Karlsruhe Institute of Technology

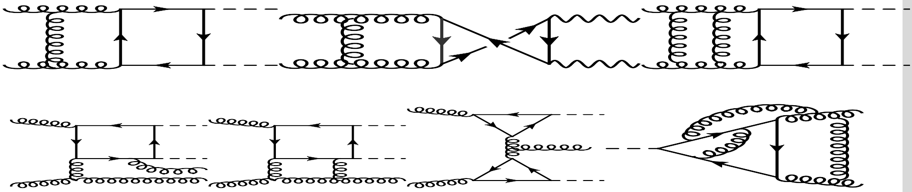
Institute for Theoretical Particle Physics

A3b: Precision predictions for Higgs boson properties as a probe for New Physics

Siegen, October 6-8, 2020

Matthias Steinhauser | Margarete Mühlleitner

TTP KIT



A3b:

Precision predictions for Higgs boson properties as a probe for New Physics

- PIs: Margarete Mühlleitner, Matthias Steinhauser
- Philipp Basler, Joshua Davies, Martin Gabelmann, Seraina Glaus, Go Mishima, Marvin Gerlach, Florian Herren, Marcel Krause, David Wellmann
- Topics:
  - precision predictions for $gg \to HH$ ($gg \to H$)
  - $H_1 \to H_2 + H_3$
  - Higgs mass in SUSY

$gg \to HH$

LO [Glover, van der Bij'88; Plehn,Spira,Zerwas'96]

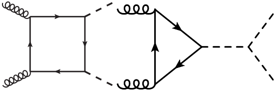
NLO exact (numerical) [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]

[Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher'18; Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira,Streicher'20]

$m_t \to \infty$ [Dawson,Dittmaier,Spira'98]

incl. $1/m_t$ terms [Grigo,Hoff,Melnikov,Steinhauser'13; Degrassi,Giardino,Gröber'16]

exact real rad. [Maltoni,Vryonidou,Zaro'14]



Padé [Gröber,Maier,Rauh'17]

small-p_T [Bonciani,Degrassi,Giardino,Gröber'18]

high energy [Davies,Mishima,Steinhauser,Wellmann'18'19; Mishima'18]

combination exact $\oplus$ high energy [Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]

NNLO $m_t \to \infty$ [de Florian,Mazzitelli'13; Grigo,Melnikov,Steinhauser'14]

incl. $1/m_t$ terms (virtual) [Grigo,Hoff,Steinhauser'15; Davies,Steinhauser'19]

$1/m_t$ terms (real) [Davies,Herren,Mishima,Steinhauser'19]

finite-m_t approx. ...,[Grazzini,Heinrich,Jones,Kallweit,Kerner,Lindert,Mazzitelli'18]

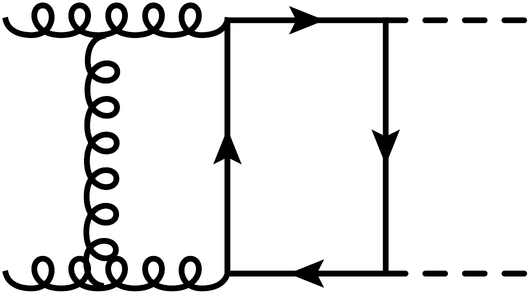
resummations [Shao,Li,Li,Wang'13],. . . ,[de Florian,Mazzitelli'18]

N^{3}LO C_{HH} [Spira'16;Gerlach,Herren,Steinhauser'18]

massless 2-loop box diagrams [Banerjee,Borowka,Dhani,Gehrmann,Ravindran'18]

σ [Chen,Li,Shao,Wang'19]

