TRR 257 - Particle Physics Phenomenology
after the Higgs Discovery

Project B1a:
Production of colour-singlet final states through N3LO QCD

Principal investigators:

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Annual Meeting of the SFB TRR 257, Siegen, 6th October 2020

Original project goals

- **Describe the production of single-particle colour-singlet final states at the LHC (H,W,Z) at N3LO in perturbative QCD**
- Higgs boson physics: understand the N3LO QCD corrections to fiducial volume cross sections defined with kinematic cuts on the Higgs boson decay products and with constraints on the QCD radiation required to suppress the backgrounds.
- Z and W bosons: get access to N3LO predictions for lepton transverse momentum distributions and charge asymmetries that are key observables for understanding e.g. the mass of the W-boson and improve on the PDF fits.

Les Houches 2019: Physics at TeV Colliders
Standard Model Working Group Report

arXiv:2003.01700 [hep-ph]

Conveners

Higgs physics: SM issues
D. de Florian (Theory), M. Donegà (CMS), M. Dührssen-Debling (ATLAS), S. Jones (Theory)

SM: Loops and Multilegs
J. Bendavid (CMS), A. Huss (Theory), J. Huston (ATLAS), S. Kallweit (Theory), D. Maître (Theory), S. Marzani (Jets contact), B. Nachman (Jets contact, ATLAS)

Tools and Monte Carlos
V. Ciulli (CMS), S. Prestel (Theory), E. Re (Theory)

process	known	desired
$pp \to H$	$\mathrm{N^3LO_{HTL}}$ (incl.) $\mathrm{N^{(1,1)}LO^{(HTL)}_{QCD\otimes EW}}$ $\mathrm{NNLO_{HTL} \otimes NLO_{QCD}}$	$\mathrm{N^3LO_{HTL}}$ (partial results available) $\mathrm{NNLO_{QCD}}$
$pp \to H + j$	$\mathrm{NNLO_{HTL}}$ $\mathrm{NLO_{QCD}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$
$pp \to H + 2j$	$\mathrm{NLO_{HTL} \otimes LO_{QCD}}$ $\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ (incl.) $\mathrm{NNLO^{(VBF^*)}_{QCD}}$ $\mathrm{NLO^{(VBF)}_{EW}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$ $\mathrm{NNLO^{(VBF)}_{QCD} + NLO^{(VBF)}_{EW}}$
$pp \to H + 3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO^{(VBF)}_{QCD}}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$
$pp \to H + V$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	$\mathrm{NLO^{(t,b)}_{gg\to HZ}}$
$pp \to HH$	$\mathrm{N^3LO_{HTL} \otimes NLO_{QCD}}$	$\mathrm{NLO_{EW}}$
$pp \to H + t\bar{t}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to H + t/\bar{t}$	$\mathrm{NLO_{QCD}}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$

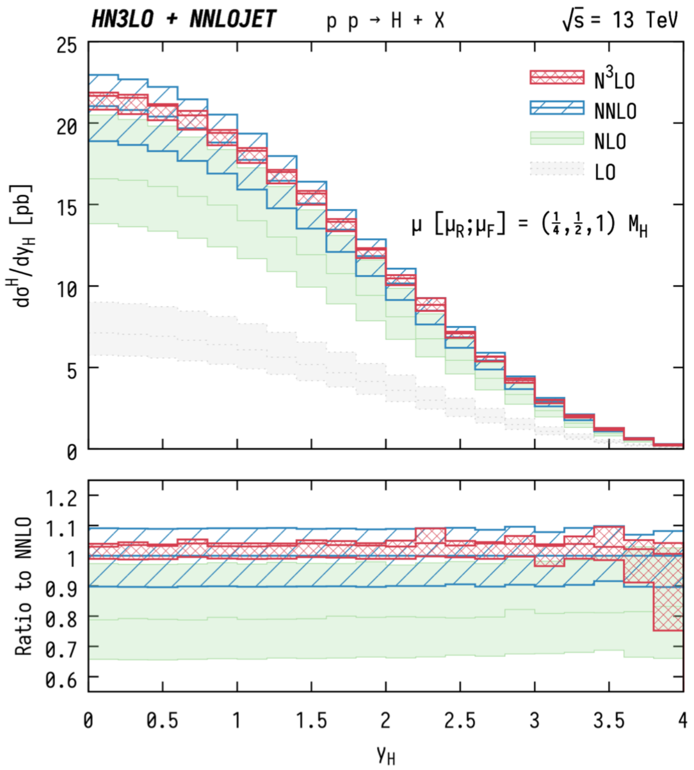
process	known	desired
$pp \to V$	$\mathrm{N^3LO^{(z\to 0)}_{QCD}}$ (incl.) $\mathrm{N^3LO_{QCD}}$ (incl., γ^*) $\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^3LO_{QCD} + N^2LO_{EW} + N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ $+\,\mathrm{NLO_{QCD}}$ (gg channel)	$\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops)
$pp \to V + j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	hadronic decays
$pp \to V + 2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to V + b\bar{b}$	$\mathrm{NLO_{QCD}}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$
$pp \to VV' + 1j$	$\mathrm{NLO_{QCD}}$ $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$
$pp \to VV' + 2j$	$\mathrm{NLO_{QCD}}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$
$pp \to W^+W^+ + 2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to W^+Z + 2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to VV'V''$	$\mathrm{NLO_{QCD}}$ $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$
$pp \to W^\pm W^+ W^-$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to \gamma\gamma$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	
$pp \to \gamma + j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	
$pp \to \gamma\gamma + j$	$\mathrm{NLO_{QCD}}$ $\mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$
$pp \to \gamma\gamma\gamma$	$\mathrm{NNLO_{QCD}}$	

Higgs at N3LO

Higgs boson production at the LHC using the q_T subtraction formalism at N^{3}LO QCD

Cieri, Chen, Gehrmann, Glover, Huss `18

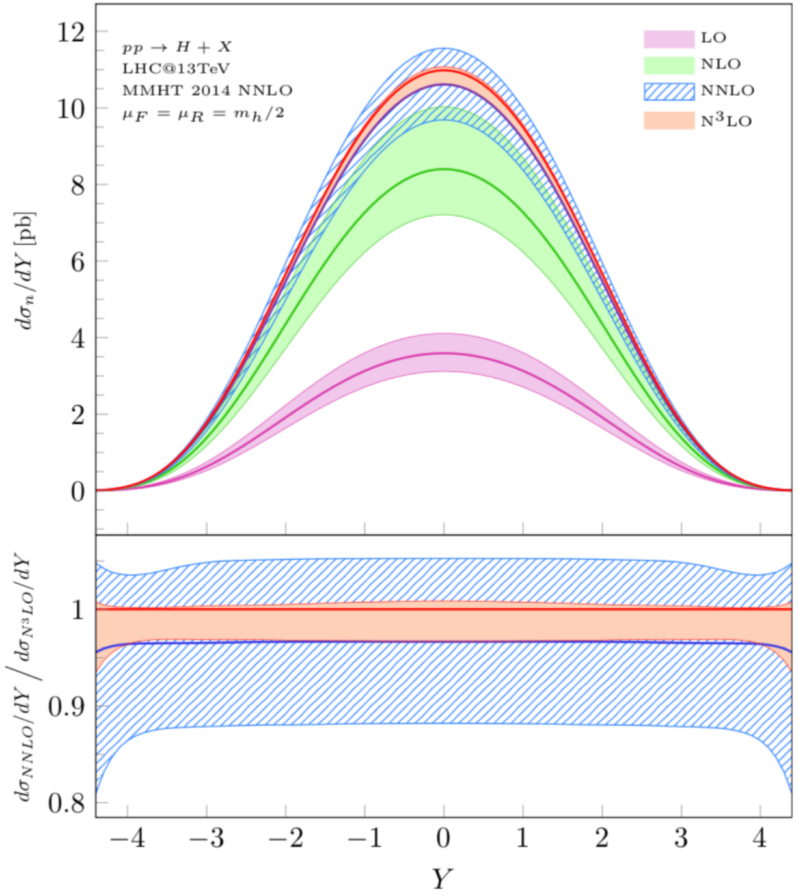
" The missing third-order collinear functions, which contribute only at $q_T = 0$, are approximated using a prescription which uses the known result for the total Higgs boson cross section at this order. "



