PROJECT B2A

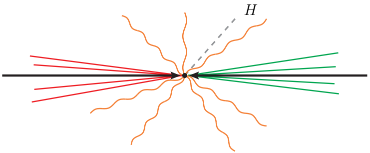
AUTOMATED CALCULATIONS IN SOFT-COLLINEAR EFFECTIVE THEORY

[GUIDO BELL]

UNIVERSITÄT SIEGEN

Project B2A

Factorisation for Higgs production at small $p_T \ll m_H$



$$\frac{d\hat{\sigma}}{dp_T} \simeq H(m_H, \mu)\, B(p_T, \mu) \otimes B(p_T, \mu) \otimes S(p_T, \mu)$$

Work program

- automated calculation of NNLO soft functions
- automated calculation of NNLO jet and beam functions
- high-precision resummations for collider observables
- further development of SCET subtraction techniques

Status

Publications

1911.04494	Three-loop soft function for heavy-to-light quark decays	Brüser, Liu, Stahlhofen
1912.13039	Resummed inclusive cross-section in ADD model at N^3LL+NNLO	Das, Kumar, Samanta
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2004.03938	Resummed inclusive cross-section in RS model at NNLO+NNLL	Das, Kumar, Samanta
2004.08396	Generic dijet soft functions at two-loop order: uncorrelated emissions	GB, Rahn, Talbert
in preparation	The N-jettiness soft function at NNLO	GB, Dehnadi, Mohrmann, Rahn

On-going projects

in progess	Generic N-jet soft functions at two-loop order	GB, Dehnadi, Mohrmann, Rahn
in progess	Automated calculation of jet functions in SCET	GB, Brune, Das, Wald
in progess	Automated calculation of beam functions in SCET	GB, Brune, Das, Wald
in progess	Towards a precision extraction of $\alpha_s(m_Z)$ from e^+e^- angularities	GB, Lee, Makris, Prager, Talbert, Yan

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Generic N-jet soft functions at two-loop order

[GB, Dehnadi, Mohrmann, Rahn]

N-jet soft functions

Definition

$$S(\tau,\mu) = \sum_{i\in X} \mathcal{M}(\tau;\{k_i\})\,\langle 0|(S_{n_1} S_{n_2}\dots S_{n_N})^\dagger|X\rangle\langle X|S_{n_1} S_{n_2}\dots S_{n_N}|0\rangle$$

- soft Wilson lines S_{n_i} with $n_i^2 = 0$
- soft function is a matrix in colour space
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$
- dijet case fully automated and implemented in `SoftSERVE` [GB, Rahn, Talbert 18,19]

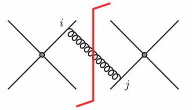
Motivation

- resummation for hadronic event shapes and boosted top production
- central ingredient for N-jettiness slicing technique [Boughezal et al 15; Gaunt et al 15]

NLO calculation

Sum over dipole contributions

$$S^{(1)}(\varepsilon) = \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{n_i \cdot n_j}{2}\right)^{\varepsilon} S_{ij}^{(1)}(\varepsilon)$$



- integration over gluon phase space

$$S_{ij}^{(1)}(\varepsilon) \sim \int d^d k \;\, \delta(k^2)\, \theta(k^0)\;\, \mathcal{M}_1(\tau; k)\;\, \frac{n_i \cdot n_j}{2\,(k \cdot n_i)\,(k \cdot n_j)}$$

- generic parametrisation of measurement function

$$\mathcal{M}_1(\tau; k) = \exp\left\{ -\tau \sqrt{\frac{n_i \cdot n_j}{2}}\; k_T\, y_k^{n/2}\, f(y_k, \theta_1, \theta_2)\right\}$$

$$k_T = \sqrt{\frac{2\,(k \cdot n_i)\,(k \cdot n_j)}{n_i \cdot n_j}} \qquad y_k = \frac{k \cdot n_i}{k \cdot n_j}$$

- perform observable-independent integrations

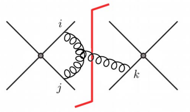
$$S_{ij}^{(1)}(\varepsilon) \sim \frac{\Gamma(-2\varepsilon)}{\Gamma(-\varepsilon)} \int_0^1 dy_k\;\, y_k^{-1+n\varepsilon} \int_{-1}^1 \frac{d\cos\theta_1}{\sin^{1+2\varepsilon}\theta_1} \int_{-1}^1 \frac{d\cos\theta_2}{\sin^{2+2\varepsilon}\theta_2}\; f(y_k, \theta_1, \theta_2)^{2\varepsilon}$$

$\Rightarrow$ soft ($k_T \to 0$) and collinear ($y_k \to 0$) singularities factorised