$gg \to HH$: NLO



$$s, t, m_t^2, m_H^2$$

$$s, t \quad \gg \quad m_t^2 \quad \gg \quad m_H^2$$

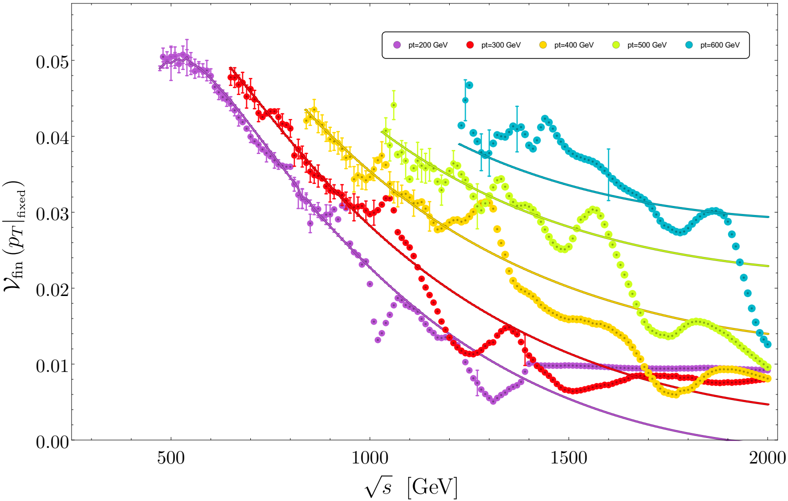
reduce to MIs
expand MIs

Taylor expansion

[Davies,Mishima,Steinhauser,Wellmann'18'19; Mishima'18]

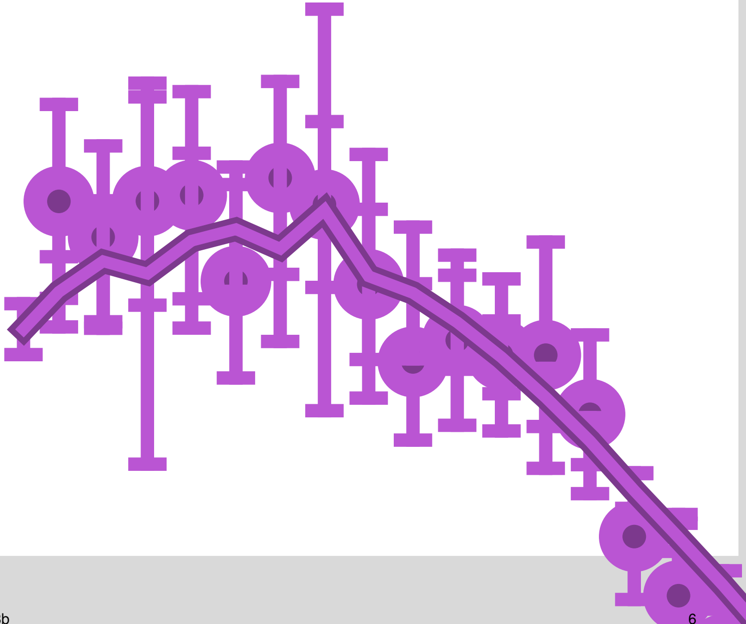
NLO $\mathcal{V}_{\text{fin}}$: grid vs. Padé

[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]



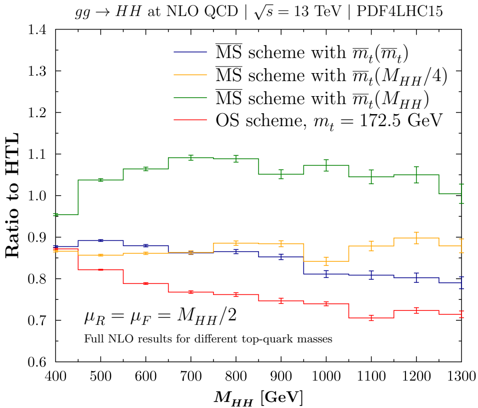
NLO $\mathcal{V}_{\rm fin}$: grid vs. Padé

[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann '19]



NLO uncertainties

[Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira'20]



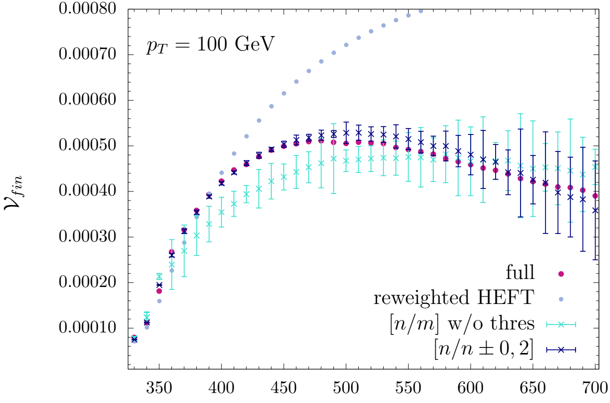
$\sigma_{\rm tot} = 31.05^{+2+4\;=+6}_{-6-18=-24}$ fb

uncertainties: "scale + scheme"

➪NNLO !?