Precision Predictions at N^{3}LO for the Higgs Boson Rapidity Distribution at the LHC

Dulat, Mistlberger, Pelloni `18

" Our approach relies on the fully analytic computation of six terms in a systematic expansion of the partonic differential cross section around the production threshold of the Higgs boson at next-to-next-to-next-to leading order (N^{3}LO) in QCD perturbation theory. "

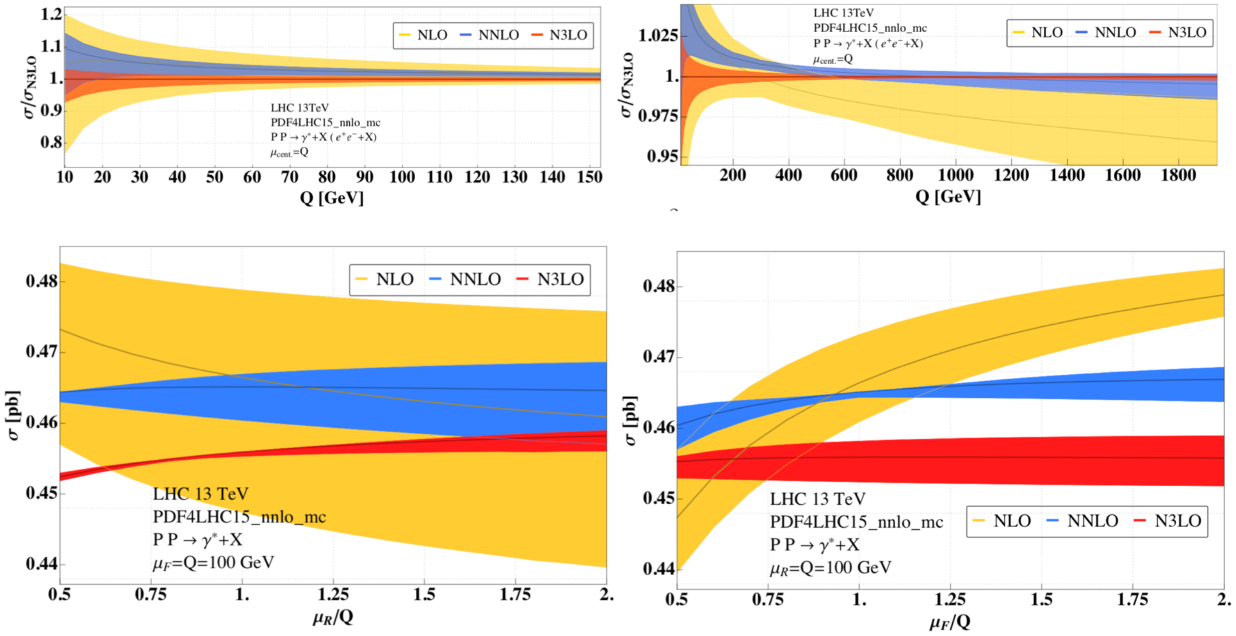


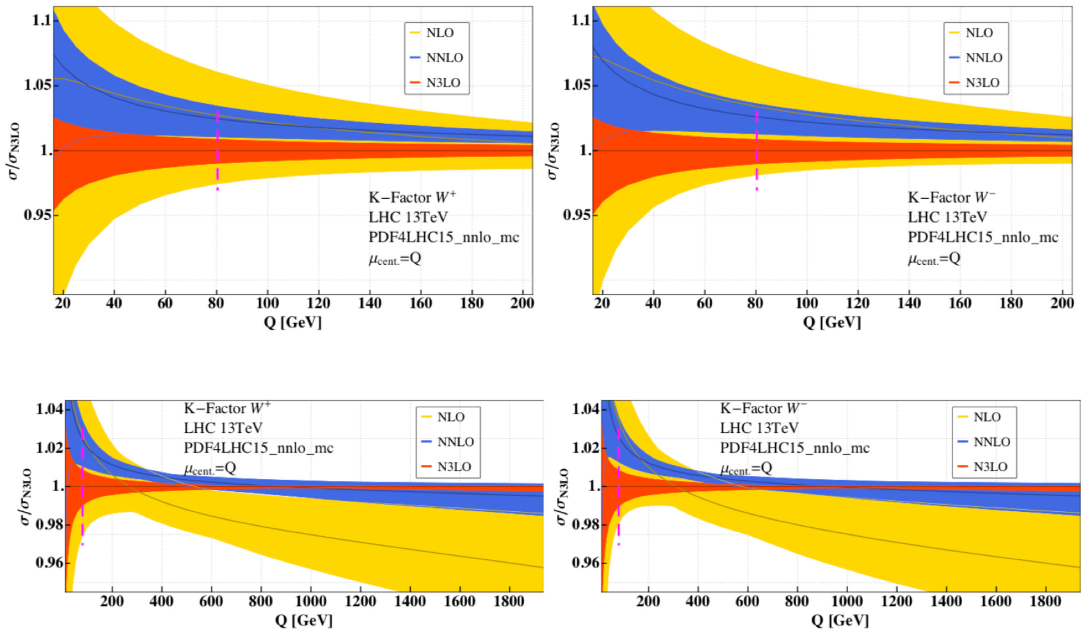
Drell-Yan at N3LO

The Drell-Yan cross section to third order in the strong coupling constant

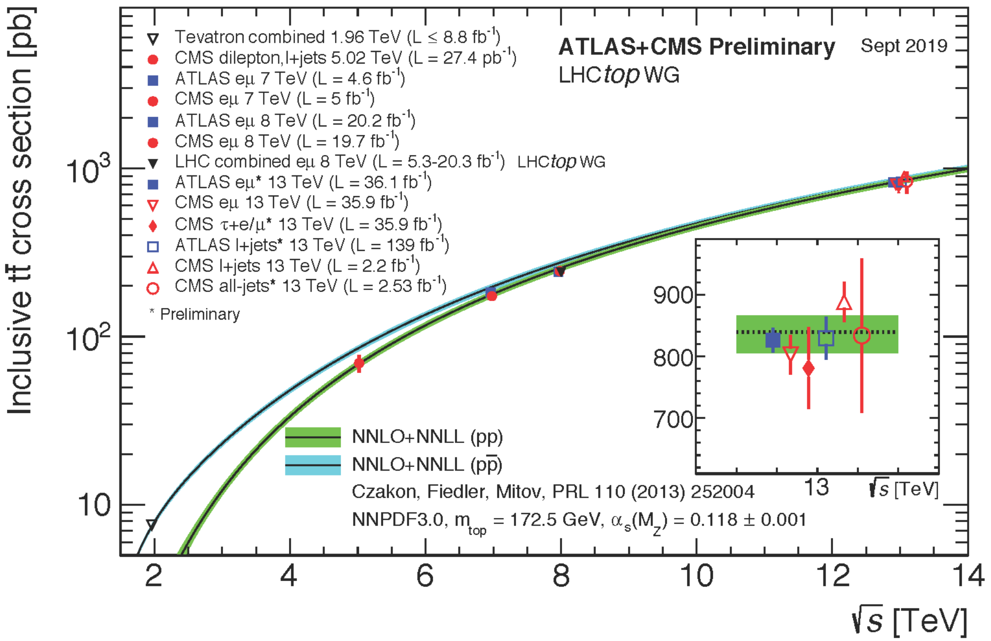
Charged Current Drell-Yan Production at N^{3}LO

Dulat, Mistlberger, Pelloni `20





Beyond colour-singlet final states



13th International Workshop on Top-Quark Physics (TOP2020)

14-18 September 2020

Total cross section in top production

Measurements by ATLAS and CMS

Olga Bessidskaia Bylund

$t\bar{t}$ dilepton, ATLAS (36.1 fb^{-1}, 13 TeV)

Results

- $\sigma^{\mathrm{fid}}_{t\bar{t}}$=14.07±0.06(stat.)±0.18(syst.)±0.27(lumi.)±0.03(beam)pb.
- $\sigma_{t\bar{t}}$=826.4±3.6(stat.)±11.5(syst.)±15.7(lumi.)±1.9(beam)pb.
- $\sigma^{theo.}_{t\bar{t}}$=832±35(pdf+$\alpha_s$)$^{+20}_{-29}$(scale)pb,(NNLO+NNLL, Top++).
- $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} = 2.4\%$, most precise inclusive measurement of $t\bar{t}$.

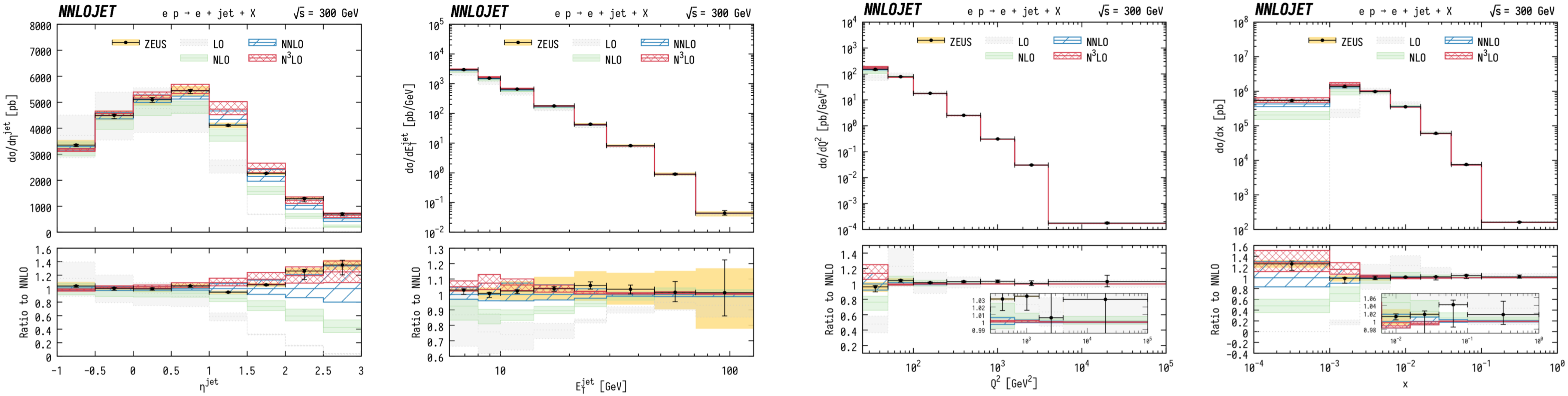
Ratios

- PDF uncertainties reduce in $\sigma^{t\bar{t}}_{13\ \mathrm{TeV}}/\sigma^{t\bar{t}}_{7\ (8)\ \mathrm{TeV}}$.
- Improvement for 7 (8) TeV: 4.9 → 3.9% (4.7 → 3.6%).

Beyond hadron colliders