NNLO real-virtual

Dipole and tripole contributions [Catani, Grazzini 00]

$$S^{(2,\mathrm{RV})}(\varepsilon) = C_A \sum_{i\neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{n_i \cdot n_j}{2}\right)^{2\varepsilon} S_{ij}^{(2,\mathrm{Re})}(\varepsilon) + \sum_{i\neq j\neq k} (\lambda_{ij} - \lambda_{ip} - \lambda_{jp})\, f_{ABC}\, \mathbf{T}_i^A\, \mathbf{T}_j^B\, \mathbf{T}_k^C\, S_{ijk}^{(2,\mathrm{Im})}(\varepsilon)$$



- dipole contribution similar to NLO case

$$\left|\mathcal{A}_{ij}^{(2,Re)}(k)\right|^2 \sim \frac{1}{\varepsilon^2} \left(\frac{n_i \cdot n_j}{2\,(k \cdot n_i)\,(k \cdot n_j)}\right)^{1+\varepsilon}$$

- tripole contribution requires at least four hard partons

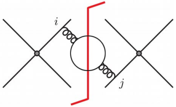
$$\left|\mathcal{A}_{ijk}^{(2,Im)}(k)\right|^2 \sim \frac{1}{\varepsilon} \left(\frac{n_i \cdot n_j}{2\,(k \cdot n_i)\,(k \cdot n_j)}\right)^{\varepsilon} \left(\frac{n_i \cdot n_k}{2\,(k \cdot n_i)\,(k \cdot n_k)}\right)$$

$\Rightarrow$ similar parametrisation in terms of $(k_T, y_k, \theta_1, \theta_2)$ as in NLO calculation

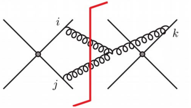
NNLO real-real

Radiation of $q\bar{q}$ pair or two gluons

[Catani, Grazzini 99]



$$S^{(2,q\bar{q})}(\varepsilon) = T_F\, n_f \sum_{i\neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{n_i \cdot n_j}{2}\right)^{2\varepsilon} S_{ij}^{(2,q\bar{q})}(\varepsilon)$$



$$S^{(2,gg)}(\varepsilon) = C_A \sum_{i\neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{n_i \cdot n_j}{2}\right)^{2\varepsilon} S_{ij}^{(2,gg)}(\varepsilon)$$

$$\underbrace{+\, \frac{1}{4} \sum_{i\neq j} \sum_{k\neq l} \{\mathbf{T}_i \cdot \mathbf{T}_j\,,\, \mathbf{T}_k \cdot \mathbf{T}_l\} \left(\frac{(n_i \cdot n_j)\,(n_k \cdot n_l)}{4}\right)^{\varepsilon} S_{ij}^{(1)}(\varepsilon)\, S_{kl}^{(1)}(\varepsilon)}_{\text{fixed by non-abelian exponentiation}}$$

- integration over two-particle phase space

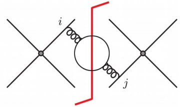
$$S_{ij}^{(2,X)}(\varepsilon) \sim \int d^d k\;\, \delta(k^2)\,\theta(k^0) \int d^d l\;\, \delta(l^2)\,\theta(l^0)\;\, \mathcal{M}_2(\tau; k, l)\;\, \left|\mathcal{A}_{ij}^X(k,l)\right|^2$$

NNLO real-real

Radiation of $q\bar{q}$ pair or two gluons

[Catani, Grazzini 99]

$$S^{(2,q\bar{q})}(\varepsilon) = T_F\, n_f \sum_{i\neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{n_i \cdot n_j}{2}\right)^{2\varepsilon} S_{ij}^{(2,q\bar{q})}(\varepsilon)$$



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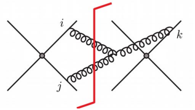
$$+ \underbrace{\frac{1}{4} \sum_{i\neq j} \sum_{k\neq l} \{\mathbf{T}_i \cdot \mathbf{T}_j\, ,\, \mathbf{T}_k \cdot \mathbf{T}_l\} \left(\frac{(n_i \cdot n_j)\,(n_k \cdot n_l)}{4}\right)^{\varepsilon} S_{ij}^{(1)}(\varepsilon)\, S_{kl}^{(1)}(\varepsilon)}_{\text{fixed by non-abelian exponentiation}}$$