NLO approximation for $gg \to HH$: large-m_t $\oplus$ threshold $\oplus$ conformal mapping $\oplus$ Padé

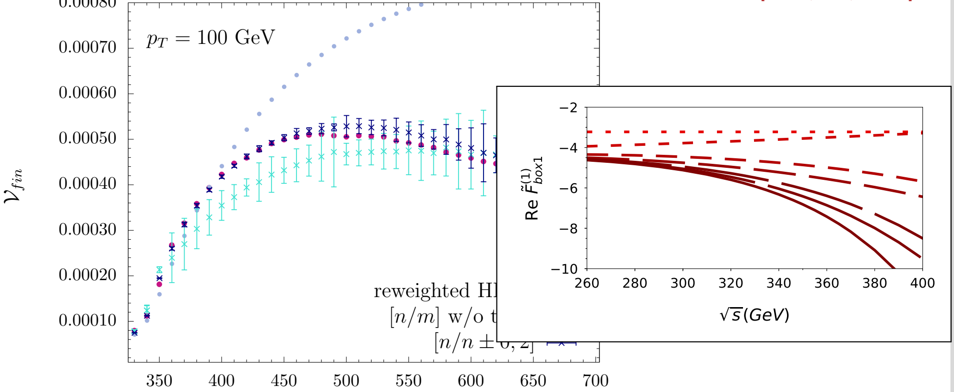
[Gröber,Maier,Rauh'17]



input for Padé:	large-m_t: up to $1/m_t^8$ threshold: non-analytic terms ("logs")

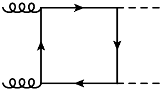
NLO approximation for $gg \to HH$: large-m_t $\oplus$ threshold $\oplus$ conformal mapping $\oplus$ Padé

[Gröber,Maier,Rauh'17]

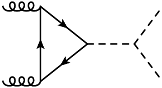


input for Padé:	large-m_t: up to $1/m_t^8$ threshold: non-analytic terms (“logs”)

