Project C1a

Inclusive semileptonic, rare and radiative decays of B mesons

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TRR 257 - Particle Physics Phenomenology
after the Higgs Discovery

Annual meeting of SFB TRR 257, Siegen, October 6-8$^{\mathrm{th}}$, 2020

Work areas

WA 1: Kinetic mass to N^{3}LO accuracy

WA 2: Charm quark mass dependence of $\bar{B} \to X_s\gamma$

WA 3: Perturbative corrections to higher-order terms in the $1/m_Q$ expansion

WA 4: Improvement of the Heavy-Quark Expansion (HQE) parameters

WA 5: Increasing the precision of $\bar{B} \to X_{s,(d)}\ell^+\ell^-$

WA 6: Multi-parton contributions to $\bar{B} \to X_{s,(d)}\gamma$

WA 7: Re-summations of the HQE

WA 8: Realistic models for Duality Violations

Publications (selection)

1. **T. Mannel** and **A. A. Pivovarov**,
"QCD corrections to inclusive heavy hadron weak decays at $\Lambda^3_{\rm QCD}/m^3_Q$,"
Phys. Rev. D **100** (2019) no.9, 093001, [arXiv:1907.09187 [hep-ph]] — WA 3

2. **T. Huber**, T. Hurth, J. Jenkins, E. Lunghi, Q. Qin and K. K. Vos,
"Long distance effects in inclusive rare B decays and phenomenology of $\bar{B} \to X_d \ell^+ \ell^-$,"
JHEP **10** (2019), 228, [arXiv:1908.07507 [hep-ph]] — QFET $\rightarrow$ WA 5

3. **M. Fael**, **T. Mannel** and K. K. Vos,
"The Heavy Quark Expansion for Inclusive Semileptonic Charm Decays Revisited,"
JHEP **12** (2019), 067, [arXiv:1910.05234 [hep-ph]]. — WA 4

4. M. Misiak, **A. Rehman** and **M. Steinhauser**,
"Towards $\overline{B} \to X_s \gamma$ at the NNLO in QCD without interpolation in m$_c$,"
JHEP **06** (2020), 175, [arXiv:2002.01548 [hep-ph]]. — WA 2

5. **T. Mannel**, **D. Moreno** and **A. Pivovarov**,
"Heavy quark expansion for heavy hadron lifetimes: completing the $1/m_b^3$ corrections,"
JHEP **08** (2020), 089, [arXiv:2004.09485 [hep-ph]]. — WA 3

$\Longrightarrow$ see Daniel Moreno's talk at YSF in June

$\longrightarrow$ see also 2004.09527 by Lenz, Piscopo, Rusov $\Longrightarrow$ see Alex Lenz' talk

Publications (selection)

6. **M. Fael**, **K. Schönwald** and **M. Steinhauser**,
"The Kinetic Heavy Quark Mass to Three Loops,"
Phys. Rev. Lett. **125** (2020) no.5, 052003, [arXiv:2005.06487 [hep-ph]]. WA 1

$\Longrightarrow$ see Kay Schönwald's talk

7. **T. Huber**, T. Hurth, J. Jenkins, E. Lunghi, Q. Qin and K. K. Vos,
"Phenomenology of inclusive $\bar{B} \to X_s \ell^+ \ell^-$ for the Belle II era,"
arXiv:2007.04191 [hep-ph]. WA 5

8. **M. Fael**, **K. Schönwald** and **M. Steinhauser**,
"Exact results for $Z_m^{\rm OS}$ and $Z_2^{\rm OS}$ with two mass scales and up to three loops,"
arXiv:2008.01102 [hep-ph]. WA 1

9. **D. Moreno**,
"Completing $1/m_b^3$ corrections to non-leptonic bottom-to-up-quark decays,"
arXiv:2009.08756 [hep-ph]. WA 3

Today's topics

- QCD corrections to inclusive semileptonic decays at $\mathcal{O}(\Lambda^3_{\rm QCD}/m_Q^3)$
- Inclusive $\bar{B} \to X_s \gamma$
- Inclusive $\bar{B} \to X_{s,(d)} \ell^+ \ell^-$