- phase-space parametrisation

$$p_T = \sqrt{\frac{2(k_i + l_i)(k_j + l_j)}{n_{ij}}} \qquad y = \frac{k_i + l_i}{k_j + l_j} \qquad a = \sqrt{\frac{k_j\, l_i}{k_i\, l_j}} \qquad b = \sqrt{\frac{k_i\, k_j}{l_i\, l_j}}$$

$$\mathcal{M}_2(\tau; k, l) = \exp\left\{ -\,\tau \sqrt{\frac{n_i \cdot n_j}{2}}\; p_T\, y^{n/2}\, F\big(a, b, y, \theta_{k1}, \theta_{k2}, \theta_{k3}, \theta_{l1}, \theta_{l2}\big) \right\}$$

Renormalisation

RG equation in Laplace space

$$\frac{\mathrm{d}S(\tau,\mu)}{\mathrm{d}\ln\mu} = \frac{1}{2}\,\Gamma_S(\tau,\mu)\,S(\tau,\mu) + \frac{1}{2}\,S(\tau,\mu)\,\Gamma_S(\tau,\mu)^\dagger$$

$$\Gamma_S(\tau,\mu) = \frac{1}{n}\left\{\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\;\Gamma_{\rm cusp}(\alpha_s)\left[2\,\ln\left(\sqrt{\frac{n_i\cdot n_j}{2}}\,\mu\bar\tau\right) - i\pi\,\lambda_{ij}\right] + 2\gamma^S(\alpha_s)\right\}$$

Two-loop solution with $L_{ij} = \ln\left(\sqrt{n_i\cdot n_j/2}\,\mu\bar\tau\right)$

$$\begin{aligned}
S(\tau,\mu) = 1 &+ \left(\frac{\alpha_s}{4\pi}\right)\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\left(\frac{\Gamma_0}{n}\,L_{ij}^2 + \frac{2\gamma_{ij}^{(0)}}{n}\,L_{ij} + c_{ij}^{(1)}\right)\\
&+ \left(\frac{\alpha_s}{4\pi}\right)^2\Bigg\{\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\left(\frac{2\beta_0\Gamma_0}{3n}\,L_{ij}^3 + \left(\frac{\Gamma_1}{n} + \frac{2\beta_0\gamma_{ij}^{(0)}}{n}\right)L_{ij}^2 + 2\left(\frac{\gamma_{ij}^{(1)}}{n} + \beta_0 c_{ij}^{(1)}\right)L_{ij} + c_{ij}^{(2)}\right)\\
&\qquad -2\pi\sum_{i\neq j\neq k} f_{ABC}\,\mathbf{T}_i^A\,\mathbf{T}_j^B\,\mathbf{T}_k^C\left(\frac{\lambda_{ij}\Gamma_0}{n^2}\left(\frac{\Gamma_0}{3}\,L_{jk}^3 + \gamma_{jk}^{(0)}L_{jk}^2 + n\,c_{jk}^{(1)}L_{jk}\right) + c_{ijk}^{(2)}\right)\\
&\qquad +\frac{1}{4}\sum_{i\neq j}\sum_{k\neq l}\{\mathbf{T}_i\cdot\mathbf{T}_j\,,\,\mathbf{T}_k\cdot\mathbf{T}_l\}\left(\frac{\Gamma_0}{n}\,L_{ij}^2 + \frac{2\gamma_{ij}^{(0)}}{n}\,L_{ij} + c_{ij}^{(1)}\right)\left(\frac{\Gamma_0}{n}\,L_{kl}^2 + \frac{2\gamma_{kl}^{(0)}}{n}\,L_{kl} + c_{kl}^{(1)}\right)\Bigg\}
\end{aligned}$$

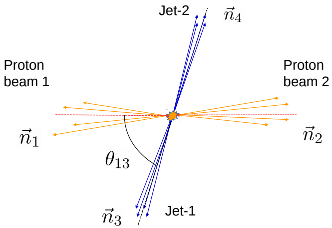
Results

N-jettiness [Stewart, Tackmann, Waalewijn 10]

$$\mathcal{T}_N(\{k_i\}) = \sum_i \min_j (n_j \cdot k_i) \qquad j = \underbrace{1,2}_{\text{beams}}, \underbrace{3,\ldots,N+2}_{\text{jets}}$$

- two independent implementations based on `pySecDec` and `SoftSERVE`
- 1-jettiness results in agreement with literature [Boughezal et al 15; Campbell et al 17]
- first 2-jettiness results for arbitrary kinematics [GB, Dehnadi, Mohrmann, Rahn 18; Jin, Liu 19]

Assume back-to-back konfiguration for illustration



$$n_{12} \equiv n_1 \cdot n_2 = n_3 \cdot n_4 = 2$$

$$n_{13} \equiv n_1 \cdot n_3 = n_2 \cdot n_4 = 1 - \cos\theta_{13}$$

$$n_{14} \equiv n_1 \cdot n_4 = n_2 \cdot n_3 = 1 + \cos\theta_{13}$$

Results

N-jettiness [Stewart, Tackmann, Waalewijn 10]

