Nonsingular bouncing cosmology from general relativity

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Also for experimentalists and engineers . . .

Outline

1. Standard cosmology
2. Bouncing cosmology
3. Discussion

General Relativity

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The basic idea of general relativity is that spacetime is curved and that the spacetime curvature is determined by the matter content. The Einstein equation reads:

$$\begin{pmatrix} \text{spacetime curvature} \\ \text{tensor} \end{pmatrix} = G_N \begin{pmatrix} \text{energy-momentum} \\ \text{tensor} \end{pmatrix}, \qquad (1)$$

with G_N Newton's gravitational coupling constant.

We will use relativistic units, so that $c = 1$.

Standard Friedmann equations

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For a spatially-flat homogeneous and isotropic universe filled with a perfect fluid, Einstein's equation reduces to the Friedmann equations

$$\left(\frac{1}{a(t)}\,\frac{da(t)}{dt}\right)^2 = \frac{8\pi G_N}{3}\rho\,, \tag{2a}$$

$$\frac{d}{da}(\rho a^3) + 3Pa^2 = 0\,, \tag{2b}$$

$$P = P(\rho)\,, \tag{2c}$$

with $a(t)$ the cosmic scale factor. With radiation ($P = \rho/3$), the solution is:

$$a(t) \propto \sqrt{t}\,, \tag{3}$$

for $t > 0$. The scale factor $a \to 0$ when $t \to 0^+$, which corresponds to the big bang singularity with **divergent** physical quantities as $t \to 0^+$.

Standard Hubble diagram

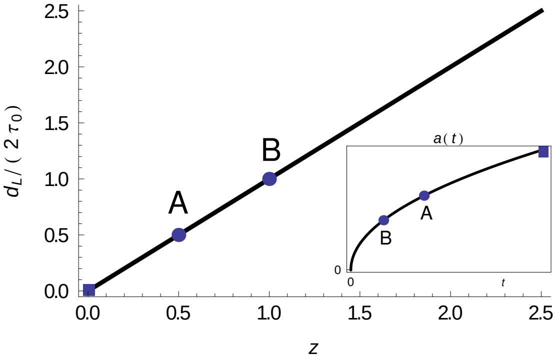

Figure 1: The main graph shows luminosity distance d_L versus redshift z for the standard radiation-dominated Friedmann universe. Two signals (A and B) are marked in the main graph, with corresponding points in the inset. Recall that $z + 1 = a(t_{\text{observed}})/a(t_{\text{emitted}})$.

Modified Friedmann equations

From a new metric *Ansatz*, Prof. Klinkhamer [1] obtained the following modified Friedmann equations:

$$\left(1+\frac{b^2}{T^2}\right)\left(\frac{1}{a(T)}\,\frac{da(T)}{dT}\right)^2 = \frac{8\pi G_N}{3}\,\rho\,, \tag{4a}$$

$$\frac{d}{da}\left(\rho a^3\right)+3Pa^2 = 0\,, \tag{4b}$$

$$P = P(\rho)\,, \tag{4c}$$

where $b>0$ is a length scale. With radiation ($P=\rho/3$), the solution is:

$$a(T)\propto\sqrt[4]{\left(b^2+T^2\right)}\,, \tag{5}$$

for $T\in(-\infty,+\infty)$. With $b\neq 0$: **finite** physical quantities at $T=0$.

[1] F. R. Klinkhamer, “Regularized big bang singularity,” Phys. Rev. D **100**, 023536 (2019), arXiv:1903.10450.

Nonsingular bounce

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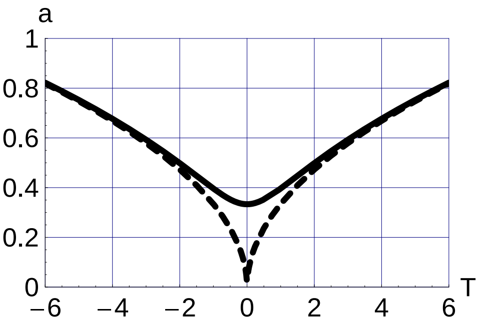

Figure 2: Dashed curve: scale factor solution from the standard Friedmann equations. Full curve: scale factor solution from the modified Friedmann equations.

