A3b: Precision Predictions for Higgs Boson Properties as a Probe of New Physics

Continued

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Overview

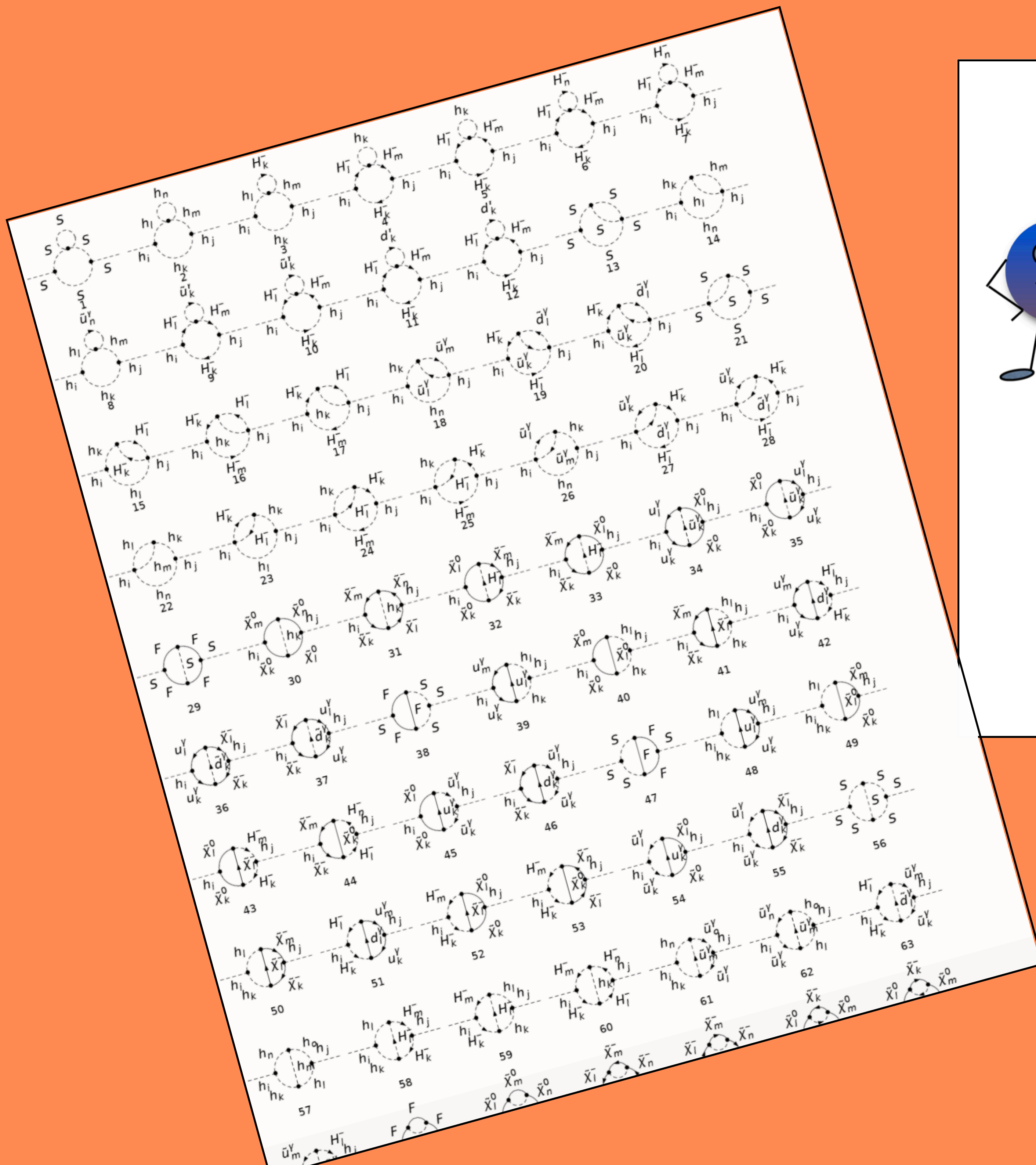
- Topics: - Higher-Order Corrections to neutral Higgs boson masses in the MSSM and the CP-violating NMSSM

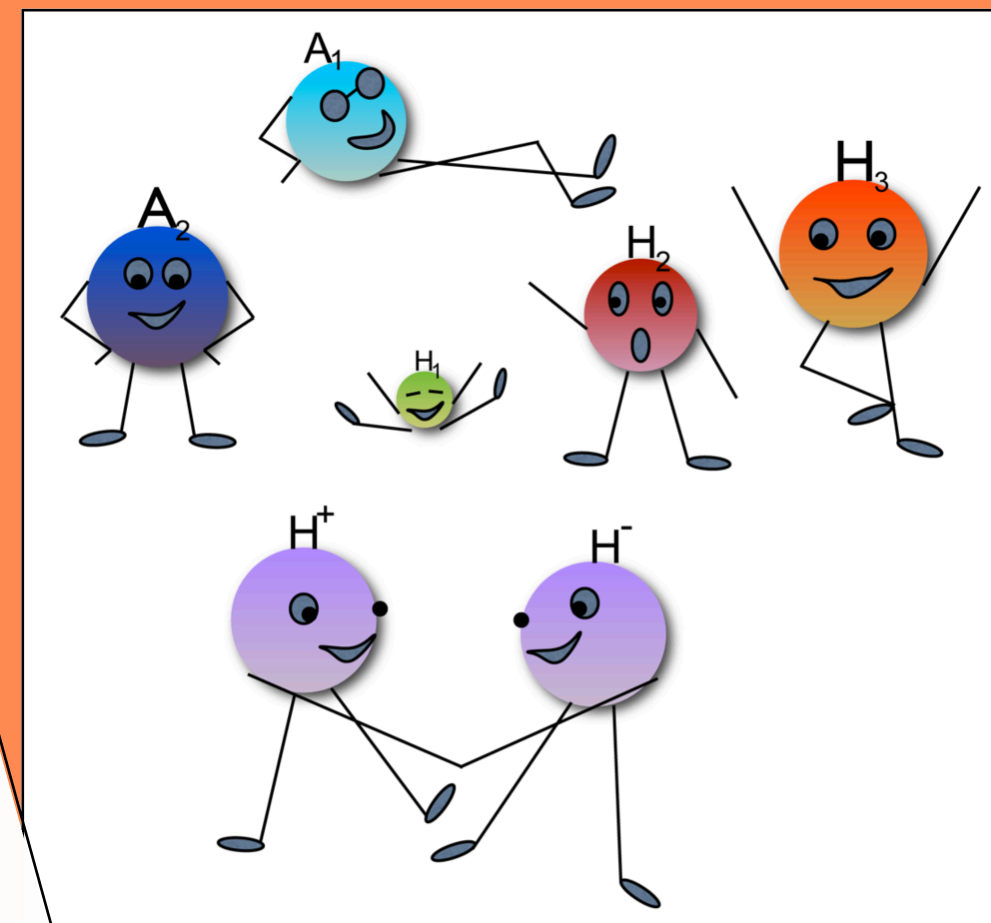
  - NLO QCD corrections to Higgs Pair Production

  - Di-Higgs Phenomenology

- Philip Basler (PhD/KIT), Martin Gabelmann (PhD/KIT), Seraina Glaus (Postdoc/KIT), Jonas Klappert (PhD/RWTH), Marcel Krause (PhD/KIT), Alexander Voigt (Postdoc/RWTH)

Higher-Order Corrections to MSSM and NMSSM Higgs Masses





$$\mathrm{Det}\left(\mathbb{1}_{5\times5}p^2 - \mathcal{M}'_{hh} + \hat{\Sigma}_{hh}(p^2)\right) = 0$$

The Role of the Higgs Boson Mass

✦ Present accuracy: [ATLAS,CMS, Phys.Rev.Lett.114(2015)191803]

$$M_H = 125.09 \pm 0.21 \text{ (stat)} \pm 0.11 \text{ (syst) GeV}$$

✦ Why precision?