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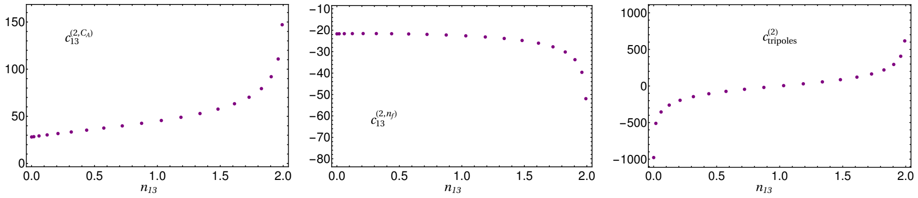
Two-loop matching correction as a function of n_{13}



Automated calculation of jet and beam functions

[GB, Brune, Das, Wald]

Definitions

Quark jet function

$$J_q(\tau,\mu) \sim \sum_{i\in X} \delta\big(Q - \textstyle\sum_i n\cdot k_i\big)\; \delta^{(d-2)}\big(\textstyle\sum_i k_i^\perp\big)\; \mathcal{M}(\tau;\{k_i\})\; \langle 0|\chi|X\rangle\langle X|\bar\chi|0\rangle$$

- collinear field operator $\chi = W^\dagger \frac{\not{n}\not{\bar n}}{4}\psi$
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$

Quark beam function

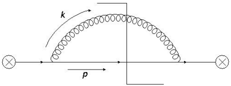
$$B_q(\tau,x,\mu) \sim \sum_{i\in X} \delta\big((1-x)P^- - \textstyle\sum_i n\cdot k_i\big)\; \mathcal{M}(\tau;\{k_i\})\; \langle P|\bar\chi|X\rangle\langle X|\chi|P\rangle$$

- as long as $\tau \ll 1/\Lambda_{\rm QCD}$ beam functions can be matched onto pdfs

$$B_q(\tau,x,\mu) = \sum_k \int_x^1 \frac{dz}{z}\, \mathcal{I}_{qk}\Big(\tau,\frac{x}{z},\mu\Big)\, f_k(z,\mu)$$

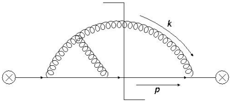
Technical aspects

NLO correction



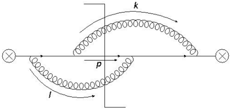
- matrix element given by LO $1 \to 2$ splitting function
- parametrise phase space in terms of (k_T, z_k, θ_k)

NNLO real-virtual correction



- matrix element given by NLO $1 \to 2$ splitting function

NNLO real-real correction



- matrix element given by LO $1 \to 3$ splitting function
- singularity structure more complicated than for soft functions

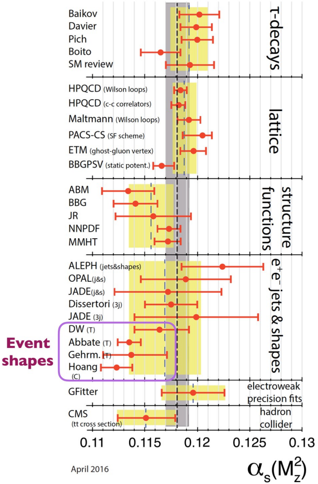
$\Rightarrow$ for details see Kevin's talk in the YSS

Towards a precision extraction of $\alpha_s(m_Z)$ from e^+e^- angularity distributions

[GB, Lee, Makris, Prager, Talbert, Yan]

e^+e^- event shapes

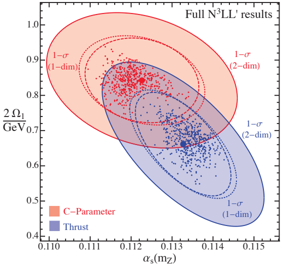
Extraction of the strong coupling $\alpha_s(M_Z)$



Precision thrust and C-parameter analyses

give significantly lower values for $\alpha_s(M_Z)$

[Hoang et al 15]



Are the non-perturbative corrections under control?