Inclusive B decays, generalities

- Main tool for inclusive decays: Heavy Quark Expansion

[Khoze,Shifman,Voloshin,Bigi,Uraltsev,Vainshtein,Blok,Chay,Georgi,Grinstein,Luke,... '80s and '90s]

$$\Gamma(B_q \to X) = \frac{1}{2m_{B_q}} \sum_X \int_{\text{PS}} (2\pi)^4 \delta^{(4)}(p_{B_q} - p_X) |\langle X|\hat{\mathcal{H}}_{eff}|B_q\rangle|^2$$

- Use optical theorem

$$\Gamma(B_q \to X) = \frac{1}{2m_{B_q}} \langle B_q|\hat{\mathcal{T}}|B_q\rangle \quad \text{with} \quad \hat{\mathcal{T}} = \text{Im}\, i \int d^4x \hat{T}\left[\hat{\mathcal{H}}_{eff}(x)\hat{\mathcal{H}}_{eff}(0)\right]$$

- Expand non-local double insertion of effective Hamiltonian in local operators

$$\begin{aligned}\Gamma \quad &= \quad \Gamma_0 \langle O_{D=3}\rangle + \Gamma_2 \frac{\langle O_{D=5}\rangle}{m_b^2} + \tilde{\Gamma}_3 \frac{\langle \tilde{O}_{D=6}\rangle}{m_b^3} + \ldots \\ &\quad +16\pi^2 \left[\Gamma_3 \frac{\langle O_{D=6}\rangle}{m_b^3} + \Gamma_4 \frac{\langle O_{D=7}\rangle}{m_b^4} + \Gamma_5 \frac{\langle O_{D=8}\rangle}{m_b^5} + \ldots\right]\end{aligned}$$

- Expand each term in perturbative series

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_i^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \Gamma_i^{(2)} + \ldots.$$

  - Tree-level terms known through to $\mathcal{O}(1/m_b^5)$.

HQE expansion parameters

- Γ_0: Decay of a free quark, known to $\mathcal{O}(\alpha_s^2)$
- Γ_1: Vanishes due to Heavy Quark Symmetry

- Two terms in Γ_2
  - Kinetic energy μ_π: $2M_B\mu_\pi^2 = -\langle B(v)|\bar{b}_v(iD)^2 b_v|B(v)\rangle$
  - Chromomagnetic moment μ_G: $2M_B\mu_G^2 = -i\langle B(v)|\bar{b}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)b_v|B(v)\rangle$
  - Both known to $\mathcal{O}(\alpha_s)$ [Becher,Boos,Lunghi'07;Alberti,Ewerth,Gambino,Nandi'13'14;Mannel,Pivovarov,Rosenthal'15]

- Two more terms in Γ_3
  - Darwin term ρ_D: $2M_H\rho_D^3 = -\langle B(v)|\bar{b}_v(iD_\mu)(ivD)(iD^\mu)b_v|B(v)\rangle$
  - Spin-orbit term ρ_{LS}: $2M_H\rho_{LS}^3 = -i\langle B(v)|\bar{b}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)b_v|B(v)\rangle$
  - ρ_D known to $\mathcal{O}(\alpha_s)$ [Mannel,Pivovarov'19]

- Number of parameters grows factorially at higher orders in $1/m$
  - Partial reduction by reparametrisation invariance [Mannel,Vos'18;Fael,Mannel,Vos'19]

QCD corrections to higher orders in $B \to X_c \ell \bar{\nu}_\ell$

- ρ_D for inclusive semileptonic $b \to c\ell\nu$ at $\mathcal{O}(\alpha_s)$ [Mannel,Pivovarov'19]

$$\Gamma(B \to X_c \ell \bar{\nu}_\ell) = \Gamma_0 |V_{cb}|^2 \left[a_0 (1 + \frac{\mu_\pi^2}{2m_b^2}) + a_2 \frac{\mu_G^2}{2m_b^2} + \frac{a_D \rho_D + a_{LS} \rho_{LS}}{2m_b^3} + \ldots \right]$$

- The HQE for the rate is constructed by using a direct matching from QCD onto HQET $(\pi_\mu = iD_\mu)$

