Project C2b

Exclusive non-leptonic and rare *b*-quark decays

Guido Bell, Thorsten Feldmann and Tobias Huber

CRC Annual Meeting, 6.-8. October 2020

DFG TRR 257
particle **p**hysics **p**henomenology
after the **H**iggs discovery

UNIVERSITÄT SIEGEN

Project C2b: People and Work Areas

Name	*Position*	*financed by*	*Period*	*contributes to*	
Marzia Bordone	postdoc	C2b	Oct 19 - Sep 20	WA 4	WA 5
Nico Gubernari	postdoc	C2b	Oct 20 –	WA 2	WA 5
Gilberto T.-X.	postdoc	C2b	Oct 19 –	WA 1	WA 4
Nicolas Seitz	PhD	C2a	Jul 19 –	WA 2	
Oscar Cata	Akad. Rat	U Siegen		WA 5	
Rusa Mandal	postdoc	A.v.H. fellow		WA 5	

WA 1: NNLO QCD corrections.

WA 2: QED corrections.

WA 3: Power corrections (and factorization).

WA 4: Phenomenology of nonleptonic decays.

WA 5: Phenomenology of rare semileptonic and radiative decays.

Publications and Preprints:

1. **G. Bell**, M. Beneke, **T. Huber** and X. Q. Li,
"Two-loop non-leptonic penguin amplitude in QCD factorization,"
JHEP **04** (2020), 055. QFET → WA1

2. A. Lenz and **G. Tetlalmatzi-Xolocotzi**,
"Model-independent bounds on NP effects in non-leptonic tree-level decays of B-mesons,"
JHEP **07** (2020), 177. WA4

3. **T. Huber**, J. Virto and K. Vos, "3-Body non-leptonic heavy-to-heavy B decays at NNLO in QCD,"
arXiv:2007.08881 [hep-ph]. **WA4**

4. **M. Bordone**, N. Gubernari, **T. Huber**, M. Jung and D. van Dyk,
"A puzzle in $\bar{B}^0_{(s)} \to D^+_{(s)}\{\pi^-, K^-\}$ decays and extraction of the f_s/f_d fragmentation fraction,"
arXiv:2007.10338 [hep-ph]. WA4

5. **O. Catà** and **T. Mannel**, "Linking lepton number violation with B anomalies,"
arXiv:1903.01799 [hep-ph] QFET → WA5

6. **M. Bordone**, **O. Catà** and **T. Feldmann**,
"Effective theory approach to NP with flavour: general framework and a leptoquark example,"
JHEP **2001** (2020) 067. QFET → **WA5**

7. **M. Bordone**, **O. Catà**, **T. Feldmann** and **Rusa Mandal**,
"Constraining flavour patterns of scalar leptoquarks in the EFT ,"
arXiv:2010.03297 [hep-ph] **WA5**

Project C2b: Outline

1. Results from WA5 (phenomenology of rare decays)
   - Bordone/Cata/Feldmann P3H-19-032
   - Bordone/Cata/Feldmann/Mandal P3H-20-046
2. Results from WA4 (phenomenology of nonleptonic decays)
   - Huber/Virto/Vos P3H-20-037

- some technical aspects relevant for WA1 – WA3
  have also been discussed at SCET workshop of the CRC, January 2020

WA 5

LQ symbol	$[SU(3), SU(2), U(1)]$	Spin	F
S_3	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	0	−2
$\tilde{S}_1$	$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	0	−2
S_1	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	0	−2
$\bar{S}_1$	$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$	0	−2
R_2	$(\mathbf{3}, \mathbf{2}, 7/6)$	0	0
$\tilde{R}_2$	$(\mathbf{3}, \mathbf{2}, 1/6)$	0	0
U_3	$(\mathbf{3}, \mathbf{3}, 2/3)$	1	0
$\tilde{U}_1$	$(\mathbf{3}, \mathbf{1}, 5/3)$	1	0
U_1	$(\mathbf{3}, \mathbf{1}, 2/3)$	1	0
$\bar{U}_1$	$(\mathbf{3}, \mathbf{1}, -1/3)$	1	0
V_2	$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$	1	−2
$\tilde{V}_2$	$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	1	−2

[leptoquark table from arXiv:1903.04977]

"Effective Theory Approach to New Physics with Flavour: General Framework and a Leptoquark Example"

[Bordone, Cata, Feldmann 2019]

SMEFT is not built as an EFT of flavour, . . .

