Project C2a

Hadronic Matrix Elements and Exclusive Semileptonic Decays

Thorsten Feldmann and Thomas Mannel

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Particle Physics Phenomenology after the Higgs Discovery

Collaborative Research Center TRR 257

UNIVERSITÄT SIEGEN

Project C2a: Work Areas and People

Work Areas:

WA 1	Factorization and Light-Cone Distribution Amplitudes
WA 2	Development of QCD Sum Rules and Related Methods
WA 3	New Channels and Multi-Hadron Final states
WA 4	Inclusive Rate from the Sum over Exclusive Channels

People:

- Jao Ji (PD) C2a
- Nicolas Seitz (PhD) C2a
- Marzia Bordone (PD) C2b
- Robin Brüser (PD) C2b
- Alex Khodjamirian (Prof.) U Siegen
- Rusa Mandal (PD) AvH

- Thorsten Feldmann U Siegen
- Thomas Mannel U Siegen

Project C2a: Papers and Current Projects

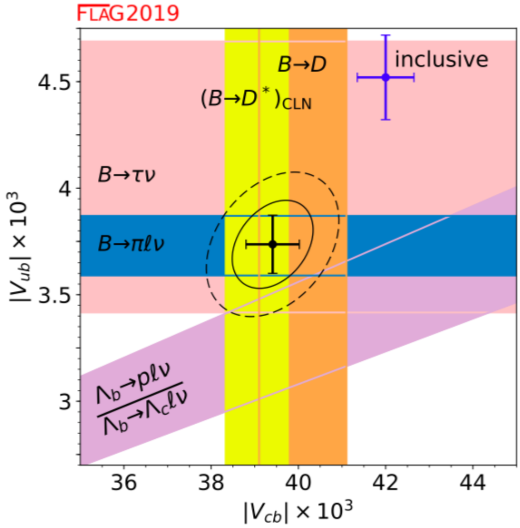
Papers:

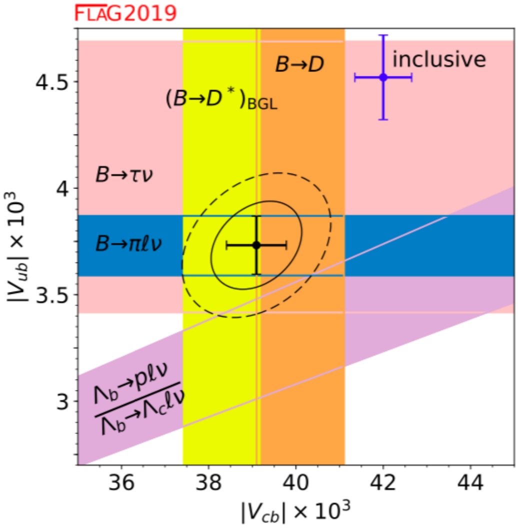
1. **M. Bordone**, N. Gubernari, D. van Dyk and M. Jung, "Heavy-Quark Expansion for $\bar{B}_s \to D_s^{(*)}$ Form Factors and Unitarity Bounds beyond the $SU(3)_F$ Limit," arXiv:1912.09335 [hep-ph].
2. **R. Brüser**, Z. L. Liu and M. Stahlhofen, "Three-loop soft function for heavy-to-light quark decays," arXiv:1911.04494 [hep-ph].
3. **M. Bordone**, M. Jung and D. van Dyk, "Theory determination of $\bar{B} \to D^{(*)}\ell^-\bar{\nu}$ form factors at $\mathcal{O}(1/m_c^2)$," Eur. Phys. J. C **80** (2020) no.2, 74
4. **A. Khodjamirian, R. Mandal and T. Mannel**, "Inverse moment of the B_s-meson distribution amplitude from QCD sum rule," [arXiv:2008.03935 [hep-ph]], JHEP in print
5. S. Cheng, **A. Khodjamirian** and A. V. Rusov, "The pion light-cone distribution amplitude from the pion electromagnetic form factor," [arXiv:2007.05550 [hep-ph]].

Current Projects:

- NLO-improved parametrization of B-meson LCDA (**TF**, van Dyk)
- Sum rule for $B_c \to J/\psi\ell\nu$ (**TM, Bordone, Khodjamirian**)
- Sum rule for $B \to D^{**}\ell\bar{\nu}$ (**TM, Khodjamirian, Mandal**)

Context: Current tensions in the determination of V_{cb}





Scrutinize the exclusive determination of V_{cb}

- Form factors, including the $1/m$ expansion
- Other effects: QED etc.