N^3LO Corrections to Jet Production in Deep Inelastic Scattering using the Projection-to-Born Method

Currie, Gehrmann, Glover, Huss, Niehues, Vogt `18



Jet production in charged-current deep-inelastic scattering to third order in QCD

Gehrmann, Huss, Niehues, Vogt, Walker `18

How do we want to evaluate the cross sections?

Slicing and subtraction

slicing parameter

Slicing:

$$\int_0^1 \mathrm{d}x\, \frac{f(x)}{x^{1+\epsilon}} \approx \int_0^\delta \mathrm{d}x\, \frac{f(0)}{x^{1+\epsilon}} + \int_\delta^1 \mathrm{d}x\, \frac{f(x)}{x} + \mathcal{O}(\epsilon)$$

$$\approx \left(-\frac{1}{\epsilon} + \ln\delta\right) f(0) + \int_\delta^1 \mathrm{d}x\, \frac{f(x)}{x} + \mathcal{O}(\epsilon)$$

large cancellations

Subtraction:

$$\int_0^1 \mathrm{d}x\, \frac{f(x)}{x^{1+\epsilon}} = \int_0^1 \mathrm{d}x\, \frac{f(0)}{x^{1+\epsilon}} + \int_0^1 \mathrm{d}x\, \frac{f(x)-f(0)}{x} + \mathcal{O}(\epsilon)$$

$$= -\frac{1}{\epsilon} f(0) + \int_0^1 \mathrm{d}x\, \frac{f(x)-f(0)}{x} + \mathcal{O}(\epsilon)$$

complicated integrand

Hybrid of the methods

A possible approach: q_T slicing, Catani, Grazzini `07

At NNLO implemented in the public codes MCFM and MATRIX

Approach chosen in Aachen

$$p + p \to F(q_T) + X$$

$$\sigma^F_{N^mLO} = \int_0^{q_{T,cut}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^F_{N^mLO}}{\mathrm{d}q_T} + \int_{q_{T,cut}}^{\infty} \mathrm{d}q_T \, \frac{\sigma^F_{N^mLO}}{\mathrm{d}q_T}$$

$$= \int_0^{q_{T,cut}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^F_{N^mLO}}{\mathrm{d}q_T} + \int_{q_{T,cut}}^{\infty} \mathrm{d}q_T \, \frac{\sigma^{F+jet}_{N^{m-1}LO}}{\mathrm{d}q_T}$$