- NP operators in SM-EFT introduce various flavour coefficients.
  $\rightarrow$ too many free parameters for generic pheno analyses
- $\mathcal{O}(1)$ couplings would require NP scales above LHC reach.
  $\rightarrow$ global-fit scan would have to include "non-natural" regions of parameter-space
- minimal flavour violation cannot explain current "B-anomalies"
  $\rightarrow$ needs extension/generalization of MFV

"Effective Theory Approach to New Physics with Flavour: General Framework and a Leptoquark Example"

[Bordone, Cata, Feldmann 2019]

SMEFT is not built as an EFT of flavour, . . . but it can account for flavour patterns:

- extending MFV with new "flavour spurions" [Feldmann, Mannel 2006]
  $\to$ NP explanations for *B*-anomalies favour leptoquark scenarios
- providing a power-counting scheme
  $\to$ simplest solution: flavour-dependent Froggatt-Nielsen charges, for quark and lepton couplings

$\Rightarrow$ Rather general and systematic framework, that contains and extends MFV.

$\Rightarrow$ Spurion scenarios and FN charges to be fixed by Phenomenology.

- consider, as an example, a four-fermion operator in SMEFT: (→ LFU violation)

$$\frac{1}{\Lambda^2} \, [\mathcal{C}_{\ell q}]^{ij\alpha\beta} \, (\bar{Q}_i \gamma_\mu Q_j) \, (\bar{L}_\alpha \gamma^\mu L_\beta)$$

- flavour tensor $[\mathcal{C}_{\ell q}]^{ij\alpha\beta}$ introduces 3^4 free real parameters:

generic EFT:	$\mathcal{O}(1)$
MFV:	
FN charges:	
LQ+FN charges:	

- SM Yukawa matrices Y_U, Y_D
- small expansion parameter $\lambda \sim \sin\theta_C \sim 0.2$
- family-dependent charges b_Q^i etc. of a hypothetical $U(1)$ symmetry
- new "leptoquark spurion" $\Delta_{QL}^{i\alpha}$

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FN charges:	$\sim \lambda^{\left\lvert b_Q^i - b_Q^j + b_L^\alpha - b_L^\beta \right\rvert}$
LQ+FN charges:	$\sim (\Delta_{QL})^{i\beta}(\Delta_{QL}^\dagger)^{\alpha j} + \ldots \sim \lambda^{\left\lvert b_Q^i - b_L^\beta \right\rvert}\,\lambda^{\left\lvert b_L^\alpha - b_Q^j \right\rvert}$

- SM Yukawa matrices Y_U, Y_D
- small expansion parameter $\lambda \sim \sin\theta_C \sim 0.2$
- family-dependent charges b_Q^i etc. of a hypothetical $U(1)$ symmetry
- new "leptoquark spurion" $\Delta_{QL}^{i\alpha}$

Features:

- independent of particular *dynamical* realization in the UV $\surd$
- exponents in the power-counting fulfill triangle inequalities, like e.g.

$$\left|b_Q^i - b_L^\beta\right| + \left|b_L^\alpha - b_Q^j\right| \geq \left|b_Q^i - b_Q^j + b_L^\alpha - b_L^\beta\right|$$

$\to$ power-counting not violated by higher-orders in spurion expansion $\surd$

- allows to play with different scenarios in SMEFT, depending on:
  - which fundamental spurions to consider ($\Delta_{QL}, \ldots$),
  - which values for the FN charges to take (see below),
  - $+$ possibly additional simplifying assumptions (see below).

$\to$ test different scenarios against NP signals in flavour observables !