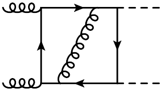
NNLO large-m_t: Feynman diagrams



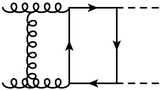
Tn_h



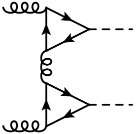
Tn_h



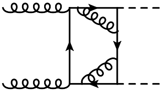
$C_F Tn_h$



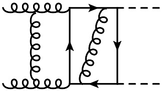
$C_A Tn_h$



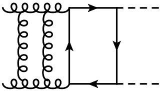
$(Tn_h)^2$



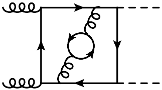
$C_F^2 Tn_h$



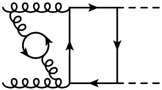
$C_A C_F Tn_h$



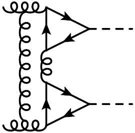
$C_A^2 Tn_h$



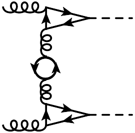
$C_F (Tn_h)^2$ and $C_F T^2 n_h n_l$



$C_A (Tn_h)^2$ and $C_A T^2 n_h n_l$



$C_A (Tn_h)^2$



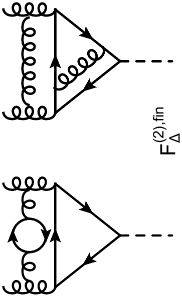
$(Tn_h)^3$

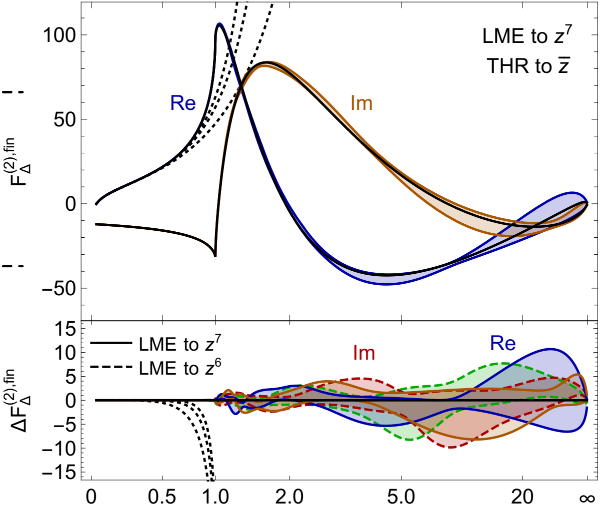
NNLO $gg \to hh$ for large-m_t

- 324 GB (gzipped) stored intermediate results (after $1/m_t$ expansion)
  $\approx$ 10 days wall time, a few $\times \geq$ 96 GB RAM, 12 cores
- apply projectors
  $\approx$ 29.000 easy tasks total time $\sim$ 4.5 yr ($\sim$ 1 month)
- heavy use of modern `(T)FORM` commands: [Ruijl,Ueda,Vermaseren'17]
  e.g. `ArgToExtraSymbols`
- $F_{\mathrm{box1}}, F_{\mathrm{box2}} \to 1/m_t^8$; $F_{\mathrm{tri}} \to 1/m_t^{14}$ [Davies,Steinhauser'19]

NNLO approximation for $gg \to h$: large-m_t + threshold + Padé

[Davies,Gröber,Maier,Rauh,Steinhauser'19]





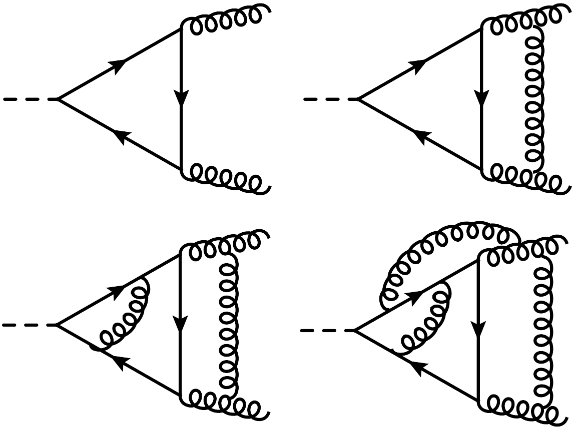
$z = \frac{s}{4m_t^2}$

$\bar{z} = 1 - z$

analytic n_l terms: [Harlander,Prausa,Usovitsch'19]; leading colour $\sim N_c^2$: [Prausa,Usovitsch'20]

num. int. of diff. eqs.: [Czakon,Niggetiedt'20]

$gg \to H$: virtual N^{3}LO corrections for finite m_t

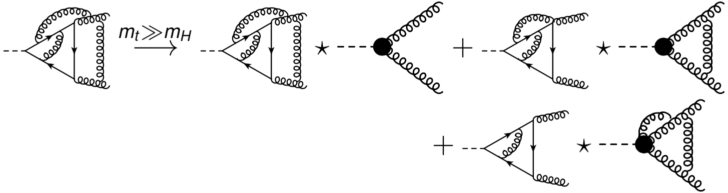


2 loops: [Harlander,Kant'05; Anastasiou,Beerli,Bucherer,Daleo,Kunszt'06; Aglietti,Bonciani,Degrassi,Vicini'07]

3 loops: expansion: [Harlander,Ozeren'09; Pak,Rogal,Steinhauser'09]

3 loops: [Davies,Gröber,Maier,Rauh,Steinhauser'19; Harlander,Prausa,Usovitsch'19; Prausa,Usovitsch'20; Czakon,Niggetiedt'20]

Asymptotic expansion



$\log(F)_{\text{finite}}$

[Davies,Herren,Steinhauser'19]