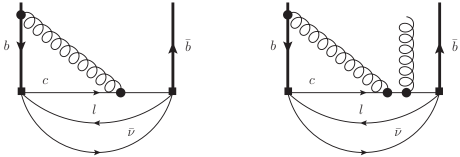
$$\mathcal{T} = C_0 \mathcal{O}_0 + C_v \frac{\mathcal{O}_v}{m_b} + C_\pi \frac{\mathcal{O}_\pi}{2m_b^2} + C_G \frac{\mathcal{O}_G}{2m_b^2} + C_D \frac{\mathcal{O}_D}{2m_b^3}$$

$$\mathcal{O}_0 = \bar{h}_v h_v \qquad \mathcal{O}_v = \bar{h}_v v\pi h_v \qquad \mathcal{O}_\pi = \bar{h}_v \pi_\perp^2 h_v$$

$$\mathcal{O}_G = \bar{h}_v \frac{1}{2} [\gamma_\mu, \gamma_\nu] \pi_\perp^\mu \pi_\perp^\nu h_v \qquad \mathcal{O}_D = \bar{h}_v [\pi_\perp^\mu, [\pi_\perp^\mu, \pi v]] h_v$$

- Project amplitude of quark to quark-gluon scattering onto HQET operators

Computational details



- Compute three-loop Feynman diagrams
  - use `LiteRed` [Lee'12,'13] for reduction
  - use REDUCE [Hearn'04] for symbolic manipuation
- Perform the renormalization
  - Heavy Quark Fields in on-shell scheme
  - Charm mass and ρ_D in $\overline{\mathrm{MS}}$
  - Subtlety: Mixing of operators of different dimensionality in HQET has to be taken into account for proper renormalization of the Darwin term.
- All poles cancel ✓

Result

[Mannel,Pivovarov'19]

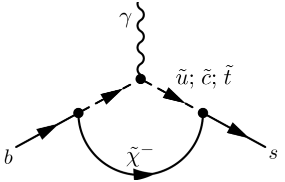
- Obtain analytic result for a_{rD}
- Numerically, have $(m_c^2/m_b^2 = 0.07,\ \alpha_s(m_b) = 0.2)$

$$
\begin{aligned}
a_D &= -57.159 + \frac{\alpha_s(m_b)}{4\pi}(-56.594\, C_A + 408.746\, C_F) \\
&= -57.159 + \frac{\alpha_s(m_b)}{4\pi} 375.213 \\
&= -57.159 \left(1 - \frac{\alpha_s(m_b)}{4\pi} 6.564\right) \\
&= -57.159\,(1 - 0.10)
\end{aligned}
$$

- $\rho_D = \rho_D(m_b)$
- Leading order has a sizeable coefficient
- QCD corrections have the expected size.
- Impact on V_{cb} will be small but visible

Inclusive $\bar{B} \to X_s\gamma$

- One of the standard candles in the search for NP in the quark flavour sector
- Flavour-changing neutral current process
- Dominant contribution is loop-induced
  - Indirectly sensitive to new particles
- Plays a prominent role in global fits



- Pre-2020 SM prediction vs. measurement of CP- and isospin-averaged BR with $E_\gamma > 1.6$ GeV

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.36 \pm 0.23) \times 10^{-4}$$ [Misiak et al.'15]

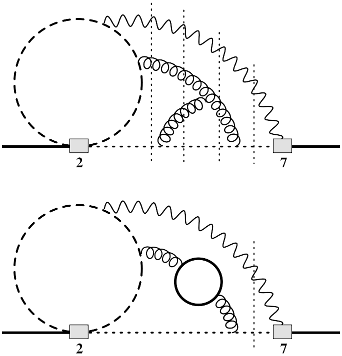
$$\mathcal{B}_{s\gamma}^{\rm exp.} = (3.32 \pm 0.15) \times 10^{-4}$$ [HFLAV'19]

- SM prediction is based on the formula

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \mathcal{B}(\bar{B} \to X_c \ell \bar{\nu}) \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\rm em}}{\pi\, C} \left[P(E_0) + N(E_0) \right],$$

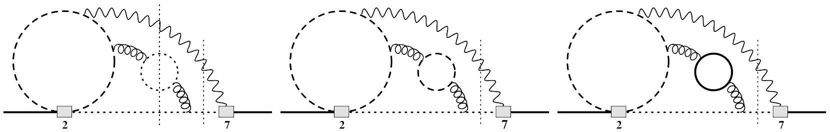
Perturbative corrections

- Many perturbative corrections known through to NNLO
- One of the largest uncertainties ($\sim 3\%$) comes from interpolation in m_c in $Q_{1,2} - Q_7$ interference
- Currently underway: Exact charm-mass dependence of $Q_{1,2} - Q_7$ interference at NNLO



- Large m_c expansion [Misiak,Steinhauser'06]
- Calculation f. $m_c = 0$ [Misiak,Steinhauser,Czakon,TH,et al.'15]
- Exact m_c dependence of fermionic part [Misiak,Rehman,Steinhauser'20]
- Exact m_c dependence of full result [Misiak,Steinhauser,TH,et al.,w.i.p.]