- $b \to sll$ processes (R_K, R_K^*, P_5')
- $b \to cl\nu$ processes (R_D, R_{D^*})
- B-meson mixing (ΔM_d, ΔM_s)
- W universality
- Z decays ($Z \to \nu\bar{\nu}$, $Z \to \ell^+\ell^-$)
- $B \to K^{(*)}\nu\bar{\nu}$
- $K^+ \to \pi^+\nu\bar{\nu}$,
- LFV kaon decays ($K_L \to \mu e$)
- LFV B-meson decays
- charged LFV ($\mu \to e\gamma$, $\tau \to \mu\gamma$)

Determining the FN charges:

- operators with (Dirac-) fermions only depend on charge differences
- left-handed quark charges from CKM angles (canonical) √

$$b_Q^1 \equiv 3\,, \qquad b_Q^2 \equiv 2\,, \qquad b_Q^3 \equiv 0$$

- L-R differences from Yukawa couplings √

$$|b_Q^1 - b_U^1| \simeq 8\,, \qquad |b_Q^2 - b_U^2| \simeq 4\,, \qquad b_U^3 = 0\,,$$
$$|b_Q^1 - b_D^1| \simeq 7\,, \qquad |b_Q^2 - b_D^2| \simeq 5\,, \qquad |b_Q^3 - b_D^3| \simeq 3$$

and

$$|b_L^1 - b_E^1| \simeq 9\,, \qquad |b_L^2 - b_E^2| \simeq 5\,, \qquad |b_L^3 - b_E^3| \simeq 3$$

$\Rightarrow$ 3 unconstrained charges plus a 2^8-fold discrete ambiguity (!)

- Further information from particular LQ scenario plus additional assumptions

Scenario with U_1 vector leptoquark: (hypercharge $y = 2/3$)

- involves two additional spurions:

  left-handed: $\Delta_{QL}^{i\alpha} \sim \lambda^{|b_Q^i - b_L^\alpha|}$, right-handed: $\Delta_{DE}^{i\alpha} \sim \lambda^{|b_D^i - b_E^\alpha|}$

- (approx.) tree-level relations for 4-fermion operators relates $SU(2)$-singlet and -triplet couplings

  $\rightarrow$ suppress e.g. rare $K \rightarrow \pi\nu\bar{\nu}$ decays

- use various precision and/or rare flavour observables
  $Z \rightarrow \nu\bar{\nu}$, $b \rightarrow s\mu^+\mu^-$, $b \rightarrow c\tau^-\nu$,
  $\bar{B}_d \rightarrow \tau^-\mu^+$, $\bar{B}_s \rightarrow \tau^\pm\mu^\mp$, $K_L \rightarrow \mu^\pm e^\mp$
  to constrain order of magnitude of spurion entries

  $\Rightarrow$ 24 possible combinations of FN charges

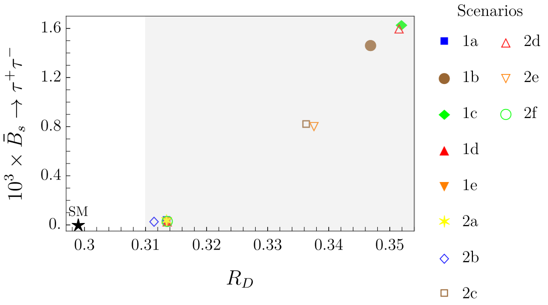
- fit of overall coefficients to experimental data
  (assuming real numbers in down-quark basis, and no fine-tuning/texture zeros)

  $\Rightarrow$ 11 phenomenologically viable scenarios for FN charges

Conclusions from U_1 vector leptoquark scenario:

- Simultaneous explanation of W universality tests and $R_D^{(*)}$ provides the strongest tensions in the fit.
- Interesting predictions for $B \to \tau^+\tau^-$ and $B \to \tau^\pm\mu^\mp$.

Example: Correlations between $B_s \to \tau^+\tau^-$ and R_D for different scenarios



Project C2b – WA 5

"Constraining flavour patterns of scalar leptoquarks in the EFT"

[Bordone, Cata, Feldmann, Mandal 2020 (today)]

Motivation:

- models with scalar leptoquarks are renormalizable (independent of UV completion)
  $\rightarrow$ consistent computation of loop-induced decays
- sensitive to decays like $B \rightarrow K^{(*)}\nu\nu$ and $K \rightarrow \pi\nu\nu$
  (which are naturally suppressed in U_1 scenario)
- EFT with FN charges $\rightarrow$ more sophisticated analyses compared to existing fits
  (without ad-hoc flavour structures and including all relevant observables)

"Which scalar leptoquark-inspired spurions can explain the B anomalies without upsetting existing low-energy constraints?"