$$\rho = m_H^2/m_t^2$$

$$
\begin{aligned}
\log(F)_{\text{finite}} =&\\
&+\frac{\alpha_s}{\pi}\left(\frac{11}{4}+\frac{1}{8}\pi^2-\frac{3}{4}l_{tH}^2+\frac{17}{135}\rho+\frac{3553}{226800}\rho^2\right)\\
&+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{523}{108}+\frac{151}{192}\pi^2-\frac{499}{48}\zeta_3+l_{tH}\left(-\frac{155}{36}+\frac{23}{48}\pi^2+\frac{9}{8}\zeta_3\right)+l_{tH}^2\left(-\frac{151}{48}+\frac{3}{16}\pi^2\right)-\frac{23}{48}l_{tH}^3\right.\\
&+\rho\left(-\frac{15765509}{829440}+\frac{7}{1080}\pi^2+\frac{7}{540}\log(2)\pi^2+\frac{1909181}{110592}\zeta_3+\frac{793}{10368}l_{tH}\right)\\
&\left.+\rho^2\left(-\frac{1013177390077}{234101145600}+\frac{857}{907200}\pi^2+\frac{857}{453600}\log(2)\pi^2+\frac{267179777}{70778880}\zeta_3+\frac{580759}{43545600}l_{tH}\right)\right]\\
&+\left(\frac{\alpha_s}{\pi}\right)^3\left[-\frac{18539405}{1119744}+\frac{441517}{62208}\pi^2-\frac{11549467}{82944}\zeta_3-\frac{50839}{311040}\pi^4-\frac{1949}{576}\pi^2\zeta_3+\frac{39307}{288}\zeta_5\right.\\
&-\frac{39}{8}\zeta_3^2-\frac{193}{7560}\pi^6+l_{tH}\left(-\frac{322955}{31104}+\frac{665}{96}\pi^2-\frac{3043}{144}\zeta_3-\frac{1801}{5760}\pi^4-\frac{15}{16}\pi^2\zeta_3-\frac{27}{4}\zeta_5\right)\\
&+l_{tH}^2\left(-\frac{58745}{3456}+\frac{1435}{576}\pi^2+\frac{25}{8}\zeta_3-\frac{33}{320}\pi^4\right)+l_{tH}^3\left(-\frac{3995}{864}+\frac{23}{96}\pi^2\right)-\frac{529}{1152}l_{tH}^4\\
&+\rho\left(-\frac{542872693595}{3218890752}+\frac{65743583}{55987200}\pi^2-\frac{4691}{9720}\log(2)\pi^2-\frac{6788585826089}{107296358400}\zeta_3\right.\\
&-\frac{11421210133}{1149603840}\log^4(2)+\frac{11364084757}{1149603840}\log^2(2)\pi^2+\frac{244657561171}{55180984320}\pi^4-\frac{11421210133}{47900160}\mathrm{Li}_4(1/2)\\
&+\frac{718337}{9979200}\log^5(2)-\frac{718337}{5987520}\log^3(2)\pi^2+\frac{46111267}{239500800}\log(2)\pi^4-\frac{10073}{25920}\pi^2\zeta_3-\frac{3254515597}{31933440}\zeta_5
\end{aligned}
$$

$$+\left(\frac{\quad}{\pi}\right)\Big[-\frac{\quad}{1119744}+\frac{\quad}{62208}\pi^2-\frac{\quad}{82944}\zeta_3-\frac{\quad}{311040}\pi^4-\frac{\quad}{576}\pi^2\zeta_3+\frac{\quad}{288}\zeta_5$$

$\log(F)_{\mathrm{finite}}$

$$-\frac{39}{8}\zeta_3^2-\frac{193}{7560}\pi^6+l_{tH}\left(-\frac{322955}{31104}+\frac{665}{96}\pi^2-\frac{3043}{144}\zeta_3-\frac{1801}{5760}\pi^4-\frac{15}{16}\pi^2\zeta_3-\frac{27}{4}\zeta_5\right)$$

$$+l_{tH}^2\left(-\frac{58745}{3456}+\frac{1435}{576}\pi^2+\frac{25}{8}\zeta_3-\frac{33}{320}\pi^4\right)+l_{tH}^3\left(-\frac{3995}{864}+\frac{23}{96}\pi^2\right)-\frac{529}{1152}l_{tH}^4$$

$$+\rho\Big(-\frac{542872693595}{3218890752}+\frac{65743583}{55987200}\pi^2-\frac{4691}{9720}\log(2)\pi^2-\frac{6788585826089}{107296358400}\zeta_3$$

$$-\frac{11421210133}{1149603840}\log^4(2)+\frac{11364084757}{1149603840}\log^2(2)\pi^2+\frac{244657561171}{55180984320}\pi^4-\frac{11421210133}{47900160}\mathrm{Li}_4(1/2)$$