NNLO calculation



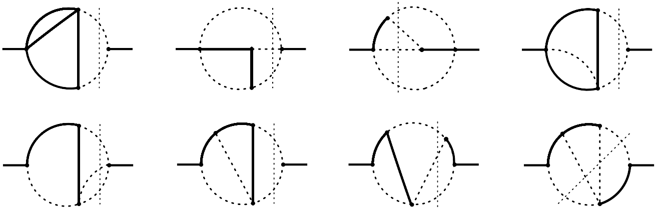
- Consider cuts of 4-loop propagator diagrams
- Reduce to master integrals $\vec{J}$ w. FIRE6[Smirnov,Chuharev'19] and KIRA[Maierhöfer,Usovitsch,Uwer'18]
- Compute masters with differential equations w.r.t. $z = m_c^2/m_b^2$

$$\frac{d}{dz}\,\vec{J} = M\,\vec{J}$$

- Solve differential equations numerically
  using half-ellipses in the complex plane

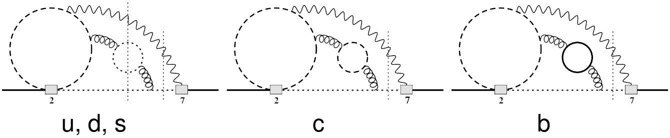
[Hindmarsch et al.; Czakon]

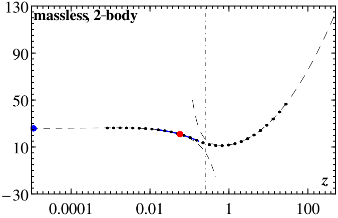
NNLO calculation

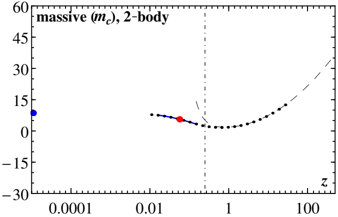


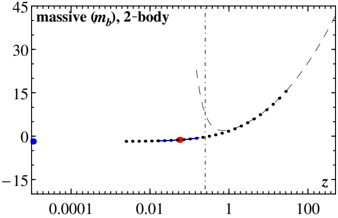
- Implement boundary condition at large z (i.e. $m_c \gg m_b$)
  - Obtain single-scale integrals with two, three and four-particle cuts
    - solve analytically (${}_pF_q$, Mellin Barnes, ...) [Misiak,Steinhauser,Mishima,Rehman,Czaja,TH]

NNLO results









- Partially known
  - Analytic small-z expansion
    [Bieri,Greub,Steinhauser'03]
  - Numerical results
    [Boughezal,Czakon,Schutzmeier'07]

Nonperturbative quantities

[Misiak,Rehman,Steinhauser'20]

- Semileptonic phase space factor

$$
\begin{aligned}
C &= \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e\bar{\nu}]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]} \\
&= g(z)\ \{0.903 - 0.588\,[\alpha_s(4.6\,\mathrm{GeV}) - 0.22] + 0.0650\,[m_{b,\mathrm{kin}} - 4.55] \\
&- 0.1080\,[m_c(2\,\mathrm{GeV}) - 1.05] - 0.0122\,\mu_G^2 - 0.199\,\rho_D^3 + 0.004\,\rho_{LS}^3\}
\end{aligned}
$$

  - Determined by using HQET methods from measurements of $\bar{B} \to X_c \ell\bar{\nu}$ decay spectra

[Alberti,Gambino,Healey,Nandi'14]

Nonperturbative quantities

[Misiak,Rehman,Steinhauser'20]

- Semileptonic phase space factor

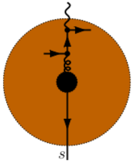
$$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e \bar{\nu}]}{\Gamma[\bar{B} \to X_u e \bar{\nu}]}$$

$$= g(z)\ \{0.903 - 0.588\,[\alpha_s(4.6\,\text{GeV}) - 0.22] + 0.0650\,[m_{b,\text{kin}} - 4.55]$$

$$- \ 0.1080\,[m_c(2\,\text{GeV}) - 1.05] - 0.0122\,\mu_G^2 - 0.199\,\rho_D^3 + 0.004\,\rho_{LS}^3\}$$