Scenario with two scalar leptoquarks S_3 and S_1 (hypercharge $y = 1/3$)

spurions: $S_{QL}^{i\alpha} = c_L^{i\alpha}\,\lambda^{|b_Q^i - b_L^\alpha|}\,,\quad \tilde{S}_{QL}^{i\alpha} = \tilde{c}_L^{i\alpha}\,\lambda^{|b_Q^i - b_L^\alpha|}\,,\quad \tilde{S}_{UE}^{i\alpha} = c_R^{i\alpha}\,\lambda^{|b_U^i - b_E^\alpha|}$

- essentially two different solutions for viable FN charge assignments

Nominal scenario (A):

$$S_{QL}^{(A)} \sim \begin{pmatrix} \lambda^8 & \lambda^4 & \lambda^2 \\ \lambda^7 & \lambda^3 & \lambda \\ \lambda^5 & \lambda & -\lambda \end{pmatrix}, \quad \tilde{S}_{QL}^{(A)} \sim \begin{pmatrix} \lambda^8 & \lambda^4 & -\lambda^2 \\ \lambda^7 & \lambda^3 & \lambda \\ \lambda^5 & -\lambda & \lambda \end{pmatrix}, \quad \tilde{S}_{UE}^{(A)} \sim \begin{pmatrix} \lambda^{25} & \lambda^{17} & \lambda^{13} \\ \lambda^{12} & \lambda^4 & -\lambda^0 \\ \lambda^{14} & \lambda^6 & \lambda^2 \end{pmatrix}$$

corresponding to FN charges:

$$b_Q = (3,2,0)\,,\ b_U = (11,-2,0)\,,\quad b_L = (-5,-1,1)\,,\ b_E = (-14,-6,-2)$$

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corresponding to FN charges:

$$b_Q = (3,2,0)\,,\ b_U = (11,-2,0)\,, \quad b_L = (-5,-1,1)\,,\ b_E = (-14,-6,-2)$$

Some phenomenologically interesting predictions:

Mode	$\mathcal{B}_{\rm SM}$	$\mathcal{B}_{\rm Exp}$	scalar LQs with FN
$\tau \to \mu\phi$	0	$< 8.4 \times 10^{-8}$	$[0.58, 1.25] \times 10^{-10}$
$B_s \to \tau^\pm \mu^\mp$	0	$< 4.2 \times 10^{-5}$	$[1.21, 2.60] \times 10^{-6}$
$B^+ \to K^+\tau^+\tau^-$	$(1.60 \pm 0.12) \times 10^{-7}$	$< 2.2 \times 10^{-3}$	$[7.9, 13.3] \times \mathcal{B}_{\rm SM}$
$\bar{B}_s \to \tau^+\tau^-$	$(7.30 \pm 0.49) \times 10^{-7}$	$< 6.8 \times 10^{-3}$	$[7.75, 13.1] \times \mathcal{B}_{\rm SM}$
$B^+ \to K^+\tau^+\mu^-$	0	$< 3.9 \times 10^{-5}$	$(1.8 \pm 0.7) \times 10^{-6}$

$\tau \to \mu\gamma$: dominated by top-quark loops,

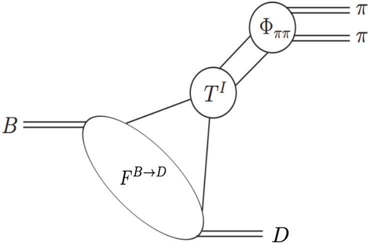
$$\mathcal{B}(\tau \to \mu\gamma) \sim g_1^2\, g_R^2\, (\tilde{c}_L^{32})^2 (c_R^{33})^2 \times 8.67 \cdot 10^{-7}\,.$$

Same combination of spurion entries also appears in $R_{D^{(*)}}$

$\Rightarrow$ Tension between $b \to c\,\tau^-\bar{\nu}$ and $\tau \to \mu\gamma$

$(g-2)_\mu$: can only be explained by bending the power-counting for c_R^{32} and/or accepting a tension with $\tau \to \mu\gamma$.