Interesting because related to factorization and resummation

manageable because lower order

Another method: N-jettiness slicing, Gaunt, Stahlhofen, Tackmann, Walsh ’15, Boughezal, Focke, Liu, Petriello ’15

At NNLO implemented in the public code MCFM

Approach chosen in Karlsruhe

Methods for cross sections at NNLO are required…

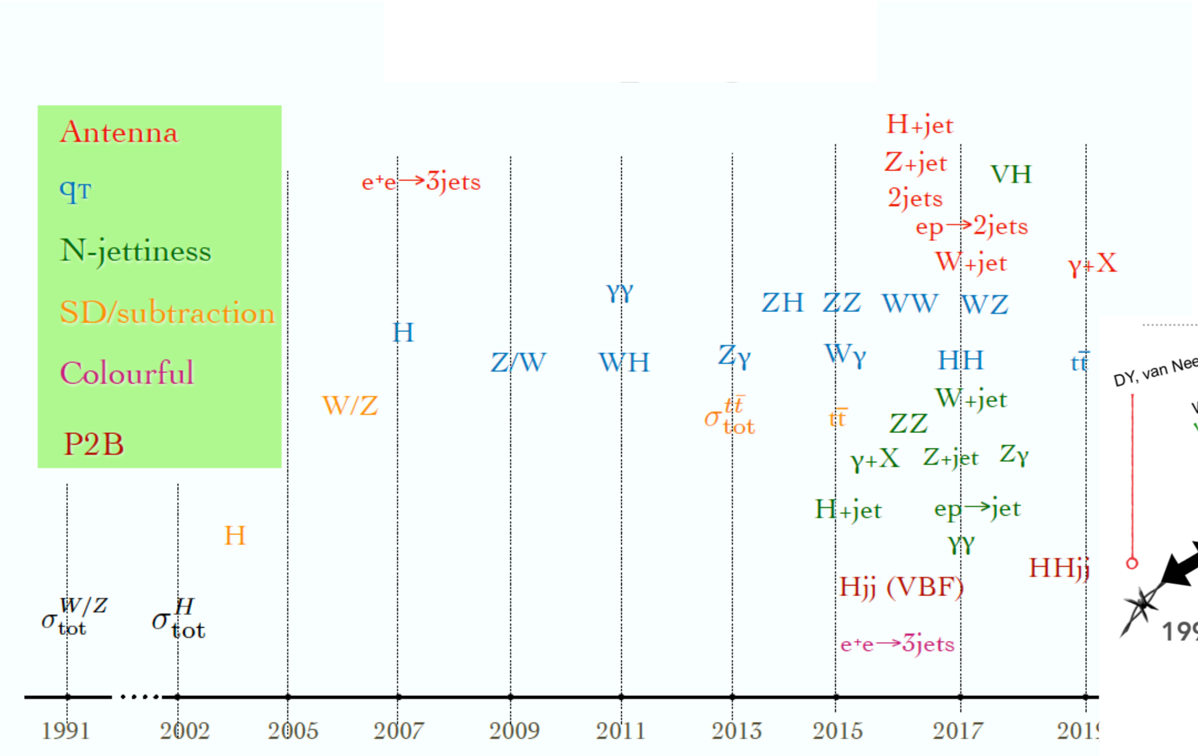
Subtraction

- sector decomposition Binoth, Heinrich `00,`04, Anastasiou, Melnikov, Petriello `04
- antenna subtraction Gehrmann, Glover `05
- colourful subtraction Somogyi, Trocsanyi, del Duca `05, `07
- sector improved residue subtraction (STRIPPER, nested soft-collinear subtraction) **MC** `10, Boughezal, Melnikov, Petriello `11, Caola, Melnikov, Rontsch `17
- local analytic sector subtraction Magnea, Maina, Pelliccoli, Signorile-Signorile, Torrielli, Uccirati `18
- geometric subtraction Herzog `18

Slicing

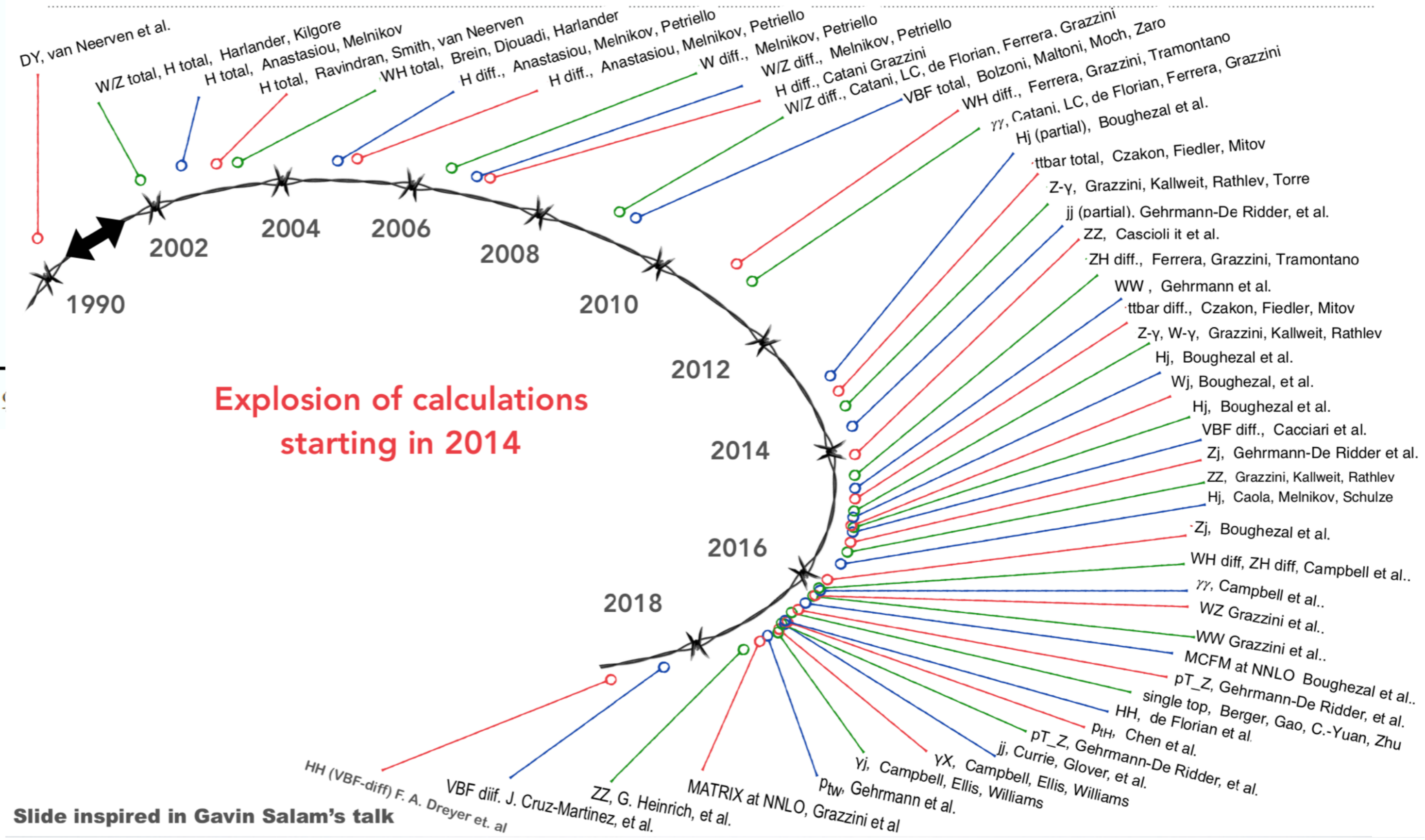
- qT slicing Catani, Grazzini `07
- N-jettiness slicing Gaunt, Stahlhofen, Tackmann, Walsh `15, Boughezal, Focke, Liu, Petriello `15
- projection to born Cacciari, Dreyer, Karlberg, Salam,Zanderighi `15

(digression: who makes the nicer summary?)