  - Determined by using HQET methods from measurements of $\bar{B} \to X_c \ell \bar{\nu}$ decay spectra [Alberti,Gambino,Healey,Nandi'14]

- Gluon-photon conversion in $Q_7 - Q_8$ interference



$$\Gamma[B^- \to X_s \gamma] \simeq A + BQ_u + CQ_d + DQ_s,$$

$$\Gamma[\bar{B}^0 \to X_s \gamma] \simeq A + BQ_d + CQ_u + DQ_s$$

$$\frac{\delta\Gamma_c}{\Gamma} \simeq \frac{Q_u + Q_d}{Q_d - Q_u}\left[1 + 2\,\frac{D - C}{C - B}\right]\Delta_{0-} = -\frac{1}{3}(1 \pm 0.3)\Delta_{0-} = (0.16 \pm 0.74)\%$$

  - Δ_{0-}: Isospin asymmetry, measured at Belle
  - $C - D$ vanishes in isospin limit, assume 30% SU(3) breaking

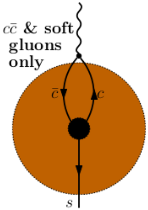
Nonperturbative quantities

- Resolved photon contribution in $Q_{1,2} - Q_7$ interference

[pics from Misiak'09]

[Voloshin'96;Buchalla,Isidori,Rey'97;Benzke,Hurth,Fickinger,Turczyk'17'20;Gunawardana,Paz'19]

$$N(E_0) \sim C_7 \left(C_2 - \frac{1}{6}C_1\right) \left(-\frac{\mu_G^2}{27m_c^2} + \frac{\Lambda_{17}}{m_b}\right) \equiv \delta N_V \, \kappa_V$$



$$\Lambda_{17} = e_c \, \mathrm{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b\,\omega_1}\right) + \frac{m_b\,\omega_1}{12m_c^2}\right] h_{17}(\omega_1, \mu)$$

  - h_{17}: soft function, modeled in [Gunawardana,Paz'19]
  - Yields $\Lambda_{17} \in [-24, 5]$ MeV $\Longrightarrow$ $\kappa_V = 1.2 \pm 0.3$

[see also Benzke,Lee,Neubert,Paz'10;Benzke,Hurth'20]

Nonperturbative quantities

- Resolved photon contribution in $Q_{1,2} - Q_7$ interference

[pics from Misiak'09]

[Voloshin'96;Buchalla,Isidori,Rey'97;Benzke,Hurth,Fickinger,Turczyk'17'20;Gunawardana,Paz'19]

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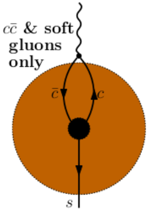

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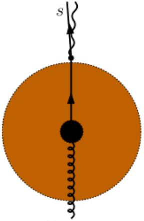
[see also Benzke,Lee,Neubert,Paz'10;Benzke,Hurth'20]

- Collinear photon radiation in $Q_8 - Q_8$ interference
  - $P(E_0)$ receives corrections $\propto |C_8|^2 \log(m_b/m_s)$
  - Small contribution (<1%), but large uncertainty
  - Vary $\log(m_b/m_s) \in [\log(10), \log(50)]$ [Czakon et al.'15]
  - Additional nonperturbative effects $\propto |C_8|^2$ impact $\mathcal{B}_{s\gamma}$ in range $[-0.3, 1.9]$% [Benzke,Lee,Neubert,Paz'10]
  - Reproduce numerically by

$$\log(m_b/m_s) \to \kappa_{88} \log(50) \quad \text{and} \quad \kappa_{88} = 1.7 \pm 1.1$$



NP constraints

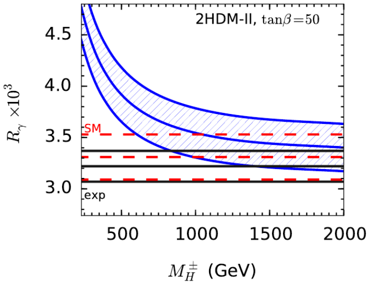
- Updated SM prediction

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.40 \pm 0.17) \times 10^{-4}$$ [Misiak,Rehman,Steinhauser'20]

$$\mathcal{B}_{s\gamma}^{\rm exp.} = (3.32 \pm 0.15) \times 10^{-4}$$ [HFLAV'19]

$$R_\gamma = (\mathcal{B}_{s\gamma} + \mathcal{B}_{d\gamma})/\mathcal{B}_{c\ell\nu} = (3.35 \pm 0.16) \times 10^{-3}$$ [Misiak,Rehman,Steinhauser'20]