* Self-consistency of SM at quantum level (e.g. Higgs loop corrections to W boson mass)

* M_H <-> stability of electroweak vacuum [Degrassi eal;Bednyakov eal]

* Higgs mass uncertainty feeds back in uncertainty on Higgs observables

* Test parameter relations in beyond the SM theories
  -> indirect constraints of beyond-SM (BSM) parameters space

✦ MSSM and NMSSM masses no free parameters: predictive power of the MSSM, NMSSM and other extensions -> important experimental test to be passed

Supersymmetric Higgs Mass Computations

✦ Status of MSSM and NMSSM Higher-Order Corrections:

* Precise predictions:

- $m_{h,\text{MSSM}}$: fixed order up to 2-loop and dominant 3-loop (real+complex MSSM) and resummed:

[Okada eal;Ellis eal;Brignole eal;Haber,Hempfling;Chankowski eal;Dabelstein eal;Pierce eal;Barbieri eal;Espinosa,Quiros;Casas eal; Carena eal;Heinemeyer eal;Zhang;Degrassi eal;Dedes eal; Martin;Borowka eal;Draper,eal;Harlander eal;Kant eal;Staub,Porod; Goodsell eal;Bahl,Hollik;Bagnaschi eal;Kunz eal;Mihaila,Zerf;Domingo,Paßehr;Harlander,Klappert,Voigt; ...]

- $m_{h,\text{NMSSM}}$: fixed order up to 2-loop (real+complex NMSSM)

[Ellwanger eal;Elliot eal;Pandita;Degrassi,Slavich;Staub eal;King eal;Graf eal;Ender eal;Drechsel eal;MM eal;Goodsell eal;Ham eal; Funakubo eal;Cheung eal;Domingo,Paßehr; ...]

✦ Estimate of uncertainty:

- MSSM: +-3GeV [Degrassi gel; Allanach eal]
- NMSSM: comparison of $\overline{\text{DR}}$ calculations [Staub eal] ; comparison of OS calculations [Drechsel eal]

✦ Numerous program packages for Higgs mass computations: FeynHiggs, FlexibleSUSY, H3m, Himalaya, NMSSMCALC, NMSSMTools, Sarah, SoftSUSY, Spheno, ...

Our projects: MSSM M_h to N^3LO+N^3LL [Harlander,Klappert,Voigt,Eur.Phys.J.C80(2020)3]
$\mathcal{O}(\alpha_t^2)$ corrections [Dao,Gröber,Krause,MM,Rzehak, JHEP08 (2019) 114]
$\mathcal{O}((\alpha_t+\alpha_\lambda+\alpha_\kappa)^2)$ corrections [Dao,Gabelmann,MM,Rzehak, in preparation]

MSSM: M_h to N^{3}LO+N^{3}LL

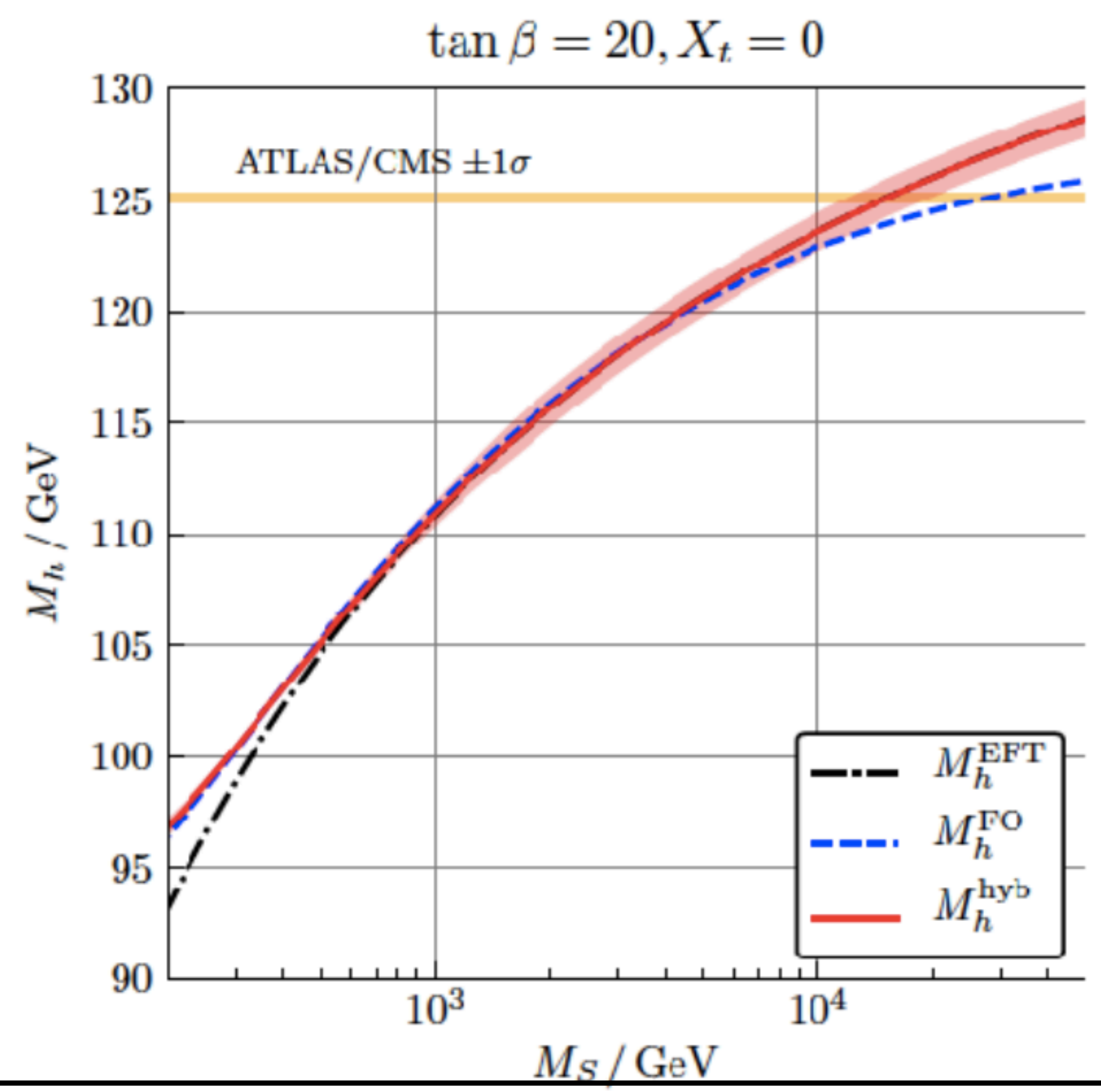
[Harlander,Klappert,Voigt,Eur.Phys.J.C80(2020)3]

Combination:

FlexibleSUSY+ Himalaya	HSSUSY+ Himalaya	FlexibleEFTHiggs
1-, 2-loop w/ O(v/Ms) 3-loop w/o O(v/Ms) w/o resummation	1-, 2-loop w/o O(v/Ms) 3-loop w/o O(v/Ms) w/ resummation	1-loop w/ O(v/Ms) w/ resummation

add O(v/Ms)

- hybrid FO+EFT result in DRbar scheme
- includes
  - tree-level + LL resummation
  - full 1-loop + NLL resummation
  - full $\alpha_t\alpha_s + \alpha_t^2$ + NNLL resummation
  - $\alpha_t\alpha_s^2$ w/o O(v/Ms) + N^{3}LL resummation
- missing higher orders sizable at low Ms
- EFT sufficient above 1-2 TeV



NMSSM: Two-Loop $\mathcal{O}((\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$ Corrections

[Dao,Gabelmann,MM,Rzehak, in preparation]

- Higher-order mass corrections implemented in the code NMSSMCALC: [Baglio,eal,'13]

  * complete one-loop (at non-zero p^2) in the CP-conserving [Ender,eal,'11] and CP-violating NMSSM [Graf eal,'12]

  * two-loop $\mathcal{O}(\alpha_t \alpha_s)$ [MM,eal,'14] and $\mathcal{O}(\alpha_t^2)$ [Dao,eal,'19] at p^2=0 and in the gaugeless approximation

  * Mixed $\overline{\text{DR}}$-OS renormalization scheme; top, stop sector: OS and $\overline{\text{DR}}$

- This project (ongoing): Martin Gabelmann, Thi Nhung Dao, MM, Heidi Rzehak

  * computation of the $\mathcal{O}((\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$ corrections at p^2=0 in the gaugeless limit in the CP-violating NMSSM