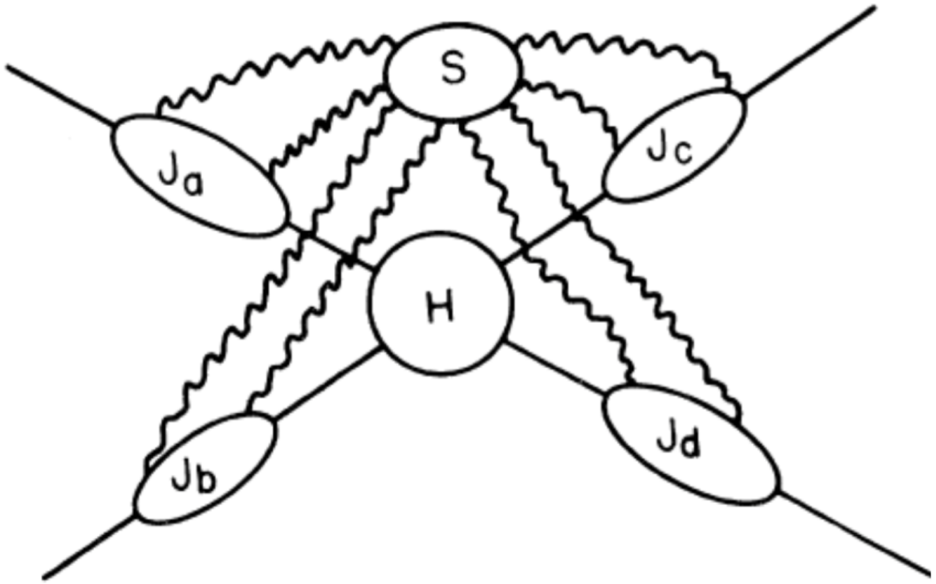
Grazzini, QCD @ LHC 2019

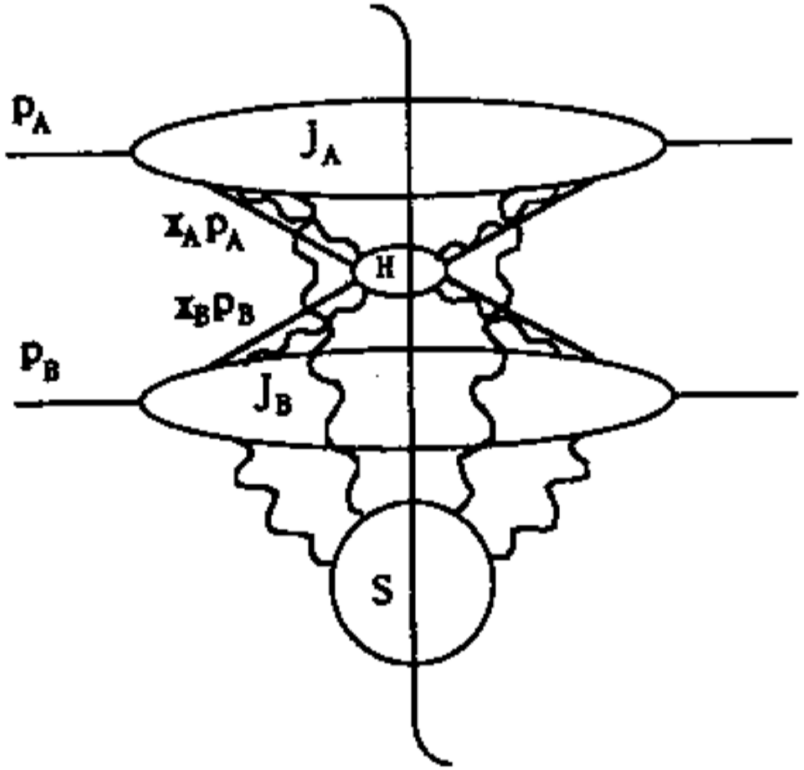
Cieri, Moriond QCD 2019



…cross section approximations are also required

Leading power factorization





Collins, Soper, Sterman `85

[hep-ph/0409313](hep-ph/0409313)

see also Collins, "Foundations of Perturbative QCD"

Example: factorization in the transverse momentum

Drell-Yan cross section in the language of Becher, Neubert `10

$$\frac{d^3\sigma}{dq^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{3N_c q^2 s} \left|C_V(-q^2 - i\epsilon)\right|^2 \frac{1}{4\pi} \int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp} \sum_q e_q^2 \left[\mathcal{S}_{q\bar{q}}(x_T^2)\, \mathcal{B}_{q/N_1}(z_1, x_T^2)\, \bar{\mathcal{B}}_{\bar{q}/N_2}(z_2, x_T^2) + (q \leftrightarrow \bar{q})\right]$$

$$\mathcal{B}_{q/N}(z, x_T^2) = \frac{1}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \sum_X \frac{\not{\bar{n}}_{\alpha\beta}}{2} \langle N(p)|\bar{\chi}_\alpha^n(t\bar{n} + x_\perp)|X\rangle \langle X|\chi_\beta^n(0)|N(p)\rangle$$

Subtleties in the definition require additional regulators

$$\lim_{\alpha\to 0} \left[\mathcal{S}(x_T^2)\mathcal{B}_{i/j}(z_1, x_T^2)\bar{\mathcal{B}}_{\bar{\imath}/k}(z_2, x_T^2)\right]_{q^2} = \left(\frac{x_T^2 q^2}{4e^{-2\gamma_E}}\right)^{-F_{i\bar{\imath}}^b(x_T^2)} B_{i/j}^b(z_1, x_T^2)\, B_{\bar{\imath}/k}^b(z_2, x_T^2)$$

