^{ab\to ij}(\kappa)\tilde{B}_a(\kappa)\tilde{B}_b(\kappa)\tilde{J}_i(\kappa)\tilde{J}_j(\kappa)$$

Factorisation requires:

$$F^{ab\to ij} = F^{ab} + F^{ij} = \frac{C_a + C_b}{2}\, F_\perp + \frac{C_i + C_j}{2}\, F'_\perp$$

RG invariance requires:

$$\gamma_{H^{ab\to ij}} + \gamma_{S^{ab\to ij}} + \gamma_{B_a} + \gamma_{B_b} + \gamma_{J_i} + \gamma_{J_j} \quad = \quad 0$$

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RG invariance requires:

$$\begin{aligned}
ab \to ij: &\quad \gamma_{H^{ab\to ij}} + \gamma_{S^{ab\to ij}} + \gamma_{B_a} + \gamma_{B_b} + \gamma_{J_i} + \gamma_{J_j} &= 0 \\
e^+e^- \to ij: &\quad \gamma_{H^{ij}} + \gamma_{S^{ij}} + \gamma_{J_i} + \gamma_{J_j} &= 0 \\
ab \to e^+e^-: &\quad \gamma_{H^{ab}} + \gamma_{S^{ab}} + \gamma_{B_a} + \gamma_{B_b} &= 0
\end{aligned}$$

consider $q\bar{q} \to e^+e^-$:

$$\gamma_{H^{q\bar{q}}} = -\gamma_{S^{q\bar{q}}} - \gamma_{B_q} - \gamma_{B_{\bar{q}}}$$

- hard anomalous dimension is known: $\gamma_{H^{q\bar{q}}} = 2\gamma^q$
- soft function is scaleless: $\gamma_{S^{q\bar{q}}} = 0$

$$\rightsquigarrow \; \gamma_{B_q} = \gamma^q$$

consider $gg \to H \to \gamma\gamma$ to obtain $\gamma_{B_g} = \gamma^g$

Anomalous Dimensions of Soft and Jet Function

consider $e^+e^- \to q\bar{q}$ and $e^+e^- \to gg$:

$$\gamma_{H^{ij}} = -\gamma_{S^{ij}} - \gamma_{J_i} - \gamma_{J_j}$$

for $F^{ij} = 0$:

- $\gamma_{H^{q\bar{q}}} = 2\gamma^q$, $\gamma_{H^{gg}} = 2\gamma^g$
- $\rightsquigarrow \gamma_{J_q} = -\gamma^q - \frac{1}{2}\gamma_{S^{q\bar{q}}}$, $\gamma_{J_g} = -\gamma^g - \frac{1}{2}\gamma_{S^{gg}}$
- extract $\gamma_{S^{q\bar{q}}}$ from fixed order result, e.g. `EVENT2` [Catani, Seymour]
- $\gamma_{S^{gg}}$ is related to $\gamma_{S^{q\bar{q}}}$ by Casimir scaling (at two loops)
- $\gamma_{S^{ab\to ij}}$ for other channels is determined by RG invariance

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for $F^{ij} = 0$:

- $\gamma_{H^{q\bar{q}}} = 2\gamma^q$, $\gamma_{H^{gg}} = 2\gamma^g$
- $\rightsquigarrow \gamma_{J_q} = -\gamma^q - \frac{1}{2}\gamma_{S^{q\bar{q}}}$, $\gamma_{J_g} = -\gamma^g - \frac{1}{2}\gamma_{S^{gg}}$
- compute $\gamma_{S^{q\bar{q}}}$ with `SoftSERVE`, see talk by Rudi Rahn
- $\gamma_{S^{gg}}$ is related to $\gamma_{S^{q\bar{q}}}$ by Casimir scaling (at two loops)
- $\gamma_{S^{ab\to ij}}$ for other channels is determined by RG invariance

for $F^{ij} \neq 0$ soft and jet anomalous dimension are not needed separately

Anomaly Coefficient

- define observable with known anomaly that
  - agrees with $\tau_\perp$ for one emission
  - differs from $\tau_\perp$ for two or more emissions
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$\tau_\perp$ in $q\bar{q} \to e^+e^-$:

$$\mathcal{T}_\perp = \sum \left|\vec{p}_{m\perp}\right| - \sum \left|\vec{p}_{m\perp} \cdot \vec{n}_\perp\right|$$

$$\mathcal{S}_\perp = \left|\sum \vec{p}_{m\perp}\right| - \left|\sum \vec{p}_{m\perp} \cdot \vec{n}_\perp\right|$$