The Model - CP-Violating NMSSM

- Next-to-Minimal Supersymmetric Extension (NMSSM) Higgs Sector:
  2 complex Higgs doublets (ensure supersymmetry and that no anomalies occur)
  plus gauge-singlet chiral superfield (relaxed stop mass bound, rich phenomenology, relaxed EDM bounds, no mu-problem)

- Higgs potential: complex/CP-violating

$$
\begin{aligned}
V_H &= (|\lambda S|^2 + m_{H_d}^2) H_d^\dagger H_d + (|\lambda S|^2 + m_{H_u}^2) H_u^\dagger H_u + m_S^2 |S|^2 \\
&+ \frac{1}{8}(g_2^2 + g_1^2)(H_d^\dagger H_d - H_u^\dagger H_u)^2 + \frac{1}{2} g_2^2 |H_d^\dagger H_u|^2 \\
&+ |-\epsilon^{ij} \lambda H_{d,i} H_{u,j} + \kappa S^2|^2 + \left[-\epsilon^{ij} \lambda A_\lambda S H_{d,i} H_{u,j} + \frac{1}{3} \kappa A_\kappa S^3 + \mathrm{h.c.} \right]
\end{aligned}
$$

- Higgs fields after electroweak symmetry breaking:

$$
H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + h_d + i a_d) \\ h_d^- \end{pmatrix}, \quad H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{1}{\sqrt{2}}(v_u + h_u + i a_u) \end{pmatrix}, \quad S = \frac{e^{i\varphi_s}}{\sqrt{2}}(v_s + h_s + i a_s)
$$

- Tadpoles (zero at tree level):

$$
(\boldsymbol{t})_\Phi = t_\Phi = \frac{\partial V_H}{\partial \Phi}, \quad \Phi = h_d, h_u, h_s, a_d, a_u, a_s
$$

Higher-Order NMSSM Higgs Boson Masses

- After EWSB: h_i (i=1,…,5) ordered by mass, no CP eigenstates <- CP-violating NMSSM
- HO Higgs masses: real parts of propagator poles; iteratively obtained from (no Goldstone boson admixture included, numerically small)

$$\mathrm{Det}\left(\mathbb{1}_{5\times 5}p^2 - \mathcal{M} \quad + \hat{\Sigma}_{hh}(p^2)\right) = 0$$

[Dao eal,'19]

- Renormalized self-energies:

$$\hat{\Sigma}_{ij} = \hat{\Sigma}^{(1)}_{ij}(p^2) + \hat{\Sigma}^{(2)}_{ij}(p^2)$$

with

$$\hat{\Sigma}^{(1)}(p^2) = \Sigma^{(1)}(p^2) + \frac{1}{2}\left[\delta^{(1)}\mathcal{Z}^\dagger\left(\mathbb{1}p^2 - \mathcal{M}^2\right) + \left(\mathbb{1}p^2 - \mathcal{M}^2\right)\delta^{(1)}\mathcal{Z}\right] + \delta^{(1)}\mathcal{M}^2 .$$

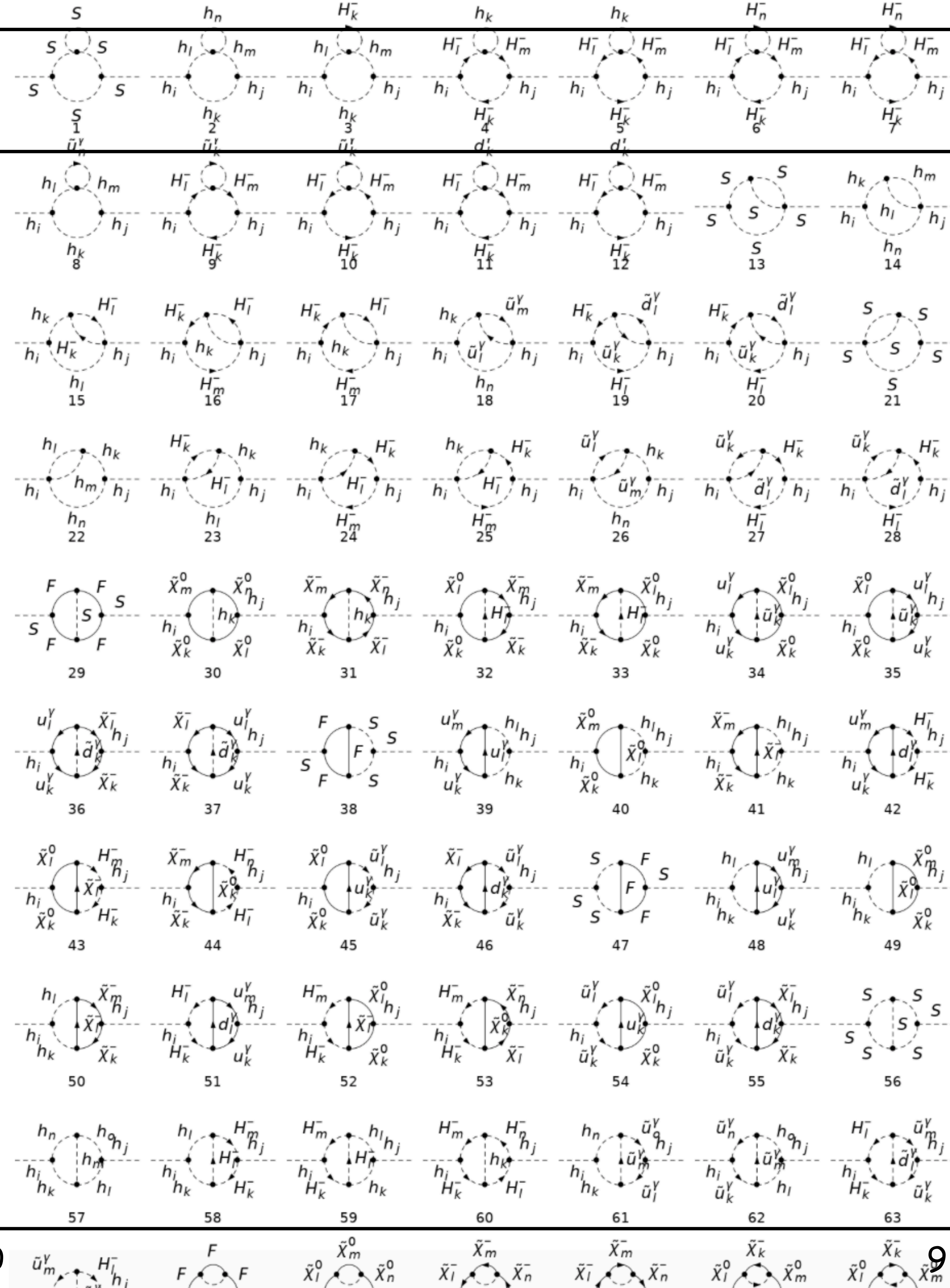
computed at non-zero p^2 in the CP-conserving and CP-violating NMSSM in [Ender eal,'11;Graf eal,'12]

$$\begin{aligned}\hat{\Sigma}^{(2)}(p^2) = \Sigma^{(2)}(p^2) + \frac{1}{2}\Big[&\delta^{(2)}\mathcal{Z}^\dagger\left(\mathbb{1}p^2 - \mathcal{M}\ \right) + \left(\mathbb{1}p^2 - \mathcal{M}\ \right)\delta^{(2)}\mathcal{Z} \\ &+\frac{1}{2}\delta^{(1)}\mathcal{Z}^\dagger\left(\mathbb{1}p^2 - \mathcal{M}\ \right)\delta^{(1)}\mathcal{Z} - \delta^{(1)}\mathcal{Z}^\dagger\delta^{(1)}\mathcal{M} - \delta^{(1)}\mathcal{M}\ \delta^{(1)}\mathcal{Z}\Big] \\ &- \delta^{(2)}\mathcal{M}\end{aligned}$$

computed at p^2=0 and in gaugeless limit in the CP-violating NMSSM at $\mathcal{O}(\alpha_t\alpha_s)$ and $\mathcal{O}(\alpha_t^2)$ in [MM eal,'14;Dao eal,'19]