- Most prominent NP application: charged-Higgs mass in type-II 2HDM

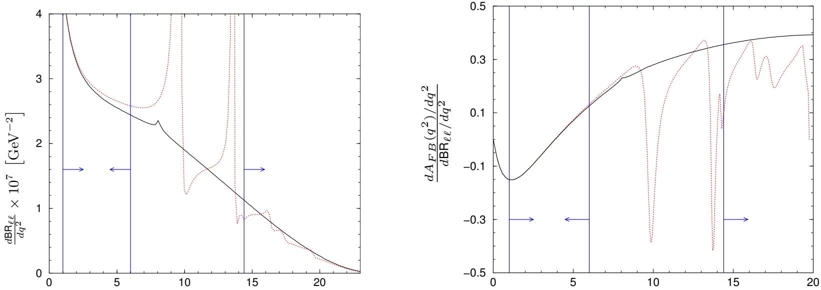


- Latest bound: $M_{H+} > 800$ GeV at 95% C.L. [Misiak,Rehman,Steinhauser'20]

- Double differential decay width ($z = \cos\theta_\ell$) [Lee,Ligeti,Stewart,Tackmann'06]

$$\frac{d^2\Gamma}{dq^2\,dz} = \frac{3}{8}\,\left[(1+z^2)\,H_T(q^2) \,+\, 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right]$$

Note: $\displaystyle \frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2)\,, \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$



q^2 [GeV^2][Ghinculov,Hurth,Isidori,Yao] q^2 [GeV^2][Ghinculov,Hurth,Isidori,Yao]

- Low-q^2 region: $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$
- High-q^2 region: $q^2 > 14.4\,\mathrm{GeV}^2$

Inclusive $\bar{B} \to X_s \ell^+ \ell^-$

- Dependence of the H_i on Wilson coefficients ($s = q^2/m_b^2$)

$$H_T(q^2) \propto 2s(1-s)^2 \left[\left| C_9 + \frac{2}{s} C_7 \right|^2 + |C_{10}|^2 \right]$$
$$H_A(q^2) \propto \; -4s(1-s)^2 \;\; \mathrm{Re}\left[C_{10} \left(C_9 + \frac{2}{s} C_7 \right) \right]$$
$$H_L(q^2) \propto (1-s)^2 \left[\left| C_9 + 2\, C_7 \right|^2 + |C_{10}|^2 \right]$$

- Normalisation $$\frac{d\, \mathcal{B}(\bar{B} \to X_s ll)}{d\, \hat{s}} = \mathcal{B}^{exp.}_{b \to c\, e\, \nu} \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{1}{C} \frac{d\Gamma(\bar{B} \to X_s\, ll)/d\hat{s}}{\Gamma(\bar{B} \to X_u e \bar{\nu})}$$

- LFU ratio $$R_{X_s}[q_m^2, q_M^2] \equiv \int_{q_m^2}^{q_M^2} dq^2 \, \frac{d\Gamma_{\ell=\mu}}{dq^2} \Big/ \int_{q_m^2}^{q_M^2} dq^2 \, \frac{d\Gamma_{\ell=e}}{dq^2}$$

- High-q^2 region, introduce the ratio $$\mathcal{R}(s_0) = \frac{\int_{\hat{s}_0}^1 \mathrm{d}\hat{s} \;\; d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s}}{\int_{\hat{s}_0}^1 \mathrm{d}\hat{s} \;\; d\Gamma(\bar{B}^0 \to X_u \ell \nu)/d\hat{s}}$$

  [Ligeti,Tackmann'07]

  - Normalize to semileptonic $\bar{B}^0 \to X_u \ell \nu$ rate with the same cut
    Need differential semi-leptonic $b \to u$ rate

Perturbative and non-perturbative corrections

$$\Gamma(\bar{B} \to X_s\,\ell\ell) = \Gamma(b \to X_s\,\ell\ell) \;+\; \text{power corrections}$$