Scrutinize the inclusive determination of V_{cb} (part of C1a)

- Contaminations from $b \to u$ and $b \to \tau$
- Duality violations

Here: Discussion of the form factors in $\bar{B} \to D^{(*)} \ell^- \bar{\nu}$

Reminder: From factors for $\bar{B} \to D^{(*)}$ transitions:

$$\langle D(k)|\,\bar{c}\gamma^\mu b\,|\bar{B}(p)\rangle = \left[(p+k)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu\right] f_+^{B\to D}(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^\mu f_0^{B\to D}(q^2)\,,$$

$$\langle D(k)|\,\bar{c}\sigma^{\mu\nu} b\,|\bar{B}(p)\rangle = \frac{2i}{M_B + M_D}(k^\mu p^\nu - p^\mu k^\nu) f_T(q^2,\mu)\,,$$

$$\langle D^*(k,\eta)|\,\bar{c}\gamma^\mu b\,|\bar{B}(p)\rangle = -\epsilon^{\mu\nu\rho\sigma}\eta_\nu^*(k)\,p_\rho\,k_\sigma \frac{2\,V(q^2)}{M_B + M_{D^*}}\,,$$

$$\langle D^*(k,\eta)|\,\bar{c}\gamma^\mu\gamma_5 b\,|\bar{B}(p)\rangle = i\eta_\nu^*\Big\{2M_{D^*}A_0(q^2)\frac{q^\mu q^\nu}{q^2} + 16\frac{M_B M_{D^*}^2}{\lambda}A_{12}\left[2p^\mu q^\nu - \frac{M_B^2 - M_{D^*}^2 + q^2}{q^2}q^\mu q^\nu\right]$$
$$+ (M_B + M_{D^*})\,A_1(q^2)\left[g^{\mu\nu} + \frac{2(M_B^2 + M_{D^*}^2 - q^2)}{\lambda}q^\mu q^\nu - \frac{2(M_B^2 - M_{D^*}^2 - q^2)}{\lambda}p^\mu q^\nu\right]\Big\}$$

$$\langle D^*(k,\eta)|\,\bar{c}\sigma^{\mu\nu} b\,|\bar{B}(p)\rangle = i\eta_\alpha^*\epsilon^{\mu\nu}{}_{\rho\sigma}\Big\{-\left[\left((p+k)^\rho - \frac{M_B^2 - M_{D^*}^2}{q^2}q^\rho\right)g^{\alpha\sigma} + \frac{2}{q^2}p^\alpha p^\rho k^\sigma\right]T_1(q^2)$$
$$-\left(\frac{2}{q^2}p^\alpha p^\rho k^\sigma - \frac{M_B^2 - M_{D^*}^2}{q^2}q^\rho g^{\alpha\sigma}\right)T_2(q^2) + \frac{2}{M_B^2 - M_{D^*}^2}p^\alpha p^\rho k^\sigma T_3(q^2)\Big\}\,.$$

Analysis up to $1/m_b, 1/m_c^2, \alpha_s$

Theoretical Machinery

- Heavy Quark Limit
- Fixed Point QCD Sum Rules
- Light-Cone Sum rules

Heavy Quark Limit relations: $q = M_B v - M_{D^{(*)}} v'$, $w = vv'$

- Only a single form factor for $m_c, m_b \to \infty$
- Corrections are of order $1/m_c$ and $1/m_b$
- Some Corrections are forbidden due to Ademollo Gatto / Luke's theorem
- Calculable hard Gluon Corrections $\alpha_s(m_c m_b)$

Analysis up to $1/m_b, 1/m_c^2, \alpha_s$

Form Factor Parametrizations

Conformal Variable:

$$z = \frac{(\sqrt{w+1} - \sqrt{2})}{(\sqrt{w+1} + \sqrt{2})}$$

- BGL (Boyd, Grinstein, Lebed) parametrization

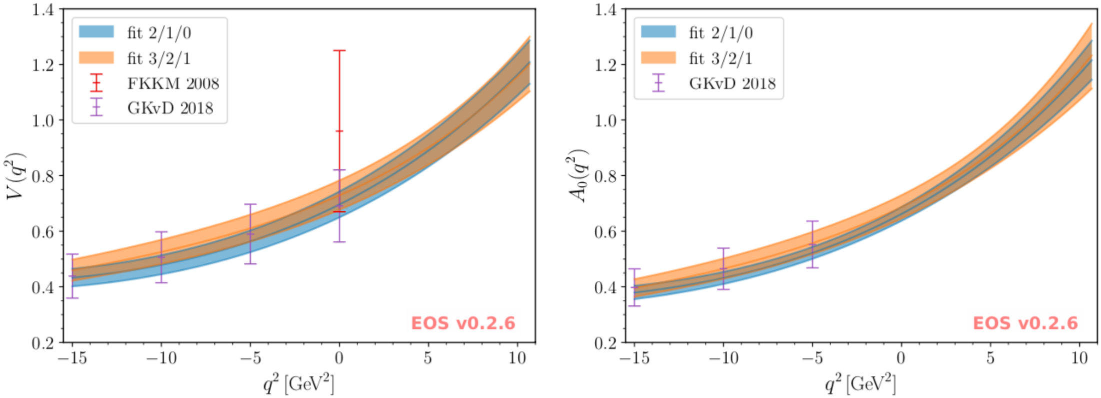
$$F(z) = \frac{1}{P_F(z)\phi_F(z)} \sum_{n=0}^{\infty} a_n z^n \qquad \sum_n a_n^2 \leq 1$$

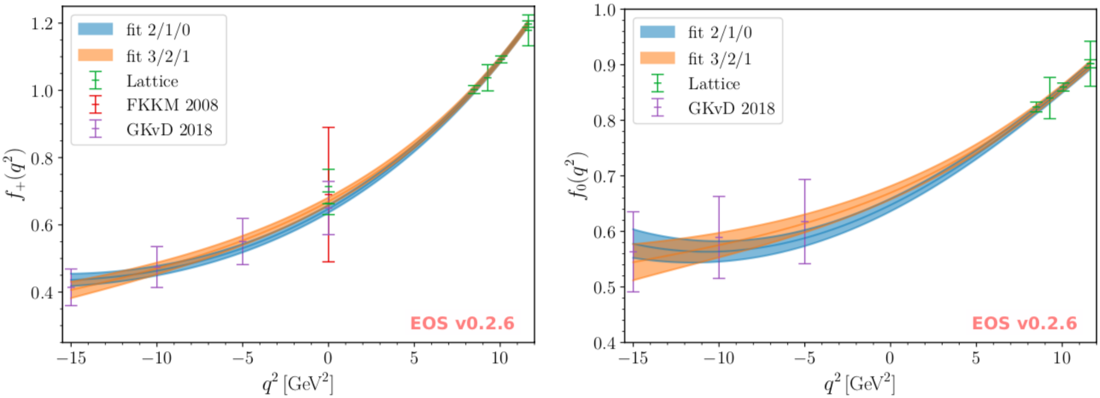
- CLN (Caprini, Lellouch, Neubert): One parameter form:

$$F(z) = F(0)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 + (231\rho^2 - 91)z^3\right]$$

Fit to theoretical predictions from

- Lattice data from most recent simulations from HPQCD and FNAL/MILC collaborations
- Subleading form factor contributions from three-point sum rules (Neubert, Ligeti Nir 93)
- Light-Cone Sum rules for the form factors (Gubernari, Kokulu, van Dyk 18)


fit 2/1/0
fit 3/2/1
FKKM 2008
GKvD 2018
$V(q^2)$
q^2 [GeV^2]
EOS v0.2.6
fit 2/1/0
fit 3/2/1
GKvD 2018
$A_0(q^2)$
q^2 [GeV^2]
EOS v0.2.6


fit 2/1/0
fit 3/2/1
Lattice
FKKM 2008
GKvD 2018
EOS v0.2.6
$f_+(q^2)$
q^2 [GeV^2]
1.2
1.0
0.8
0.6
0.4
−15
−10
−5
0
5
10
fit 2/1/0
fit 3/2/1
Lattice
GKvD 2018
EOS v0.2.6
$f_0(q^2)$
q^2 [GeV^2]
1.0
0.9
0.8
0.7
0.6
0.5
0.4

	scenarios				
model	2/1/0	3/2/1	3/2/1	3/2/1	3/2/1
exp. likelihood	—	—	2017	2018	all exp.
$\mathcal{B}(\bar{B}^0 \to D^+\{e^-,\mu^-\}\bar{\nu})/\|V_{cb}\|^2$	12.99 ± 0.35	13.48 ± 0.37	—	—	13.56 ± 0.35
$\mathcal{B}(\bar{B}^0 \to D^{*+}\{e^-,\mu^-\}\bar{\nu}/\|V_{cb}\|^2$	32.33 ± 1.28	33.16 ± 2.15	31.74 ± 1.46	32.19 ± 1.03	32.00 ± 1.03
correlation	0.34	0.14	—	—	0.10
$\|V_{cb}\| \times 10^3$ from $\bar{B} \to D\{e^-,\mu^-\}\bar{\nu}$	41.5 ± 1.2	40.7 ± 1.2	—	—	40.6 ± 1.1
$\|V_{cb}\| \times 10^3$ from $\bar{B} \to D^*\{e^-,\mu^-\}\bar{\nu}$	39.8 ± 1.2 —	39.3 ± 1.7 —	40.1 ± 1.3 (39.5 ± 1.9)	39.8 ± 1.0 (39.0 ± 1.3)	40.0 ± 1.1 —
$\|V_{cb}\| \times 10^3$ combined incl. corr.	40.7 ± 1.0	40.2 ± 1.0	—	—	40.3 ± 0.8