WA 4



"3-body non-leptonic heavy-to-heavy B decays at NNLO in QCD"

[Huber/Virto/Vos 2020]

QCD Factorization for two-body $\bar{B}_d \to D^+\pi^-$ decays well established

$$\langle D^+\pi^-|\mathcal{Q}_i|\bar{B}_d\rangle = \sum_j F_j^{B\to D}(m_L^2)\int_0^1 du\, T_{ij}(u)\phi_\pi(u) + \mathcal{O}(\Lambda_{\rm QCD}/m_b)$$

- Only vertex terms of colour-allowed tree-amplitude contribute
  (no colour-suppressed tree-amplitude, no penguin amplitudes)
- Hard spectator scattering and weak annihilation power-suppressed
- hadronic information in terms of form factors $F_j^{B\to D}$ at $q^2 = m_\pi^2$ and light-cone distribution amplitude ϕ_π for pion

Idea:
Establish factorization for $\bar{B}^0 \to D^+ K^- \pi^0$ and $\bar{B}^0 \to D^+ \pi^- \pi^0$ decays in phase-space region of small invariant mass, $k^2 = M^2_{K\pi,\pi\pi} \ll m_B^2$

- Hard kernels $T_{ij}(u)$ identical to two-body decay and known to NNLO [Huber,Kränkl,Li'16]
- new hadronic information in di-meson distribution amplitudes, e.g.

$$\Phi_{\pi\pi}(u) = 6u(1-u) \sum_{n=0}^{\infty} \alpha_n^{\pi\pi} \, C_n^{3/2}(2u-1)$$

where Gegenbauer coefficients $\alpha_n^{\pi\pi}$ now depend on $(k^2, \cos\theta)$

→ first NNLO study of non-leptonic 3-body decays
→ numerical impact of higher-order corrections
→ estimate of finite-width effects in quasi-2-body kinematics

Gegenbauer coefficients can be expanded in partial waves:

- in case of $\bar{B}^0 \to D^+\pi^-\pi^0$ case:

$$\alpha_n^{\pi\pi}(k^2, \theta_\pi) = \sum_{\ell=1,3,\cdots}^{n+1} B_{n\ell}^{\pi\pi}(k^2)\, P_\ell(\cos\theta_\pi) \qquad (n \text{ even})$$

with normalization for $n = 0$ fixed by (time-like) form factors, (from data)

$$B_{01}^{\pi\pi}(k^2) \propto f_+^{\pi\pi}(k^2)$$

- in case of $\bar{B}^0 \to D^+K^-\pi^0$:

$$\alpha_n^{K\pi}(k^2, \theta_\pi) = \sum_{\ell=0}^{n+1} B_{n\ell}^{K\pi}(k^2)\, P_\ell(\cos\theta_\pi) \qquad (\text{all } n)$$

with normalization for $n = 0$ fixed by two form factors (from data)

$$B_{00}^{K\pi}(k^2) \propto f_0^{K\pi}(k^2)\,, \qquad B_{01}^{K\pi}(k^2) \propto f_+^{K\pi}(k^2)$$

The set of functions $B_{n\ell}^L(k^2)$ determines the k^2 spectrum of each partial wave.

Perturbative expansion of the decay amplitude:

$$\mathcal{A} \propto (V_{ux}^* V_{cb})\, F_0^{B\to D} \sum_{n\geq \max(\ell-1,0)} B_{nl}^{\pi/K}\, \mathcal{G}_n$$

- process-dependent information in short-distance functions

$$\mathcal{G}_n \equiv C_1(\mu)\, \mathcal{V}_{1n}(\mu) + C_2(\mu)\, \mathcal{V}_{2n}(\mu)$$

- The functions $\mathcal{V}_{in}$ are known to NNLO in QCDF [TH/Kränkl/Li 2016]

$$\mathcal{G}_0(\mu_b) = 1.034_{\rm LO} + (0.026 + i\,0.020)_{\rm NLO} + (0.013 + i\,0.027)_{\rm NNLO}\,,$$

$$\mathcal{G}_1(\mu_b) = (-0.013 + i\,0.030)_{\rm NLO} + (-0.044 + i\,0.018)_{\rm NNLO}\,,$$

$$\mathcal{G}_2(\mu_b) = (0.0023 - i\,0.0017)_{\rm NLO} + (0.0017 - i\,0.0054)_{\rm NNLO}\,.$$