- compute $\mathcal{S}_\perp$ from known Drell-Yan results [Becher, Neubert]
- $d_2^{\mathsf{DY}} - d_2^{\perp}$ is determined by rapidity divergences of soft function

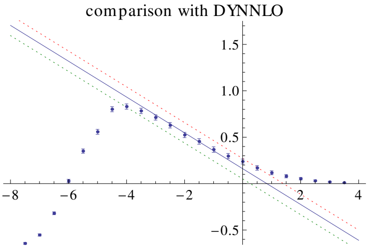
Anomaly Coefficient

$$F_\perp^{q\bar{q}}(L_\perp) = \frac{\alpha_s}{4\pi} C_F \Gamma_0 L_\perp + \left(\frac{\alpha_s}{4\pi}\right)^2 C_F \left(\Gamma_0 \beta_0 \frac{L_\perp^2}{2} + \Gamma_1 L_\perp + d_2^\perp\right)$$

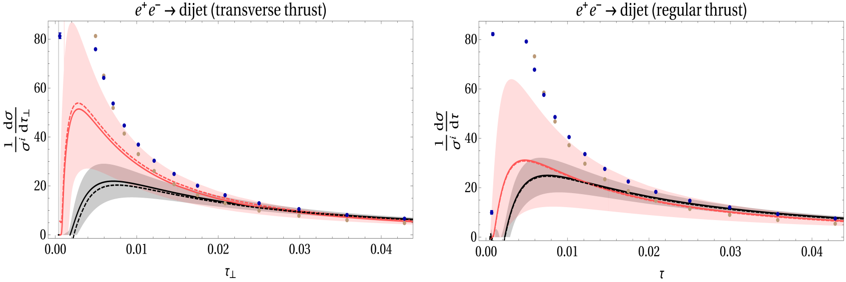
from numerical evaluation of tree-level two-emission soft amplitude:

$$d_2^\perp = (208.0 \pm 0.1)\, C_A + (-37.191 \pm 0.006)\, T_F n_f$$

- agrees with `SoftSERVE` [Bell, Rahn, Talbert]
- agrees with DYNNLO [Grazzini]



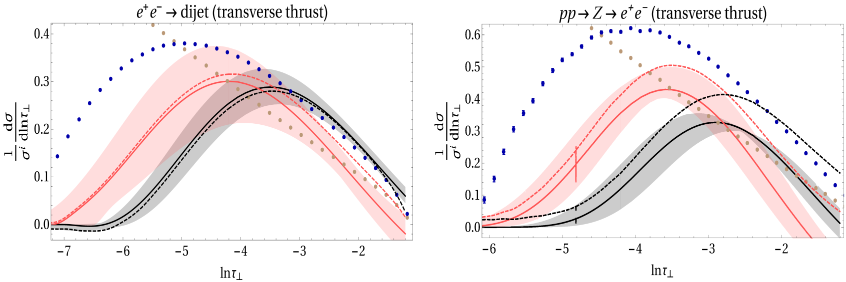
Results: Transverse vs. Regular Thrust



NLL NNLL $\mathcal{O}(\alpha_s)$ fixed order $\mathcal{O}(\alpha_s^2)$ fixed order

$$\mu_{\text{soft}} = 4M_Z\tau_\perp,\ \mu_{\text{jet}} = 2M_Z\sqrt{\tau_\perp}$$

Results: Transverse Thrust



NLL NNLL $\mathcal{O}(\alpha_s)$ fixed order $\mathcal{O}(\alpha_s^2)$ fixed order

$$\mu_{\text{soft}} = 4M_Z\tau_\perp,\ \mu_{\text{jet}} = 2M_Z\sqrt{\tau_\perp},\ \mu_{\text{beam}} = 2e^{4G/\pi}M_Z\tau_\perp$$

MSTW 2008 NNLO PDFs, $\alpha_S(M_Z) = 0.11707$

Summary

- we determined all ingredients for NNLL resummation of transverse thrust by exploiting universality properties of factorised cross section
- we find large corrections at NNLL
- transverse thrust in $pp \to e^+e^-$ could be used as a probe of underlying event
- the method can be applied to other observables and can be extended to an automated resummation framework

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still missing:

- full implementation for hadronic processes
- Glauber gluons?