$$V_{cb}^{\text{excl}} = (40.3 \pm 0.8) \times 10^{-3}$$

Calcuation for non-strange B mesons: Brain, Ivanov, Korchemski 2004

Light Cone distribution for the B_s

$$\langle 0|\bar{s}(tn)\, \not{n}[tn,0]\gamma_5 h_v(0)|\bar{B}_s(v)\rangle = iF_{B_s}(\mu)\int_0^\infty dk\, e^{-ikt}\phi_+^{B_s}(k,\mu)\,,$$

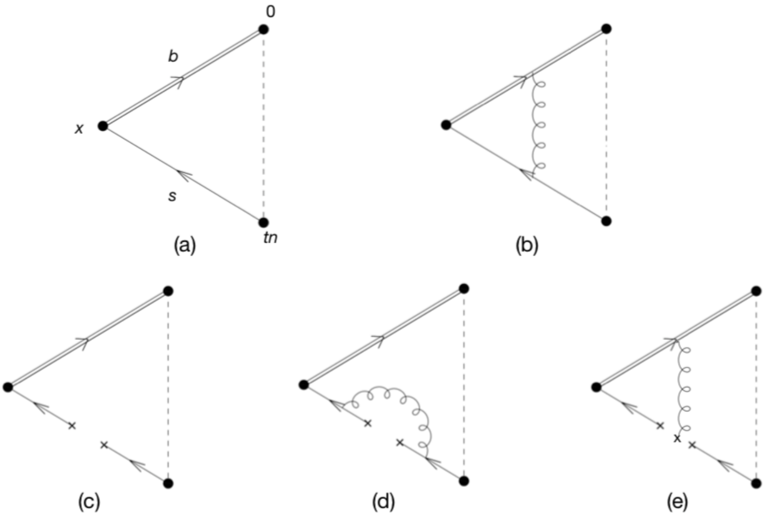
Inverse moment

$$\lambda_{B_{(s)}}^{-1}(\mu) = \int_0^\infty \frac{dk}{k}\,\phi_+^{B_{(s)}}(k,\mu)\,,$$

Correlation function in HQET $\to$ sum rule

$$\mathcal{P}_{5s}(\omega,t) = i\int d^4x e^{-i\omega v\cdot x}\langle 0|T\{\bar{s}(tn)\, \not{n}[tn,0]\gamma_5 h_v(0)\bar{h}_v(x) i\gamma_5 s(x)\}|0\rangle$$

$$= \frac{\langle 0|\bar{s}(tn)\, \not{n}[tn,0]\gamma_5 h_v(0)|B_s(v)\rangle\langle B_s(v)|\bar{h}_v i\gamma_5 s|0\rangle}{\bar{\Lambda}_s-\omega} + \cdots = \frac{[F_{B_s}(\mu)]^2}{2(\bar{\Lambda}_s-\omega)}\int_0^\infty dk\, e^{-ikt}\phi_+^{B_s}(k,\mu) + \ldots$$


0
b
x
s
tn
(a)
(b)
(c)
(d)
(e)

Condensate Contibutions are singular, need to be modeled by non-local condensates

$$\langle 0|\bar{q}(x)[x,0]q(0)|0\rangle = \langle \bar{q}q\rangle f_S(x^2) \qquad f_S(x^2) = \int_0^\infty \tilde{f}_S(\nu) e^{\nu x^2} d\nu \,,$$

Local OPE yields

$$\tilde{f}_S(\nu) = \delta(\nu) - \frac{m_0^2}{16}\delta'(\nu) + \dots$$

Two models (Braun, Ivanov, Korchemsky)

- Model 1: $\tilde{f}_S(\nu) = \delta(\nu - m_0^2/16)$
- Model 2: $\tilde{f}_S(\nu) = \frac{\lambda^{p-2}}{\Gamma(p-2)} \nu^{1-p} e^{-\lambda/\nu} \,, \quad p = 3 + \frac{16\lambda}{m_0^2}$