$$+\frac{718337}{9979200}\log^5(2)-\frac{718337}{5987520}\log^3(2)\pi^2+\frac{46111267}{239500800}\log(2)\pi^4-\frac{10073}{25920}\pi^2\zeta_3-\frac{3254515597}{31933440}\zeta_5$$

$$-\frac{718337}{83160}\mathrm{Li}_5(1/2)+\frac{5327119}{11197440}l_{tH}+\frac{25639}{746496}l_{tH}^2\Big)$$

$$+\rho^2\Big(-\frac{105579436141788248 7061}{668136655521054720 0}+\frac{4077367559}{23514624000}\pi^2+\frac{23157917500539717053}{117837152649216000}\zeta_3$$

$$-\frac{110153}{1632960}\log(2)\pi^2+\frac{2712037738087}{2391175987200}\log^4(2)-\frac{2729355664999}{2391175987200}\log^2(2)\pi^2-\frac{150868470717581}{229552894771200}\pi^4$$

$$+\frac{2712037738087}{99632332800}\mathrm{Li}_4(1/2)+\frac{46902913}{202176000}\log^5(2)-\frac{46902913}{121305600}\log^3(2)\pi^2-\frac{8632107859}{33965568000}\log(2)\pi^4$$

$$-\frac{1233223}{21772800}\pi^2\zeta_3+\frac{49563452909}{4528742400}\zeta_5-\frac{46902913}{1684800}\mathrm{Li}_5(1/2)+\frac{103150403081}{658409472000}l_{tH}+\frac{18740929}{3135283200}l_{tH}^2\Big)\Big]$$

$\log(F)_{\text{finite}}$ numerical results

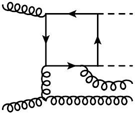
$\mu = m_t^{\text{OS}}$:

$$\begin{aligned}\log(F)_{\text{finite}} \approx\ & a_t\,[(11.07 - i3.06) + 0.07 + 0.004] \\ & + a_t^2\,[(22.59 + i13.24) + (1.02 + i0.13) + (0.07 + i0.01)] \\ & + a_t^3\,[(-73.18 + i51.55) + (7.61 + i0.85) + (0.70 + i0.14)]\end{aligned}$$

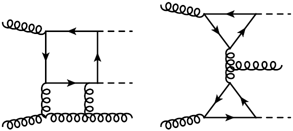
$$a_t = \alpha_s(m_t)/\pi$$

- ρ^1 at 4 loops: 10%
  ρ^2: $< 1\%$ ➪ check convergence
- $m_t^{\overline{\text{MS}}}$: ➪ $1/m_t$ corrections smaller

NNLO real corrections for large m_t

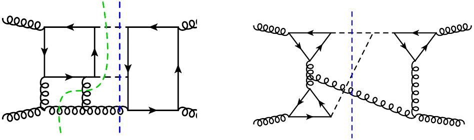
- 

  1-loop $2 \to 4$

  2-loop $2 \to 3$

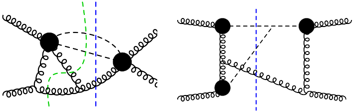
- use optical theorem

- asymptotic expansion for $m_t^2 \gg m_H^2, s$ ➪

tadpoles $\otimes$ "phase-space" integrals

1 and 2 loops



m_t^2

m_H^2, s

3-loop 3- and 4-particle cuts [DHMS WIP]

and 2-loop 3-particle cut [DHMS'19]

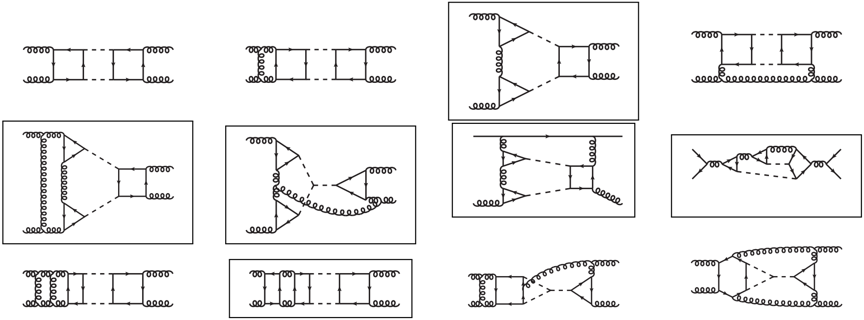
[DHMS = Davies,Herren,Mishima,Steinhauser]