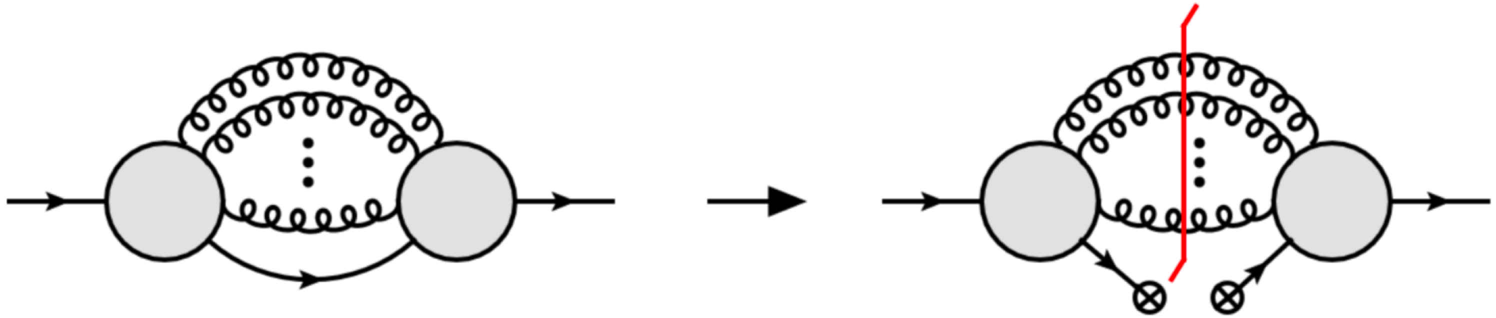
Analytic regulator studied in Becher, Bell `11

perturbative (that's what we want in this project)

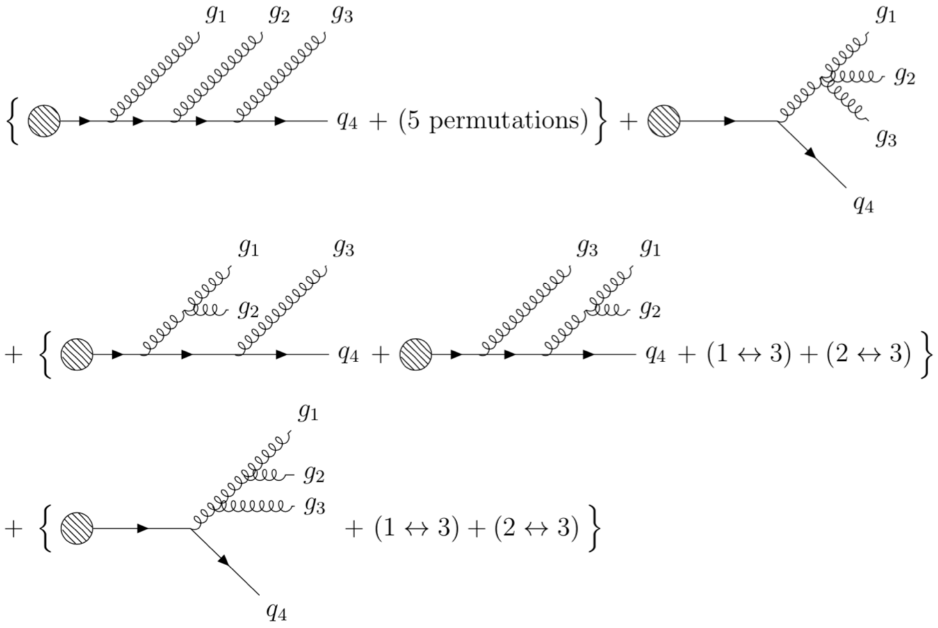
non-perturbative (PDF)

$$B_{i/j}(z, x_T^2, \mu) = \sum_k I_{i/k}(z, x_T^2, \mu) \otimes \phi_{k/j}(z, \mu)$$

The integrand of $I_{j/k}$ is a splitting function



Recently published del Duca, Duhr, Haindl, Lazopoulos, Michel `19, `20



results too long to print in an article…

https://people.phys.ethz.ch/~pheno/quadruple_collinear

Quadruple Collinear Splitting Amplitudes

Supplementary material from arXiv:1912.06425 and arXiv:2007.05345

Vittorio Del Duca, Claude Duhr, Rayan Haindl, Achilleas Lazopoulos and Martin Michel.

Differences between factorization in transverse momentum and jettiness

Transverse momentum: rapidity regulator required, soft function trivial in some cases

Jettiness: no need for rapidity regulator, soft function required

Main achievements so far

Double-real contribution to the quark beam function at N^{3}LO QCD

Melnikov, Rietkerk, Tancredi, Wever `18

Triple-real contribution to the quark beam function in QCD at next-to-next-to-next-to-leading order

Melnikov, Rietkerk, Tancredi, Wever `19

Quark beam function at next-to-next-to-next-to-leading order in perturbative QCD in the generalized large-N_c approximation

Behring, Melnikov, Rietkerk, Tancredi, Wever `19

Main achievements so far

Method: reverse unitarity Anasatisiou, Melnikov `02, analytic integration

$$D_k(z) = [\ln^k(1-z)/(1-z)]_+$$

$$1/\mu^2[\ln^k(t/\mu^2)/(t/\mu^2)]_+$$

$$\mathcal{I}_{qq}^{(n)} = \sum_{k=0}^{2n-1} L_k\left(\frac{t}{\mu^2}\right) F_+^{(n,k)}(z) + \delta(t) F_\delta^{(n)}(z) \qquad F_\delta^{(n)}(z) = C_{-1}^{(n)}\delta(1-z) + \sum_{k=0}^{2n-1} C_k^{(n)} D_k(z) + F_{\delta,\mathrm{h}}^{(n)}(z)$$

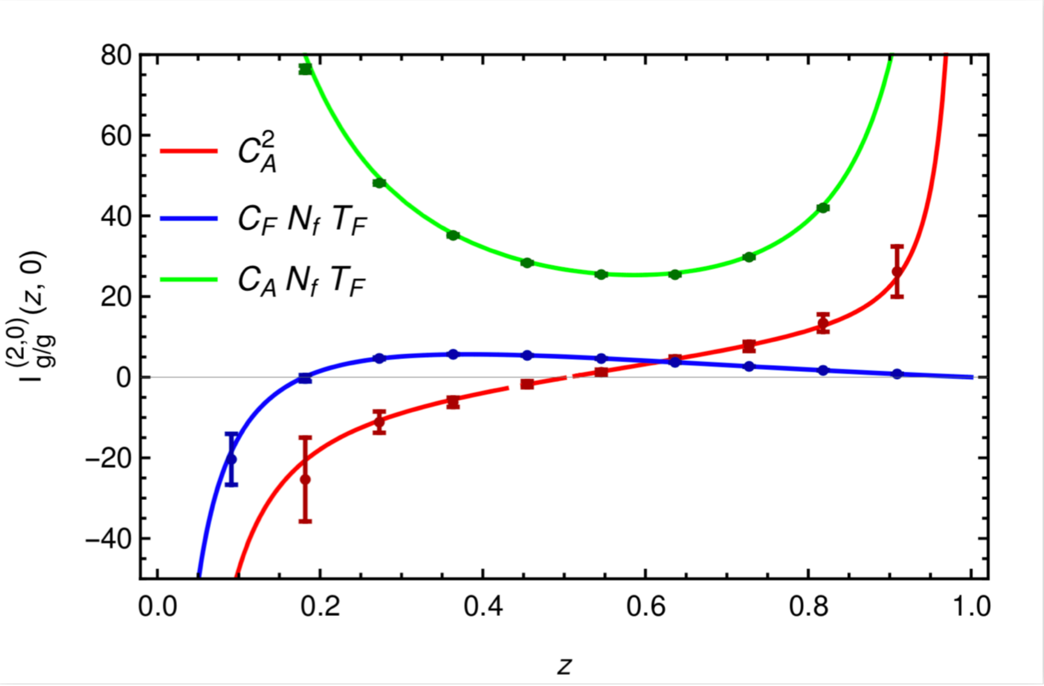
$$F_{\delta,\mathrm{h}}^{(3)} = N_f^2 N_c T_R^2 F_1 + N_f N_c^2 T_R F_2 + N_c^3 F_3$$