Results

$$\lambda_{B_s} = 438 \pm 150 \text{ MeV}\,, \quad \lambda_B = 383 \pm 153 \text{ MeV}$$

$$\frac{\lambda_{B_s}}{\lambda_B} = 1.19 \pm 0.14$$

Snapshot on some Work in Progress

Start from the correlation function

$$T(q,P) = i \int d^4x\, e^{iqx} \langle B_c(P)| T \left[\bar{b}(x)\Gamma_1 c(x)\, \bar{c}(0)\Gamma_2 c(0)\right] |0\rangle$$

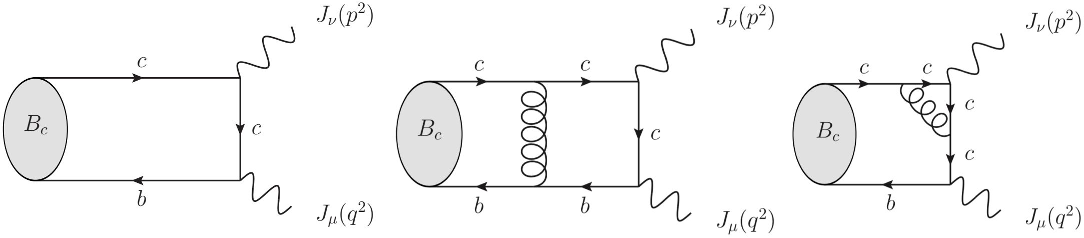
The dispersion relation in the variable p^2 has the spectral function

$$W_{\mu\nu}(q,P) = i \int d^4x\, e^{iqx} \langle B_c(P)|\bar{b}(x)\gamma_\mu c(x)\, \bar{c}(0)\gamma_\nu c(0)|0\rangle$$

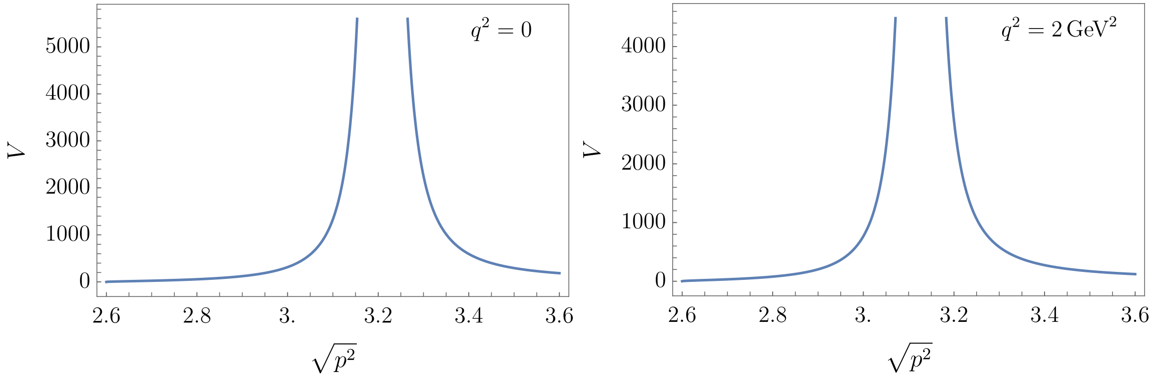
from which we get the sum rule

$$\begin{aligned} W_{\mu\nu}(q,P) &= \int d^4x\, e^{iqx} \sum_n \langle B_c(P)|\bar{b}(x)\gamma_\mu c(x)|n\rangle \, \langle n|\bar{c}(0)\gamma_\nu c(0)|0\rangle \\ &= \int \widetilde{dp}(2\pi)^4 \delta^4(P-p-q)\langle B_c(P)|\bar{b}\gamma_\mu c|J/\psi(p)\rangle \, \langle J/\psi(p)|\bar{c}\gamma_\nu c|0\rangle + \cdots \end{aligned}$$

For heavy b and c quark we can compute perturbatively



Spectral function for $m_c = 1.3$ GeV (Preliminary!!):



To get the $B_c \to J/\psi$ Form Factors

- Fix the duality threshold
- Numerical Evaluation
- Study the q^2 dependence

Outlook / ToDo List for C2a

WA1:

- generic parametrizations for higher-twist B-meson LCDAs
- implications for phenomenology, in particular for $B \to \gamma \ell \nu$ [→ C2b]
- applying the same ideas to shape functions for inclusive decays [→ WA4 + C1a]

WA2:

- Study the new sum rules for B_c form factors
- Study the subleading terms in the new B_c sum rule

WA3:

- semileptonic processes with bottom baryons
- B_c phenomenology

WA4:

- study of non-resonant contributions in $b \to c$ and $b \to u$
- decays into excited hadrons