NNLO real corrections for large m_t (2)

	amplitude	$\rightarrow$	map to integral family	$\rightarrow$	IBP reduction `FIRE` [Smirnov]	$\rightarrow$	minimal set of MIs
$\rightarrow$	diff. eqs. in $x = m_H^2/s$	$\rightarrow$	Fuchsian form `epsilon` [Prausa'17]	$\rightarrow$	ϵ form `LIBRA` [Lee] [Henn'13]	$\rightarrow$	solution Goncharov polylogs `GiNaC` [Vollinga,Weinzierl'05]
$\rightarrow$	expand in $\delta = \sqrt{1 - 4m_H^2/s}$ `PolyLogTools` [Duhr,Dulat'19]		alternative: use diff. eqs. to obtain expansion in δ ($>$500 terms possible)				

NNLO real corrections for large m_t (3)

Cross section for n_h^3 diagrams

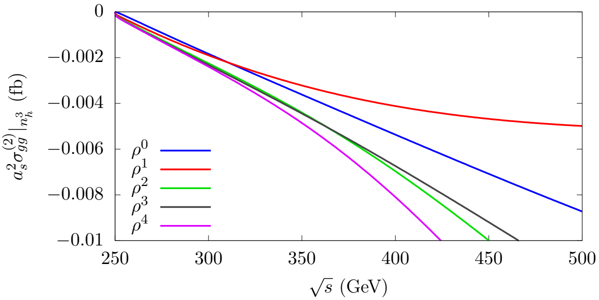


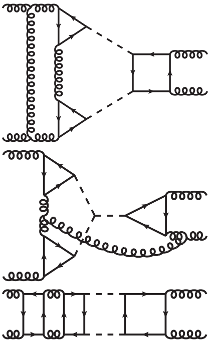
- expansion up to $1/m_t^8$
- combine with virtual corrections [Grigo,Hoff,Steinhauser'15; Davies,Steinhauser'19]

NNLO real corrections for large m_t (3)

Cross section for n_h^3 diagrams

- expansion up to $1/m_t^8$
- combine with virtual corrections [Grigo,Hoff,Steinhauser'15; Davies,Steinhauser'19]

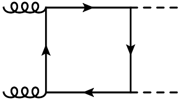




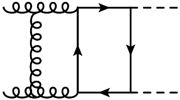
$\rho = m_H^2/m_t^2$

C_{HH} to 4 loops

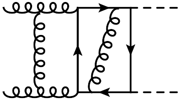
$$\mathcal{L}_{\text{eff}} = -\frac{H}{v} C_H \mathcal{O}_1 + \frac{1}{2}\frac{H^2}{v^2} C_{HH} \mathcal{O}_1 \qquad \mathcal{O}_1 = -G_{\mu\nu}^2/4 \qquad C_{HH} = C_H + \Delta_{HH}$$



$$\Delta_{HH} = 0$$



$$\Delta_{HH} = 0$$

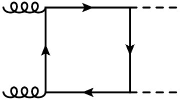


$$\Delta_{HH} = \frac{7}{8} C_A^2 - \frac{11}{8} C_A C_F - \frac{5}{6} C_A T_F + \frac{1}{2} C_F T_F + n_l C_F T_F$$

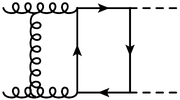
[Grigo,Melnikov,Steinhauser'14]

C_{HH} to 4 loops

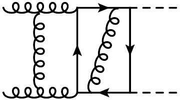
$$\mathcal{L}_{\mathrm{eff}} = -\frac{H}{v} C_H \mathcal{O}_1 + \frac{1}{2}\frac{H^2}{v^2} C_{HH} \mathcal{O}_1 \qquad \mathcal{O}_1 = -G_{\mu\nu}^2/4 \qquad C_{HH} = C_H + \Delta_{HH}$$



$$\Delta_{HH} = 0$$

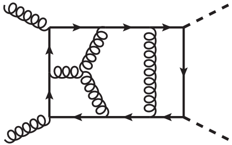


$$\Delta_{HH} = 0$$



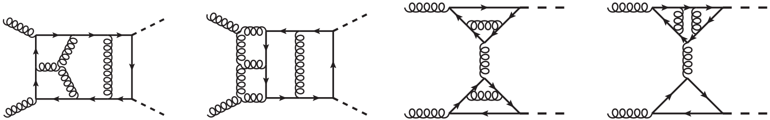
$$\Delta_{HH} = \frac{7}{8} C_A^2 - \frac{11}{8} C_A C_F - \frac{5}{6} C_A T_F + \frac{1}{2} C_F T_F + n_l C_F T_F$$