This work: $\mathcal{O}((\alpha_t+\alpha_\lambda+\alpha_\kappa)^2)$

Set-Up



- Two independent calculations:
  - * dimensional reduction in D=4-2*ε dimensions
  - * generation of FeynArts model file including vertex counterterms: SARAH 4.14.3
  - * fermion traces, gamma matrices, reduction of one-loop amplitudes and counterterm-inserted diagrams: FeynCalc 9.2.0
  - * Diagram generation: FeynArts 3.1 (patched version that comes with FeynCalc)
  - * reduction to two-loop master integrals including full momentum dependence: TARCER 2.0 (patched version which comes with FeynCalc)
  - * loop integrals defined in TSIL (second calc. own code)

Goldstone Boson Catastrophe - is over

* Goldstone boson catastrophe: appearance of IR divergences and imaginary parts due to massless Goldstone bosons (circumvented in gaugeless limit in the MSSM, but not the NMSSM) [Martin ,02,'03]

  see also [Espinosa eal,'16,'18;Kumar,Martin,'16;Braathen,Goodsell,'16] for proposed solutions
  and [Braathen eal,'17] for NMSSM application

* Origin of IR divergences:
  1) vanishing loop-momentum 2) spurious IR divergences (also at non-zero p^2) [Braathen,Goodsell,'16]

* This work: 3 different ways to regulate IR divergences
  a) use mass regulator everywhere
  b) use mass regulator only in subset with spurious IR divergences, in the others with ´true´ IR divergences turn on external momentum and use analytically known results for low-momentum expansion of these loop integrals
  c) as in b) but include full external momentum dependence in all Feynman diagrams using TSIL

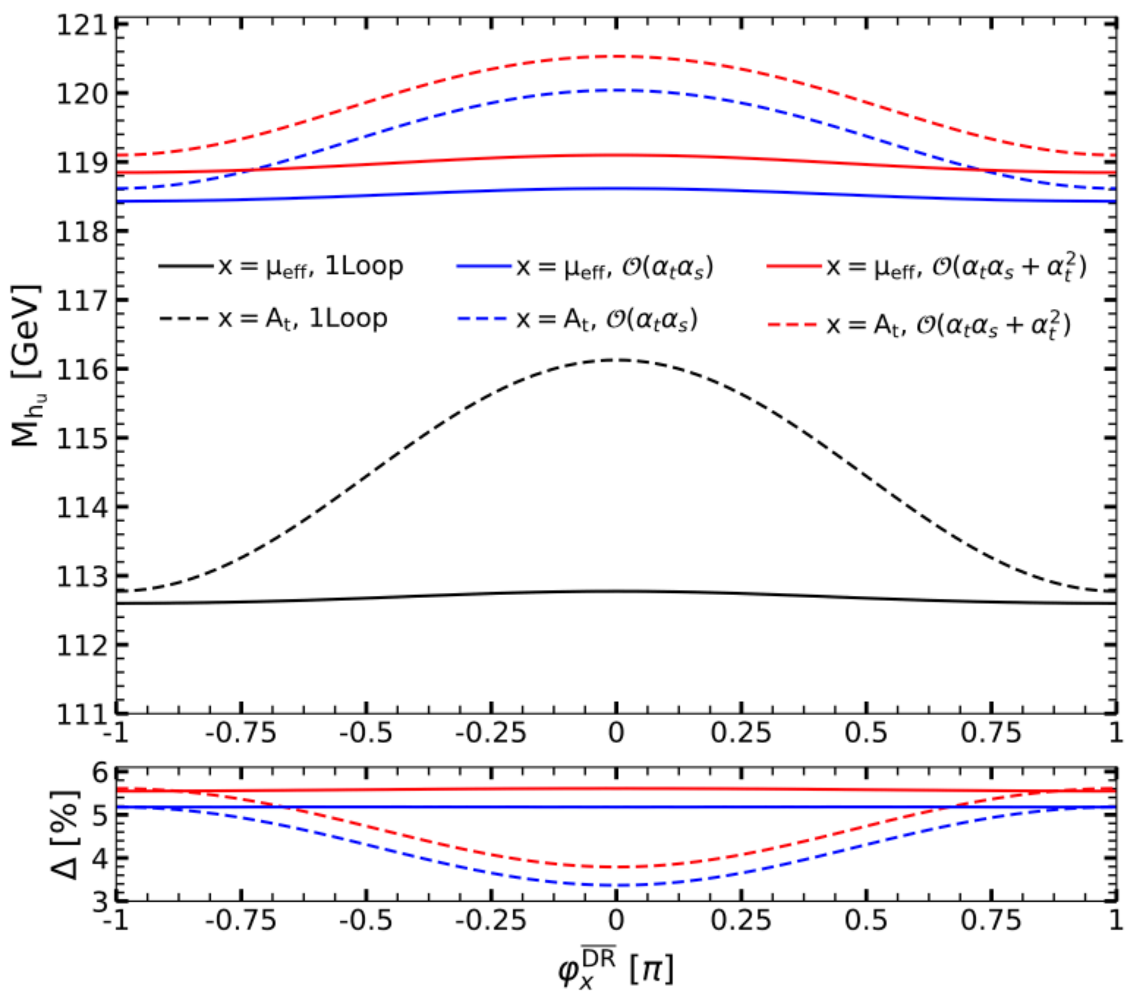
* Status: All required self-energies calculated and cross-checked

* Next steps:
  - perform numerical analysis

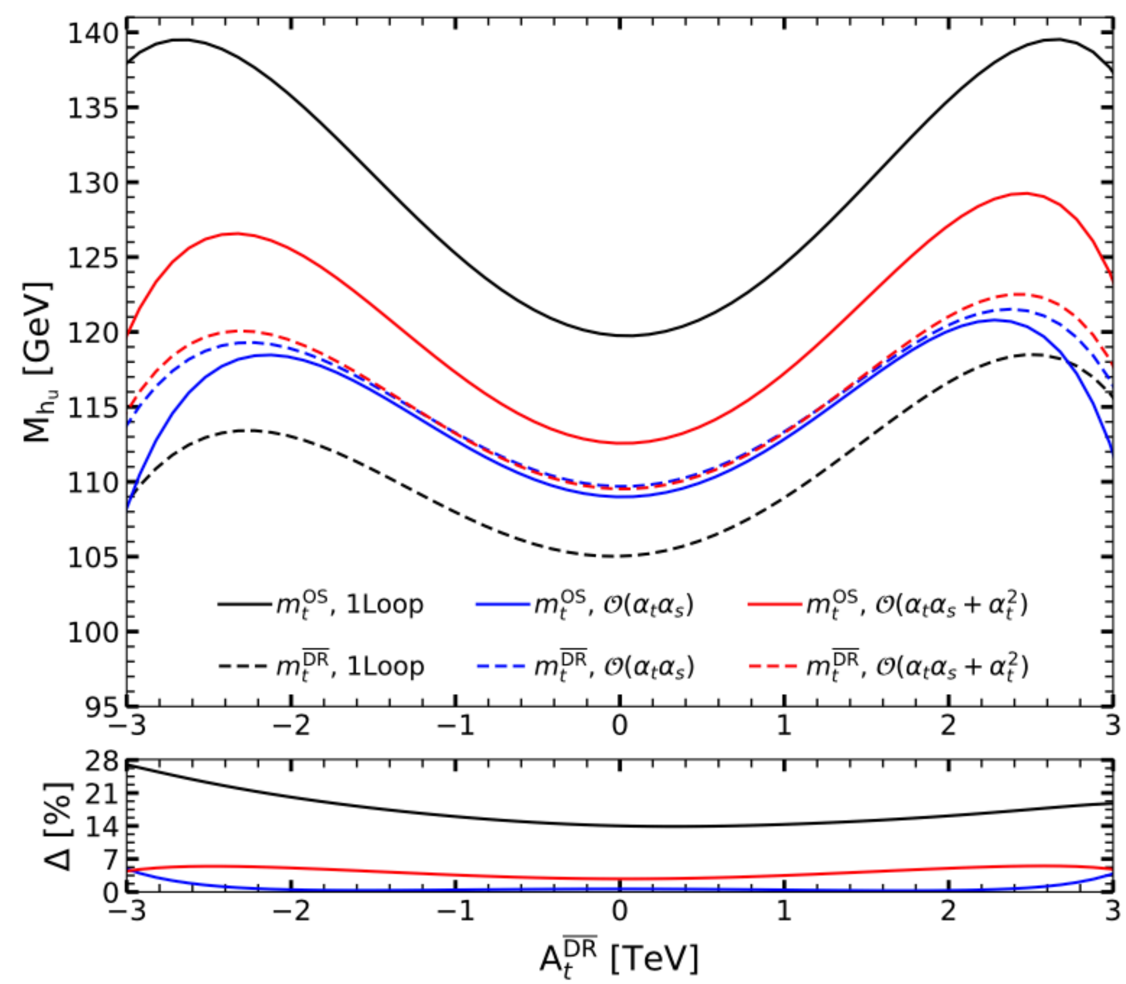
Results for $\mathcal{O}(\alpha_t^2)$ Corrections