Modified Hubble diagram [2]

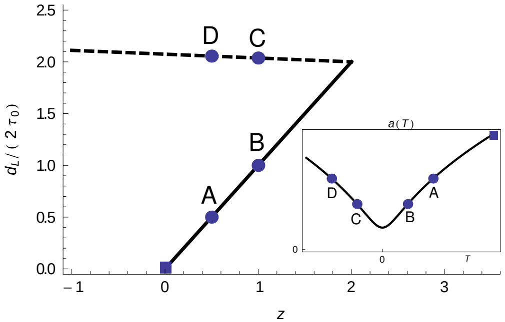

Figure 3: The main graph shows luminosity distance d_L versus redshift z for the radiation-dominated bouncing universe obtained from the modified Friedmann equations. The dashed curve corresponds to signals from the contracting phase. Four signals (A, B, C, and D) are marked in the main graph, with corresponding points in the inset.

[2] F. R. Klinkhamer and Z. L. Wang, "Nonsingular bouncing cosmology from general relativity," Phys. Rev. D **100**, 083534 (2019), arXiv:1904.09961.

Cosmological perturbations

Recently [3], we have also shown that:

- the nonsingluar bounce is stable under small perturbations of the metric and the matter;
- a small-amplitude gravitational wave from tensor perturbations in the pre-bounce phase can pass across the bounce into the post-bounce phase.

[3] F. R. Klinkhamer and Z. L. Wang,"Nonsingular bouncing cosmology from general relativity: Scalar metric perturbations," arXiv:1911.06173.

Discussion

Most of the bouncing models in the scientific literature [4] require a **violation of the Null Energy Condition** (NEC) [5].

The bounce model discussed here is based on the assumption that a **spacetime defect** (with length parameter $b \neq 0$) exists at the bounce. The physical origin of b is still an open question.

Thanks for your attention!

[4] M. Novello and S. E. P. Bergliaffa, "Bouncing Cosmologies," Phys. Rept. **463**,127 (2008), arXiv:0802.1634.

[5] For a perfect fluid, NEC $\Leftrightarrow \rho + P \geq 0$,

Backup – Metric *Ansatz*

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To get the modified Friedmann equation (4a), the following metric *Ansatz* [1] is used:

$$ds^2 = -\frac{T^2}{b^2+T^2}\, dT^2 + a^2(T)\, \delta_{kl}\, dx^k\, dx^l\,, \tag{6a}$$

$$b > 0\,, \tag{6b}$$

$$T \in (-\infty,\, \infty)\,, \tag{6c}$$

$$x^k \in (-\infty,\, \infty)\,. \tag{6d}$$

Observe that the metric from (6a) is **degenerate**: $\det\, g_{\mu\nu} = 0$ at $T = 0$.

[1] F. R. Klinkhamer, “Regularized big bang singularity,” Phys. Rev. D **100**, 023536 (2019), arXiv:1903.10450.

Backup – Spacetime defect

With the coordinate transformation

$$\tau(T) = \begin{cases} +\sqrt{b^2 + T^2}\,, & \text{for } T \geq 0\,, \\ -\sqrt{b^2 + T^2}\,, & \text{for } T \leq 0\,, \end{cases} \tag{7}$$

the metric (6a) takes the standard spatially-flat Robertson-Walker form,

$$d\tilde{s}^2 = -d\tau^2 + \tilde{a}^2(\tau)\, \delta_{kl}\, dx^k\, dx^l\,, \tag{8}$$

where $\tau \in (-\infty,\, -b] \cup [b,\, \infty)$.

The corresponding $T = 0\, (\tau = \pm b)$ slice may be interpreted as a 3-dimensional "defect" of spacetime with topology $\mathbb{R}^3$.

The differential structure of metric (8) is different from the one of (6a), because the coordinate transformation (7) is not a diffeomorphism.