[Grigo,Melnikov,Steinhauser'14]



$$\Delta_{HH} = ?$$

Renormalization and matching

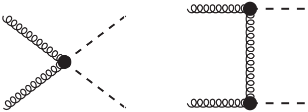


- Tricky renormalization:

$$\langle \mathcal{O}_1 \mathcal{O}_1 \rangle^{\mathrm{ren}} = (Z_{\mathcal{O}_1})^2 \langle \mathcal{O}_1 \mathcal{O}_1 \rangle^{\mathrm{bare}} + Z_{11}^L \langle \mathcal{O}_1 \rangle^{\mathrm{bare}} \qquad Z_{11}^L = 1 + \mathcal{O}(\alpha_s^2)$$

[Zoller'16]

- Matching



$$(C_{HH} Z_{\mathcal{O}_1} + C_H^2 Z_{11}^L) \mathcal{A}^{\mathrm{eff}}_{\mathrm{LO,1PI}} + C_H^2 Z_{\mathcal{O}_1}^2 \mathcal{A}^{\mathrm{eff}}_{\mathrm{LO,1PR},\lambda=0} = \mathcal{A}^h_{\mathrm{1PI}} + \mathcal{A}^h_{\mathrm{1PR},\lambda=0}$$

[Gerlach,Herren,Steinhauser'18]

Δ_{HH}

[Gerlach,Herren,Steinhauser'18]

$$\begin{aligned}
\Delta_{HH}^{(4)} &= \frac{1993}{576} C_A^3 - \frac{1289}{144} C_A^2 C_F - \frac{3191}{864} C_A^2 T_F + \frac{165}{32} C_A C_F^2 + \frac{67}{18} C_A C_F T_F + \frac{5}{72} C_A T_F^2 \\
&- \frac{3}{2} C_F^2 T_F + \frac{1}{9} C_F T_F^2 + \left[\frac{77}{48} C_A^3 - \frac{121}{48} C_A^2 C_F - \frac{7}{12} C_A^2 T_F + \frac{11}{12} C_A C_F T_F \right] \ln\left(\frac{\mu^2}{M_t^2}\right) \\
&+ n_l T_F \left[-\frac{55}{144} C_A^2 + \frac{55}{18} C_A C_F + \frac{109}{216} C_A T_F - \frac{11}{4} C_F^2 + \frac{19}{36} C_F T_F \right] \\
&+ n_l^2 T_F^2 \left[\frac{5}{72} C_A + \frac{1}{9} C_F \right] + n_l T_F \left[-\frac{7}{12} C_A^2 + \frac{11}{4} C_A C_F - \frac{2}{3} C_F T_F \right] \ln\left(\frac{\mu^2}{M_t^2}\right) \\
&- \frac{2}{3} n_l^2 C_F T_F^2 \ln\left(\frac{\mu^2}{M_t^2}\right)
\end{aligned}$$

[Spira'16] **Low-energy theorem:**

$[C_H = -\frac{m_t}{\zeta_{\alpha_s}} \frac{\partial}{\partial m_t} \zeta_{\alpha_s}$ [Chetyrkin,Kniehl,Steinhauser'98]]

$$C_{HH} = \frac{m_t^2}{\zeta_{\alpha_s}} \frac{\partial^2}{\partial m_t^2} \zeta_{\alpha_s} - 2 \left(\frac{m_t}{\zeta_{\alpha_s}} \frac{\partial}{\partial m_t} \zeta_{\alpha_s} \right)^2$$

$$\alpha_s^{(5)} = \zeta_{\alpha_s} \alpha_s^{(6)}$$

Conclusions/Outlook

- Promising: combine numerical results and analytic expansions
- $gg \to H$, N^{3}LO-virtual: finite-m_t corrections available
- $gg \to HH$, NNLO: first steps towards incorporating systematic finite-m_t