[Dao,Gröber,Krause,MM,Rzehak, JHEP08 (2019) 114]

Size of relative corrections



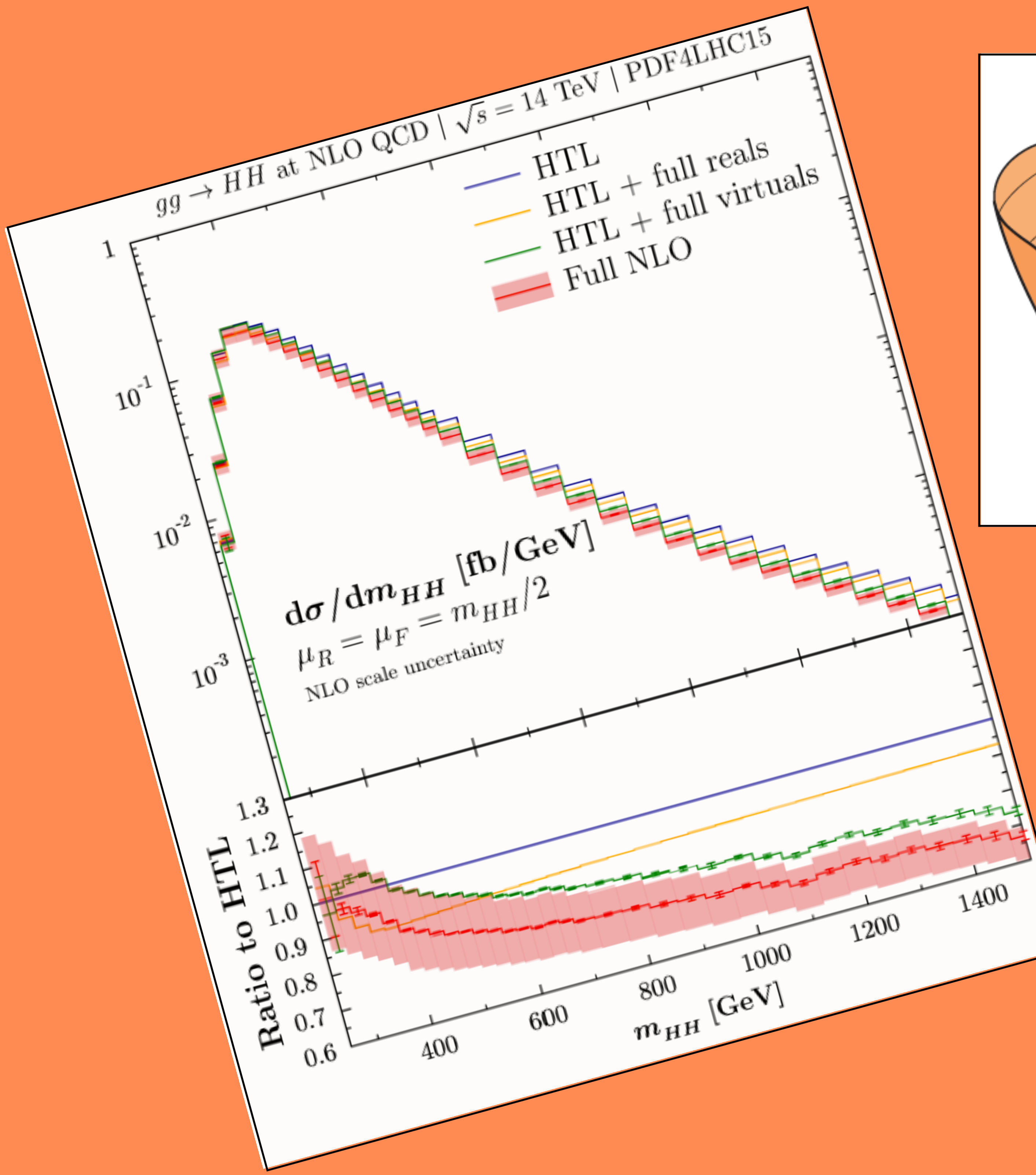
Renormalization scheme dependence

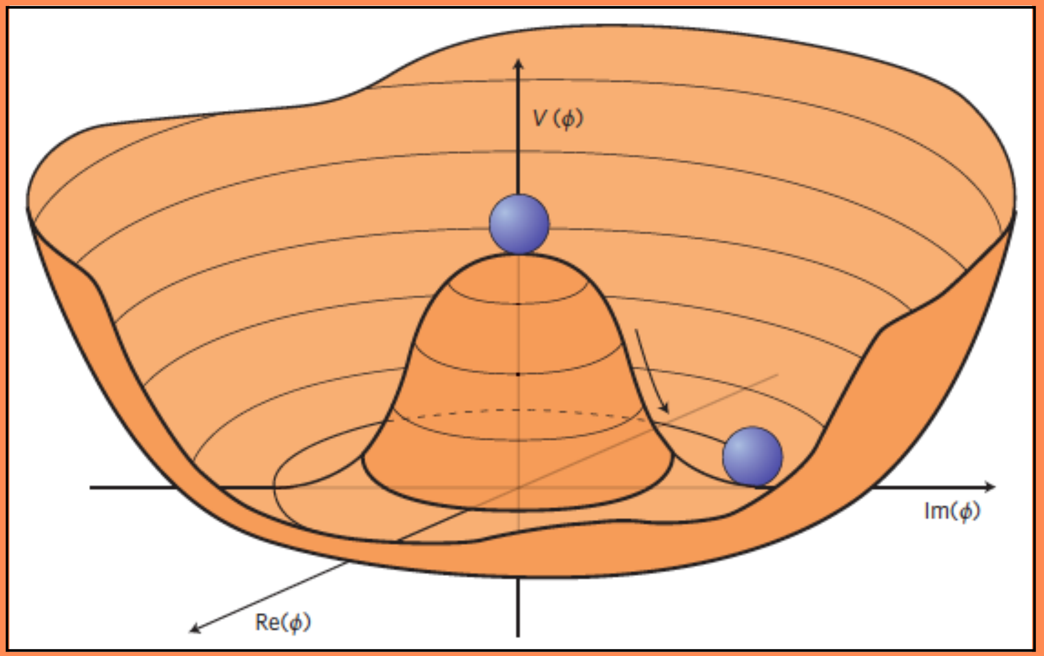


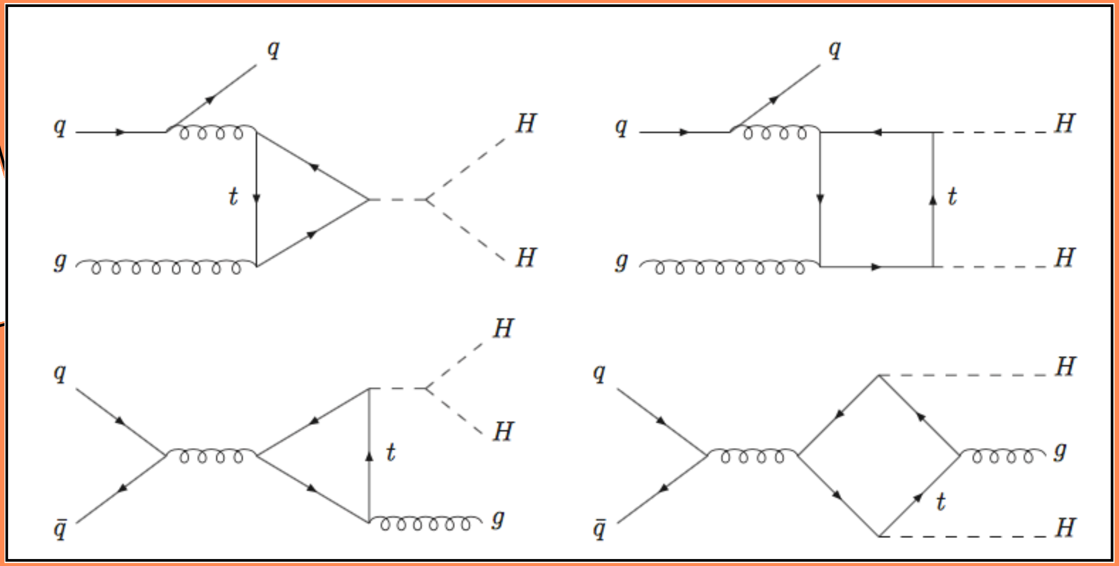
* tree-level CP-violating phase kept zero
* $\Delta = \frac{|M_{h_u}^{(2,x)} - M_{h_u}^{(1)}|}{M_{h_u}^{(1)}}$ and $x = \mathcal{O}(\alpha_t\alpha_s),\ \mathcal{O}(\alpha_t\alpha_s + \alpha_t^2)$