- NNLO contributions large relative to NLO (← colour factors!)
- NNLO essential for contributions of higher terms ($n \geq 1$) in Gegenbauer expansion

Application: Probing higher-order QCD effects,
higher Gegenbauer moments, and higher partial waves:

- Define ratios of angular-integrated decay widths

$$\mathcal{R}_{MM}[z_1, z_2; z_1', z_2'](k^2) \equiv \frac{\int_{z_1}^{z_2} dz \, \frac{d\Gamma(\bar{B} \to D^+ M^- \pi^0)}{dk^2 \, dz}}{\int_{z_1'}^{z_2'} dz \, \frac{d\Gamma(\bar{B} \to D^+ M^- \pi^0)}{dk^2 \, dz}}$$

- dependence on form factors, CKM elements cancel.

These ratios are being analyzed at the Belle-2 experiment.

Modelling the functions $B^{L}_{n\ell}(k^2)$ from sum over individual resonances R:

- Yields for S- and P-wave coefficients

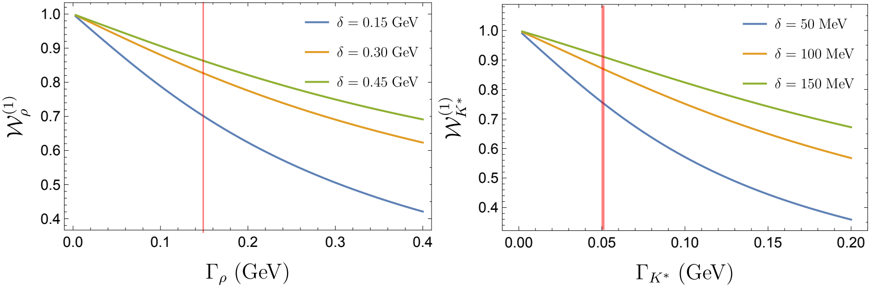
$$
\begin{aligned}
B_{00}^{K\pi}(s) &= \sum_{R_0} \frac{m_{R_0}\, f_{R_0}\, g_{R_0 K\pi}\, e^{i\varphi_{R_0}}}{\sqrt{2}[m_{R_0}^2 - s - i\sqrt{s}\,\Gamma_{R_0}(s)]} \\
B_{01}^{\pi\pi}(s) &= \sum_{R} \frac{m_R\, f_R\, g_{R\pi\pi}\, e^{i\varphi_R}}{\sqrt{2}[m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)]} \\
B_{01}^{K\pi}(s) &= \frac{\sqrt{\lambda_{K\pi}(s)}}{s} \sum_{R} \frac{m_R\, f_R\, g_{RK\pi}\, e^{i\varphi_R}}{\sqrt{2}[m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)]}
\end{aligned}
$$

- Satisfies narrow-width limit in case of stable vector resonance (ρ or K^*)

Application: leading corrections to narrow-width approximation:

- Define resonance width as a function of bin size 2δ:

$$\Gamma_{[R]}(\delta) \equiv \int_{(m_R-\delta)^2}^{(m_R+\delta)^2} ds \, \frac{d\Gamma(\bar{B}\to D^+ M^- \pi^0)}{ds} = \sum_\ell \Gamma^{(\ell)}_{[R]} \,, \qquad \mathcal{W}^{(\ell)}_R \equiv \Gamma^{(\ell)}_{[R]} / \Gamma^{(\ell)}_{[R]\,,\mathrm{NWL}}$$



Typically, finite-width corrections are of the order 20%.

Ongoing projects:

- NNLO corrections to penguin coefficient a_6 (**TH, GTX**) → WA1
- mixed QCD/QED corrections in B-decays (**TF, TH, Gubernari, Seitz**) → WA2
- factorization of endpoint divergencies (**GB**, Böer, **TF**) → WA3
- QCD factorization and flavour symmetries (**TH, GTX**) → WA4
- rare decays of Λ_b baryons (**TF, Gubernari**) → WA5