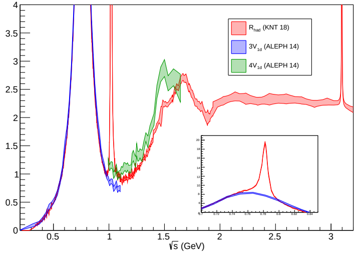
- Pert. corrections at quark level are known to NNLO QCD + NLO QED
  [Misiak,Buras,Münz,Bobeth,Urban,Asatrian,Asatryan,Greub,Walker,Bobeth,Gambino,Gorbahn,Haisch,Blokland]
  [Czarnecki,Melnikov,Slusarczyk,Bieri,Ghinculov,Hurth,Isidori,Yao,Greub,Pilipp,Schüpbach,Lunghi,TH]
  - Fully differential QCD corrections at NNLO for $Q_{9,10}$ also known [Brucherseifer,Caola,Melnikov'13]
- $1/m_b^2$, $1/m_b^3$ and $1/m_c^2$ non-pert. corrections [Falk,Luke,Savage'93] [Ali,Hiller,Handoko,Morozumi'96] [Bauer,Burrell'99; Buchalla,Isidori,Rey'97]
- New in 2020 update [Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]
  - SM prediction of all angular observables + LFU ratio R_{X_s}
  - More sophisticated implementation of factorizable $c\bar{c}$ contributions via KS approach [Krüger, Sehgal'96]
  - Resolved contributions from charm loops [Benzke,Hurth,Fickinger,Turczyk'17-'20]
  - Monte Carlo study of collinear photon radiation, tailored for Belle II analysis
  - Comprehensive model-independent new-physics analysis

Resonances

[Hurth,Jenkins,Lunghi,Qin,Vos,TH'19]

- Idea of Krüger and Sehgal: [Krüger,Sehgal'96]
  - Assume factorization of $b \to s(d)$ and $c\bar{c}$ current
  - Replace one-loop perturbative function by dispersion integral over $R_{\rm had}$
- Use data from BES, BaBar, ALEPH in the resonance regions

  [see Keshavarzi,Nomura,Teubner'18]



- Use perturbation theory (program `rhad`) for asymptotics at large s [Harlander,Steinhauser'02]
- In the replacement of the perturbative function,

$$h_q(s) \to h_q(s_0) + \frac{s - s_0}{\pi} \int_{s_0}^{\infty} dt \, \frac{\text{Im}[h_q(t + i\epsilon)]}{(t - s_0)(t - s - i\epsilon)}.$$

adjust subtraction point to $s_0 = -(5\ \text{GeV})^2$

SM predictions

- Branching ratio low-q^2 region [Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

$$\mathcal{B}[1,6]_{\mu\mu} = (17.29 \pm 0.76_{\text{scale}} \pm 0.19_{m_t} \pm 0.39_{C,m_c} \pm 0.20_{m_b} \pm 0.09_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.26_{\text{BR}_{\text{sl}}} \pm 0.12_{\lambda_2} \pm 0.86_{\text{resolved}}) \times 10^{-7} = (17.29 \pm 1.28) \times 10^{-7}$$

  - Total error 7.5%, dominated by scale uncertainty and resolved contributions

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[Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

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  - Total error 7.5%, dominated by scale uncertainty and resolved contributions
- Ratio R_{X_s} has small uncertainty, $R_{X_s}[1,6] = 0.971 \pm 0.003$
- Branching ratio, high-q^2 region

$$\mathcal{B}[> 14.4]_{\mu\mu} = (2.38 \pm 0.27_{\text{scale}} \pm 0.03_{m_t} \pm 0.04_{C,m_c} \pm 0.21_{m_b} \pm 0.002_{\text{CKM}} \pm 0.04_{\text{BR}_{\text{sl}}} \pm 0.006_{\alpha_s} \pm 0.12_{\lambda_2} \pm 0.57_{\rho_1} \pm 0.54_{f_{u,s}}) \times 10^{-7} = (2.38 \pm 0.87) \times 10^{-7}$$

  - Total error >30%, dominated by HQET annihilation matrix elements

SM predictions

- Branching ratio low-q^2 region

[Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

$$\mathcal{B}[1,6]_{\mu\mu} = (17.29 \pm 0.76_{\text{scale}} \pm 0.19_{m_t} \pm 0.39_{C,m_c} \pm 0.20_{m_b} \pm 0.09_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.26_{\text{BR}_{\text{sl}}} \pm 0.12_{\lambda_2} \pm 0.86_{\text{resolved}}) \times 10^{-7} = (17.29 \pm 1.28) \times 10^{-7}$$