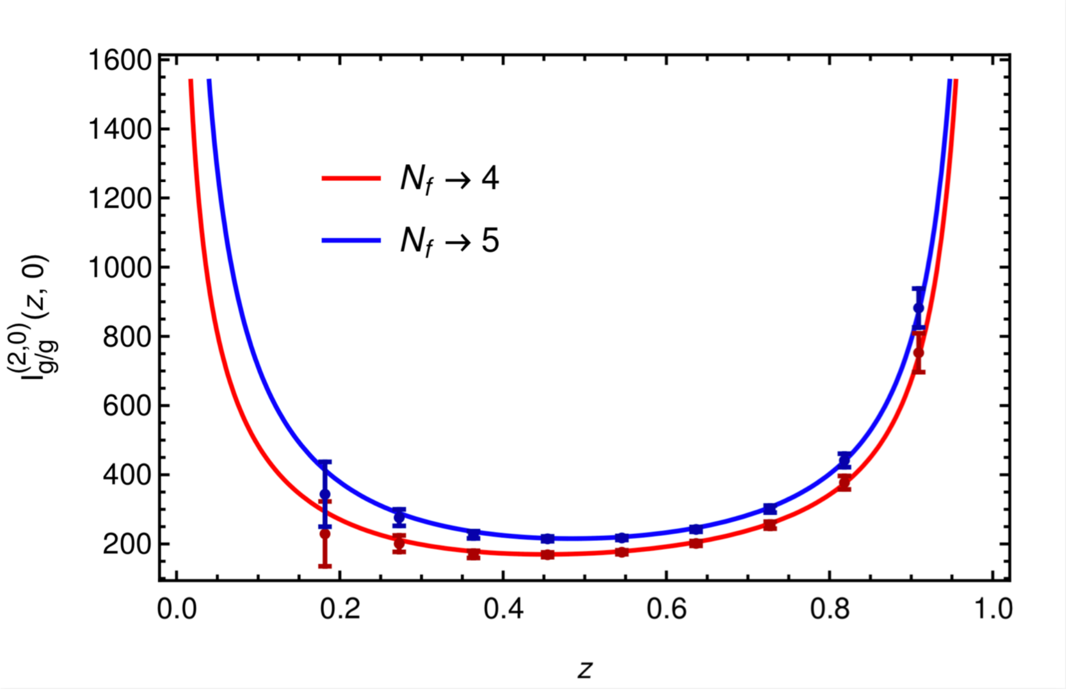
$$\begin{aligned}
F_1(z) = {} & \frac{32}{729}(157z-41) + \frac{80}{81}(11z-1)H_1 + \frac{64}{27}(4z+1)H_{1,1} + \frac{32}{9}(z+1)H_{1,1,1} - \frac{16}{27}(z+1)\pi^2 H_1 \\
& + \frac{1}{1-z}\left[-\frac{32}{81}(49z^2-32z+34)H_0\right] + \frac{1}{1-z}\left[-\frac{32}{27}(16z^2-9z+13)H_2 - \frac{64}{27}(4z^2-3z+4)H_{1,0}\right. \\
& \left. - \frac{16}{81}(133z^2-60z+97)H_{0,0} + \frac{16}{81}\pi^2(16z^2-9z+3)\right] + \frac{1}{1-z}\left[-\frac{32}{3}(z^2+1)H_3 - \frac{64}{9}(z^2+1)H_{2,1}\right. \\
& - \frac{64}{9}(z^2+1)H_{2,0} - \frac{32}{9}(z^2+1)H_{1,2} - \frac{32}{9}(z^2+1)H_{1,1,0} - \frac{32}{9}(z^2+1)H_{1,0,0} - \frac{368}{27}(z^2+1)H_{0,0,0} \\
& \left. + \frac{16}{9}(z^2+1)\pi^2 H_0 + \frac{64}{27}(z^2+2)\zeta_3\right],
\end{aligned}$$

What we’ve been doing at RWTH Aachen

Together with Philipp Müllender (PhD Student): numerical calculation of the perturbative beam functions for transverse momentum resummation known analytically to ϵ^0 at NNLO