* $\Delta = \frac{|M_{h_u}^{m_t(\overline{\rm DR})} - M_{h_u}^{m_t({\rm OS})}|}{M_{h_u}^{m_t(\overline{\rm DR})}}$

Higher-Order Corrections to Higgs Pair Production



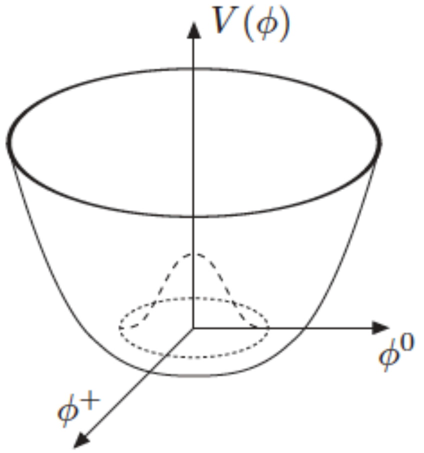


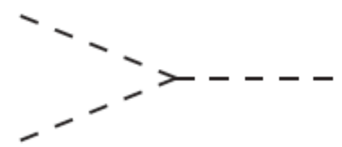
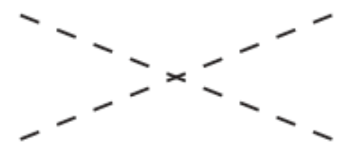


Higgs Pair Production - Ultimate Test of the Higgs Mechanism

The EWSB potential:

$$V(H) = \frac{1}{2!}\lambda_{HH} H^2 + \frac{1}{3!}\lambda_{HHH} H^3 + \frac{1}{4!}\lambda_{HHHH} H^4$$



Trilinear coupling	$\lambda_{HHH} = 3\frac{M_H^2}{v}$	
Quartic coupling	$\lambda_{HHHH} = 3\frac{M_H^2}{v^2}$	

Measurement of the scalar boson self-couplings
and
Reconstruction of the EWSB potential
} Experimental verification
Of the scalar sector of the
EWSB mechanism

Determination of the scalar boson self-couplings at colliders:

λ_{HHH} via pair production

λ_{HHHH} via triple production

radiation off W/Z, $t\bar{t}$, WW/ZZ fusion, gg fusion

NLO Calculation

[Baglio,Campanario,Glaus,MM,Ronca,Spira,Streicher, Eur.Phys.J. C79 (2019) 6, 459; JHEP 04 (2020) 181; 2008.11626]

NLO HH production with full top quark mass dependence: 2 independent calculations

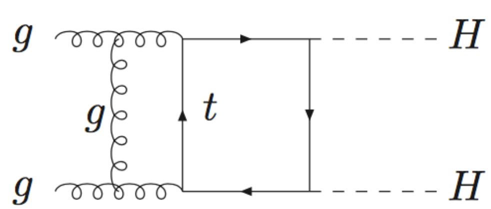
Borowka *et al.*	Baglio *et al.*
tensor reduction	no tensor reduction
sector decomposition	IR, end-point subtraction
contour deformation	IBP, Richardson extrapolation
$m_t = 173$ GeV	$m_t = 172.5$ GeV

[Borowka,Heinrich,Greiner,Jones,Kerner,Schlenk,Schubert,Zirke; Baglio,Campanario,Glaus,MM,Ronca,Spira,Streicher]

IBP: integrations by parts

treatment of thresholds: $m_t^2 \to m_t^2(1 - i\bar{\epsilon})$
narrow-width limit $\bar{\epsilon} \to 0$ w/ Richardson extrapolation