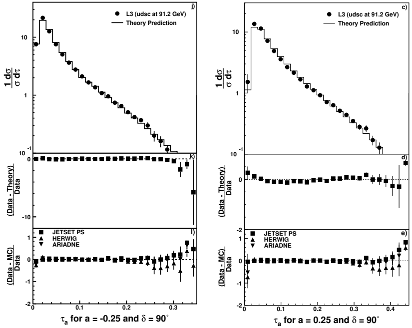
Angularities

e^+e^- event shape that depends on a continous parameter a [Berger, Kucs, Sterman 03]

$$\tau_a(\{k_i\}) = \sum_i |k_\perp^i| \, e^{-|\eta_i|\,(1-a)}$$

- interpolates between thrust ($a = 0$) and total broadening ($a = 1$)

L3 data for $a = \{-1, -0.75, -0.5, -0.25, 0, +0.25, +0.5, +0.75\}$ at $Q = \{91.2, 197\}$ GeV



- quality of data not competitive with thrust and C-parameter analyses

$\Rightarrow$ highlight the potential of angularities and motivate a reanalysis of LEP data

Theory status

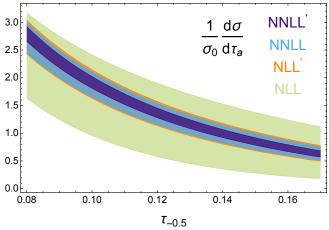
[GB, Hornig, Lee, Talbert 18]

Factorisation theorem for $\tau_a \to 0$

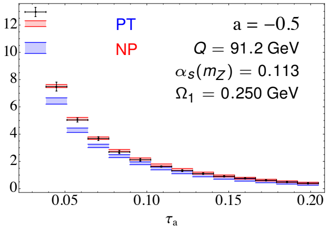
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a} \simeq H(Q,\mu) \int d\tau_n\, d\tau_{\bar{n}}\, d\tau_s\;\; J_n(\tau_n,\mu)\, J_{\bar{n}}(\tau_{\bar{n}},\mu)\, S(\tau_s,\mu)\, \delta(\tau_a - \tau_n - \tau_{\bar{n}} - \tau_s)$$

- NNLL′ resummation using input from `SoftSERVE`
- matched to fixed-order α_s^2 calculation
- non-perturbative effects modelled by a renormalon-free gapped shape function

convergence



non-perturbative effects



Non-perturbative corrections

Dominant effects related to soft function modelling

$$S(k,\mu) = \int dk' \underbrace{S_{\mathrm{PT}}(k-k',\mu)}_{\text{perturbative soft function}} \quad \underbrace{f_{\mathrm{mod}}(k'-2\overline{\Delta}_a)}_{\text{non-pert. shape function}}$$

- expand shape function in complete set of orthogonal basis functions

$$f_{\mathrm{mod}}(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} b_n\, f_n\left(\frac{k}{\lambda}\right)\right]^2$$

- OPE predicts a shift of the distribution in the tail region [Lee, Sterman 06]

$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow{\mathrm{NP}} \frac{d\sigma}{d\tau_a}\left(\tau_a - \frac{2}{1-a}\frac{\overline{\Omega}_1}{Q}\right) \qquad \frac{2\overline{\Omega}_1}{1-a} = 2\overline{\Delta}_a + \int dk\, k\, f_{\mathrm{mod}}(k)$$

- gap parameter $\overline{\Delta}_a$ and perturbative soft function S_{PT} share a renormalon ambiguity

  $\Rightarrow$ switch to the renormalon-free Rgap scheme [Hoang, Stewart 07]

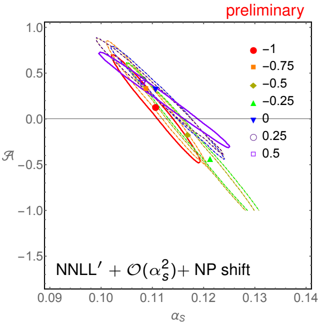
First look at α_s fits

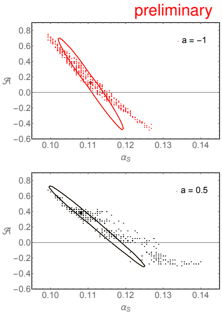
[GB, Lee, Makris, Prager, Talbert, Yan in preparation]

Setup

- χ^2 fit with minimal overlap assumption for experimental uncertainties
- theory uncertainties from scan over 500 random input parameters

Individual angularities





First look at α_s fits

[GB, Lee, Makris, Prager, Talbert, Yan in preparation]

Setup

- χ^2 fit with minimal overlap assumption for experimental uncertainties
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Global fit

- crucial to take correlations between different angularities into account

  $\Rightarrow$ construct covariance matrix with PYTHIA
- rather strong dependence on the fit window

  $\Rightarrow$ include matching to fixed-order α_s^3 calculation
- issues related to renormalon subtraction in the Rgap scheme

Summary

Work program

- automated calculation of NNLO soft functions
- automated calculation of NNLO jet and beam functions
- high-precision resummations for collider observables
- further development of SCET subtraction techniques