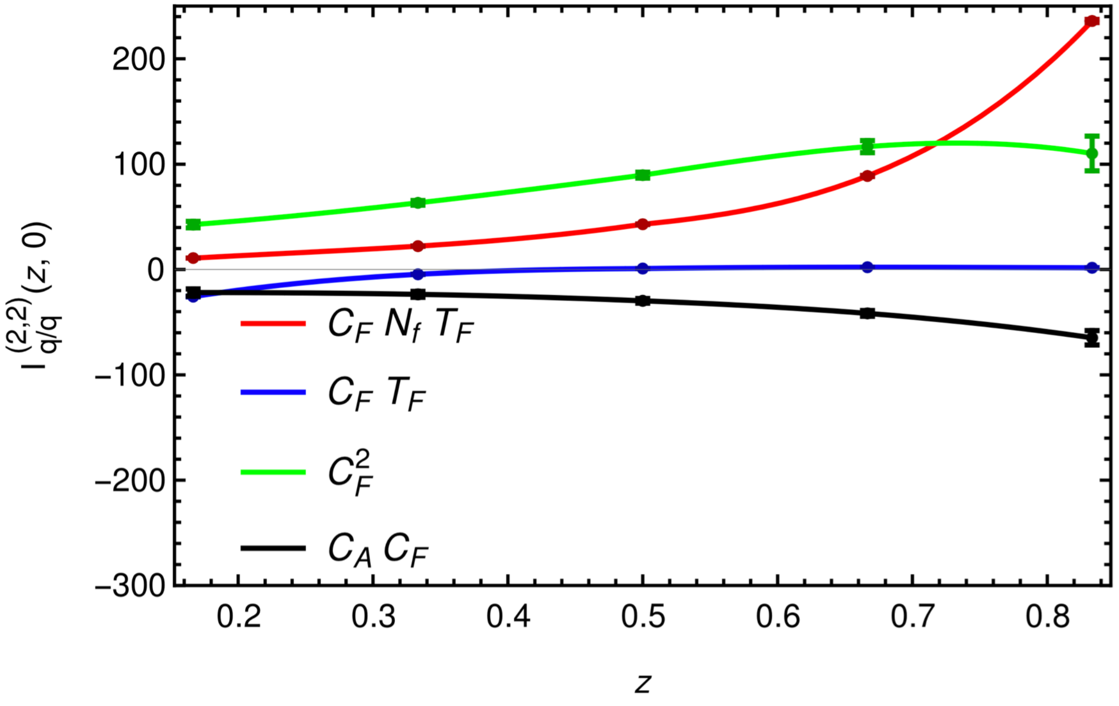
Gehrmann, Thomas Lübbert, Yang `14

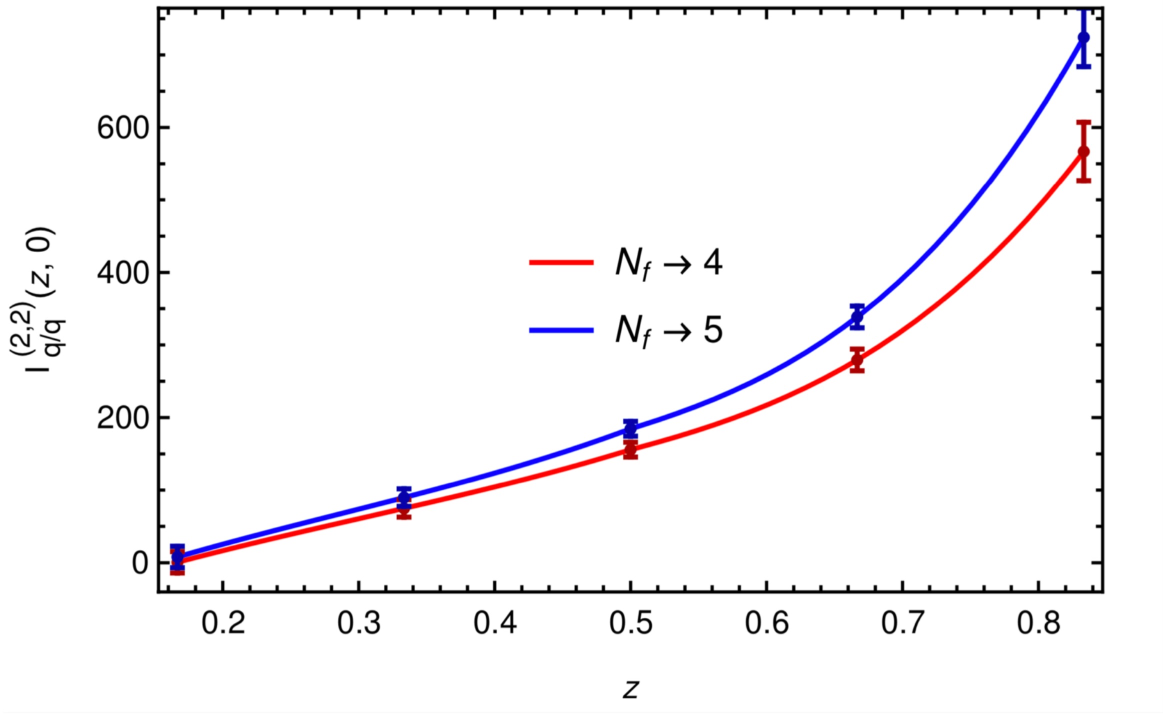




What we've been doing at RWTH Aachen

Together with Philipp Müllender (PhD Student): numerical calculation of the perturbative beam functions for transverse momentum resummation to ϵ^2 at NNLO



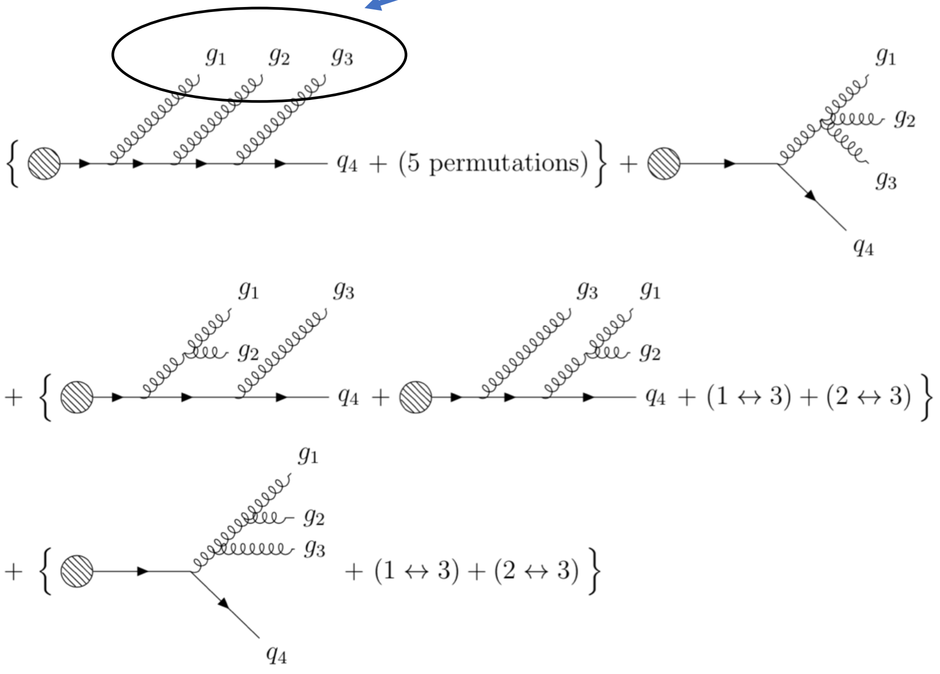


What we’ve been doing at RWTH Aachen

Together with Sebastian Sapeta (IFJ PAN Cracow): numerical calculation of the perturbative beam functions for transverse momentum resummation at N3LO

Methodology based on physical sector decomposition similar to STRIPPER at NNLO

non-vanishing transverse momentum, are the singularities just as at NNLO ?



unfortunately not quite

$$l^\mu = (l_+, l_-, l_T) = \left(\frac{(1-\rho)\chi}{\rho}, \frac{\rho\chi}{(1-\rho)}, \chi \right)$$

rapidity divergence at non-vanishing transverse momentum

$$\rho = 0$$

Others have been busy as well

Beam functions in the soft limit: Billis, Ebert, Michel, Tackmann `19

A Toolbox for q_T and 0-Jettiness Subtractions at N^{3}LO

Complete solution: Ebert, Mistlberger, Vita `20

Collinear expansion for color singlet cross sections

N-jettiness beam functions at N^{3}LO

Transverse momentum dependent PDFs at N^{3}LO

result for quarks: Luo, Yang, Zhu, Zhu `19

different regulator, Li, Neill, Zhu `16, with non-vanishing soft function known to sufficient order for N3LO applications, Echevarria, Scimemi, Vladimirov `16, Li, Zhu` 16, Lübbert, Oredsson, Stahlhofen `16

Others have been busy as well

Complete solution: Ebert, Mistlberger, Vita `20

Collinear expansion for color singlet cross sections

N-jettiness beam functions at $\mathrm{N}^3\mathrm{LO}$

"The class of functions appearing in the matching coefficents for all channels includes iterated integrals with non-rational kernels, thus going beyond the one of harmonic polylogarithms"

Transverse momentum dependent PDFs at $\mathrm{N}^3\mathrm{LO}$

"perturbative matching kernels for all channels are expressed in terms of simple harmonic polylogarithms up to weight five "

Where do we go from here?

- Check/calculate beam functions at N3LO to the end
- Calculate the 0-jettiness soft function at N3LO
- Improve NNLO methods for speed/stability as basis for N3LO
- Are we falling behind? Will there be phenomenology to contribute?