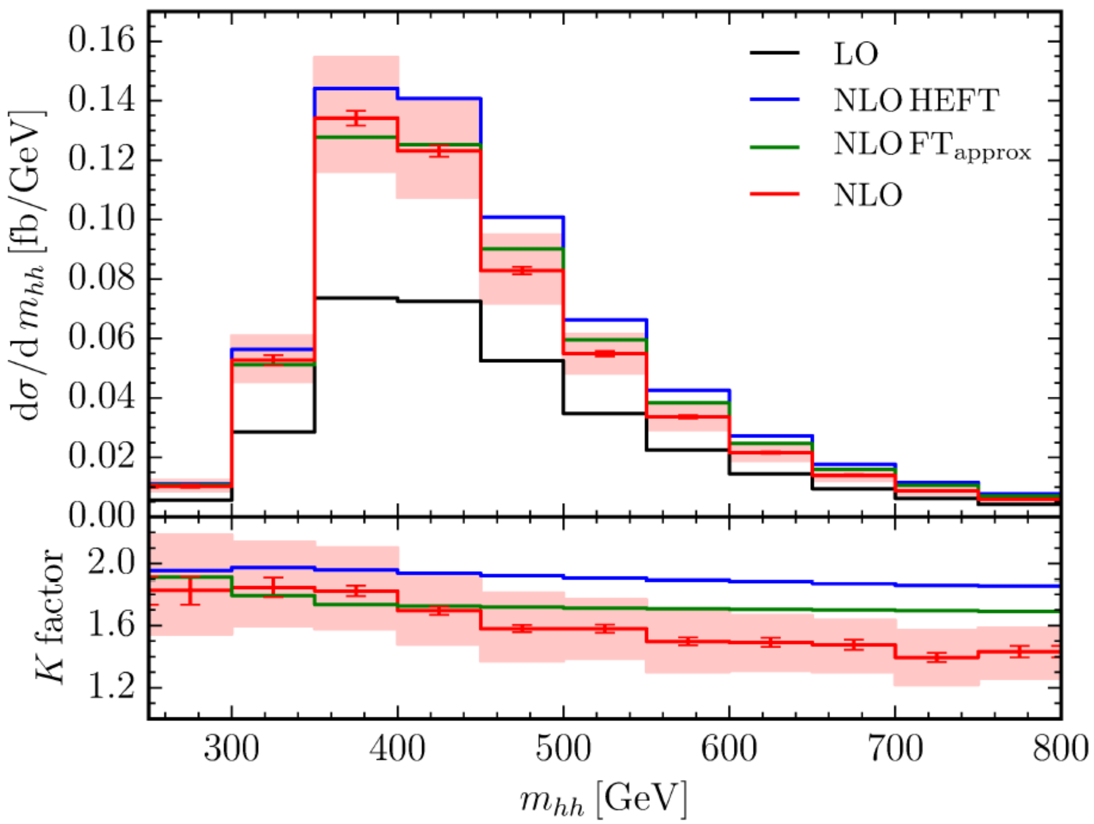
IR: IR divergence in



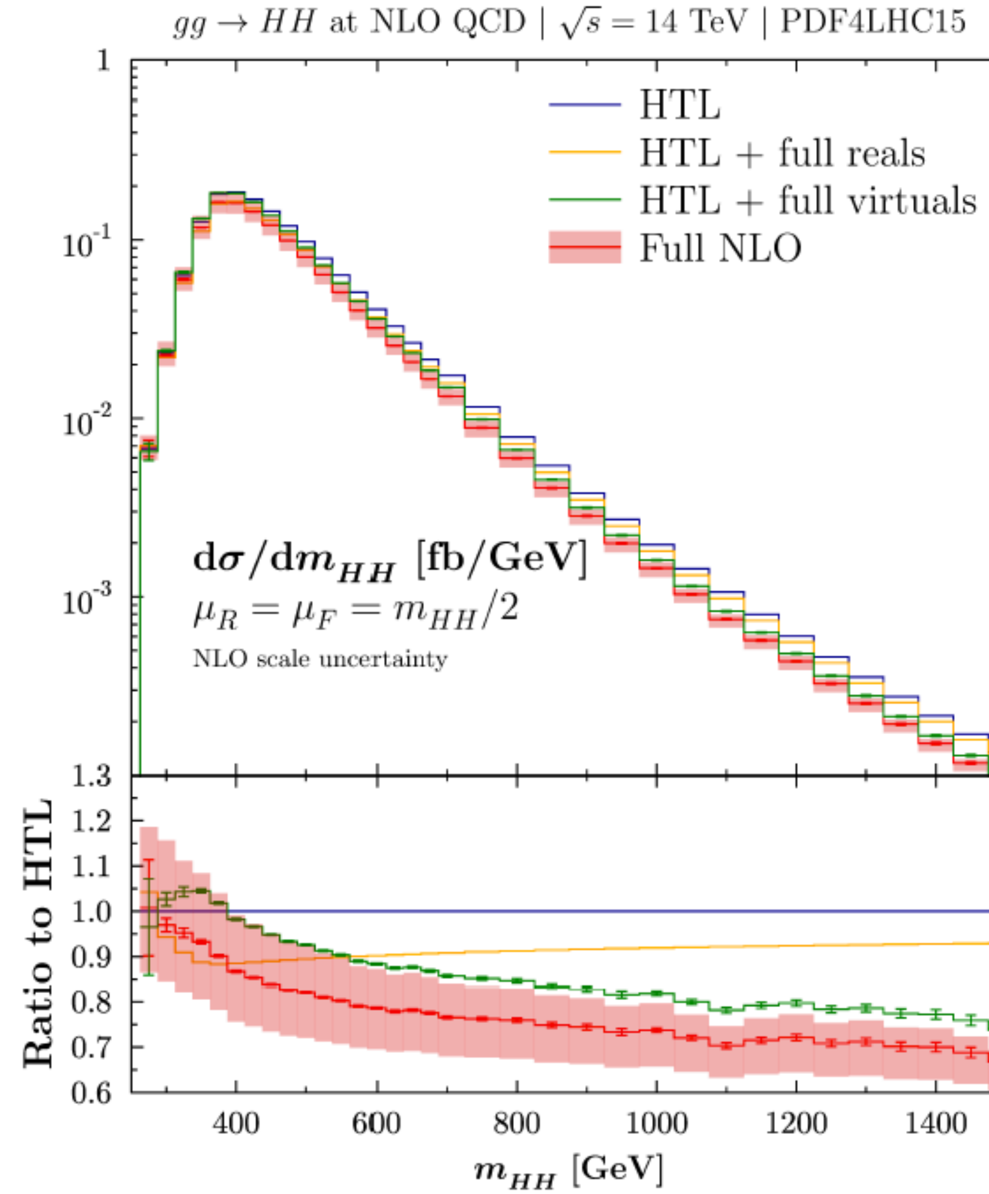
has to be isolated before end-point subtraction (suitable subtraction term -> exact integration of one Feynman integral)

Results

[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke]



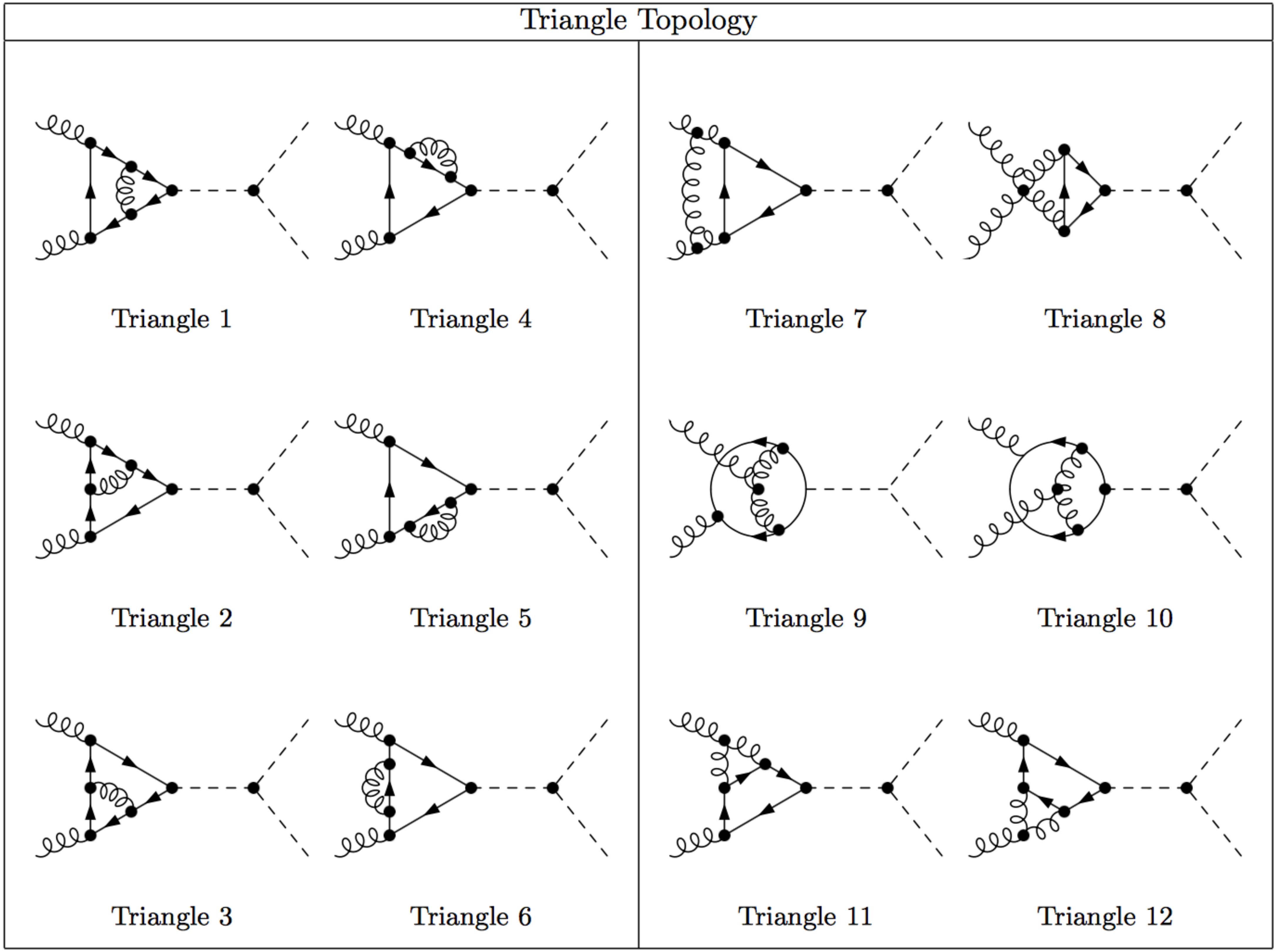
[Baglio,Glaus,Campanario,MM,Ronca,Spira,Streicher]



$\sigma_{NLO} =$	$32.91(10)^{+13.8\%}_{-12.8\%}\ fb$	$32.81(7)^{+13.5\%}_{-12.5\%}\ fb$
$\sigma^{HTL}_{NLO} =$	$38.75^{+18\%}_{-15\%}\ fb$	$38.66^{+18\%}_{-15\%}\ fb$