  - Total error 7.5%, dominated by scale uncertainty and resolved contributions
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  - Total error >30%, dominated by HQET annihilation matrix elements
- Different normalisation

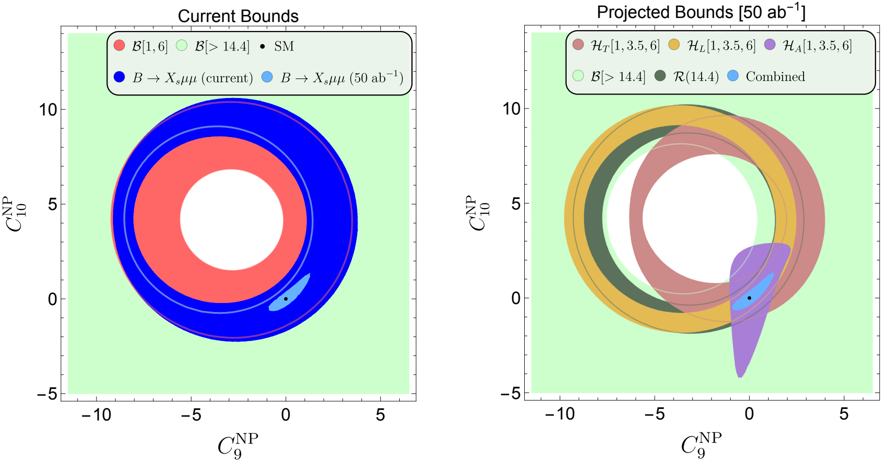
$$\mathcal{R}(14.4)_{\mu\mu} = (25.33 \pm 0.27_{\text{scale}} \pm 0.29_{m_t} \pm 0.14_{C,m_c} \pm 0.03_{m_b} \pm 0.07_{\alpha_s} \pm 1.09_{\text{CKM}} \pm 0.04_{\lambda_2} \pm 0.83_{\rho_1} \pm 1.29_{f_{u,s}}) \times 10^{-4} = (25.33 \pm 1.93) \times 10^{-4}$$

  - Total error <10%

NP constraints

[Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

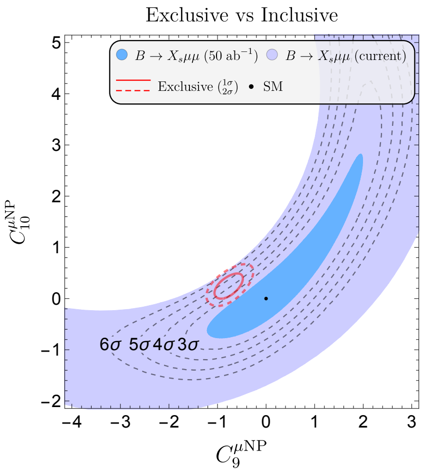
- Current and projected bounds from inclusive $\bar{B} \to X_s \ell^+ \ell^-$

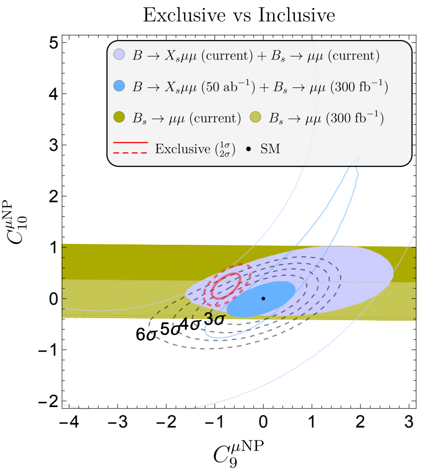


NP constraints

[Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

- Projected bounds using interplay with exclusive $\bar{B} \to K^{(*)}\ell^+\ell^-$ and $\bar{B}_s \to \mu^+\mu^-$





Ongoing projects

- Many-body contributions to $\bar{B} \to X_s\gamma$ (Huber, Moos)
- Full m_c-dependence of $\bar{B} \to X_s\gamma$ at NNLO (Huber, Steinhauser et al.)
- $\bar{B} \to X_s\ell^+\ell^-$ with an M_{X_s}-cut (Huber et al.)
- HQE for charm (Mannel, Moreno et al.)
- Improvement of inclusive determination of $|V_{cb}|$ and $|V_{ub}|$ (Mannel et al.)