$\Rightarrow$ -15% mass effects on top of LO

Topologies - Sample Diagrams - Triangles



Topologies - Sample Diagrams - Boxes and Real Corrections

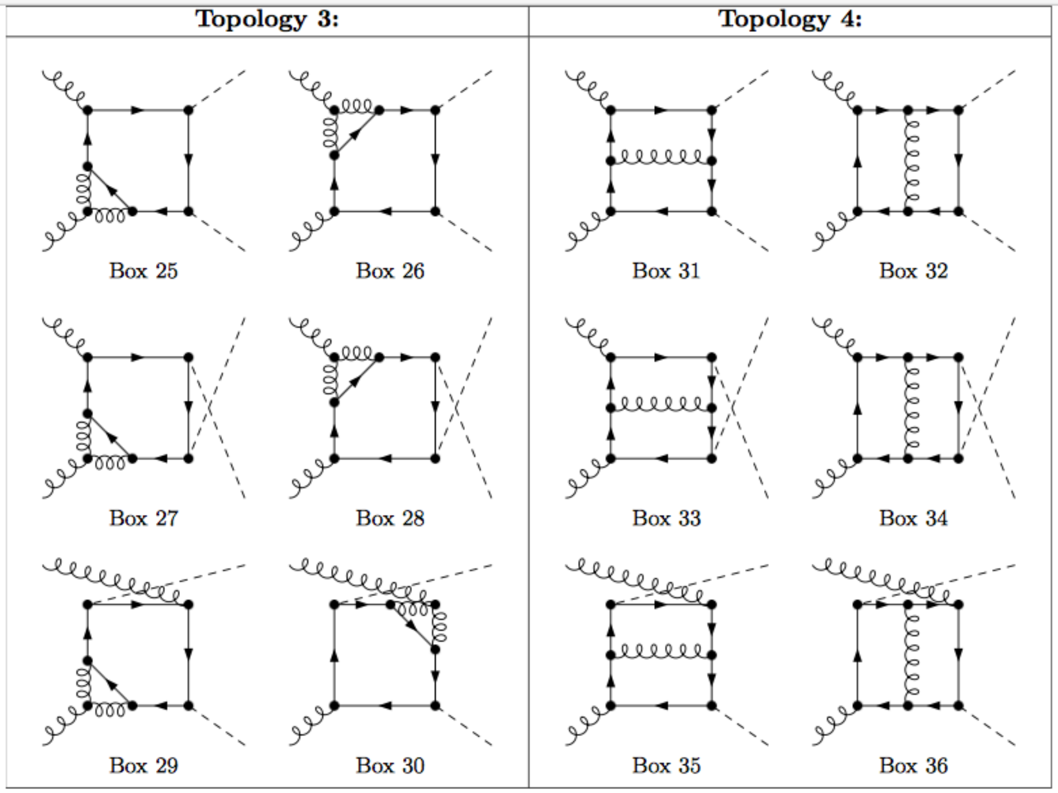
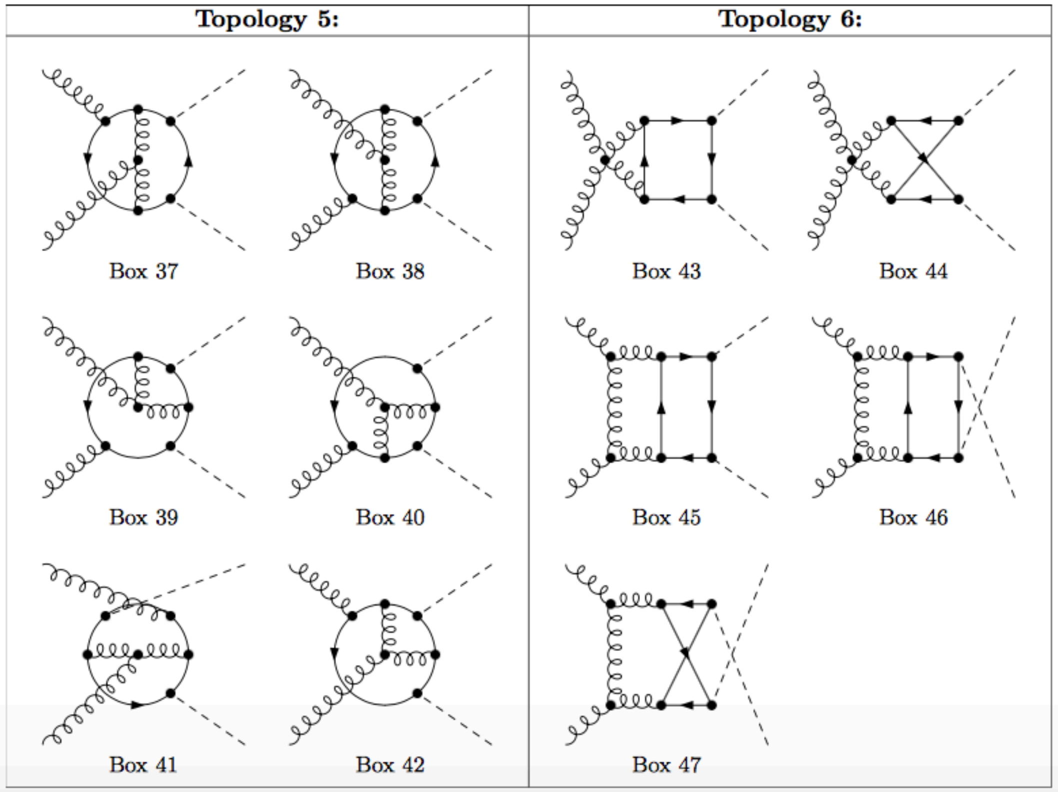
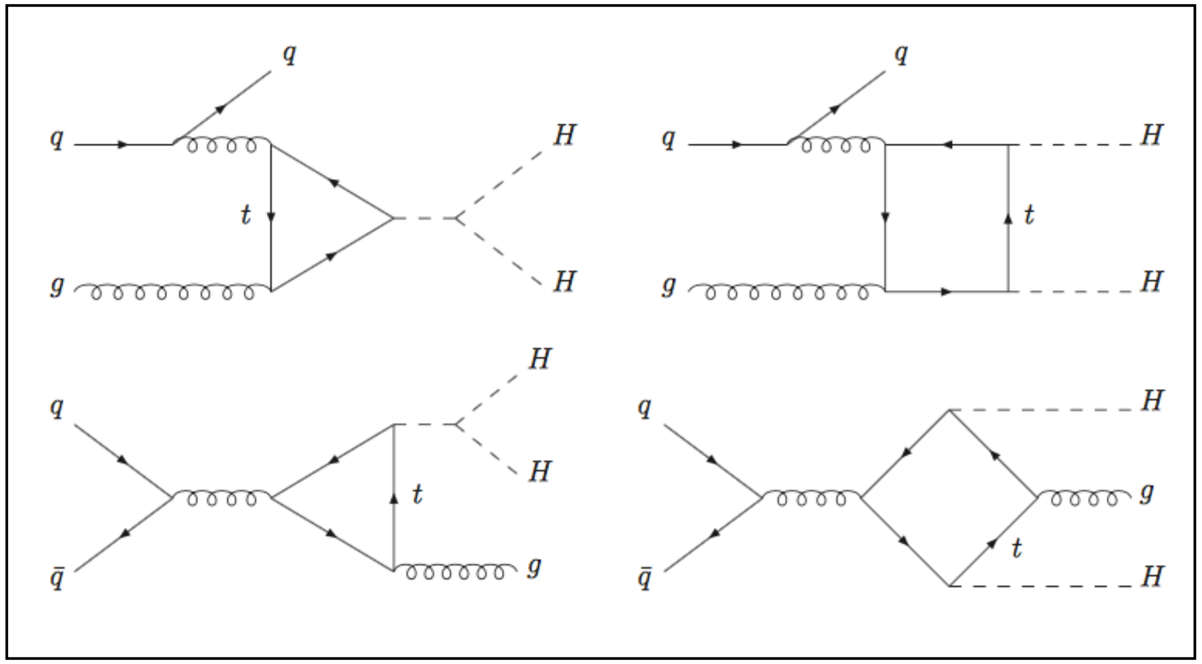


Figure 20: *Two-loop box diagrams: topologies 3 and 4.*





Renormalization

* α_s renormalized in $\overline{\text{MS}}$ scheme with top quark decoupled:

$$\alpha_{s,0} = \alpha_s(\mu_R) + \delta\alpha_s \,,$$
$$\frac{\delta\alpha_s}{\alpha_s} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_0^2}{\mu_R^2}\right)^\epsilon \left\{-\frac{33-2(N_F+1)}{12\epsilon} + \frac{1}{6}\log\frac{\mu_R^2}{m_t^2}\right\}$$

* m_t default scheme: OS:

$$m_{t,0} = m_t - \delta m_t \,,$$
$$\frac{\delta m_t}{m_t} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_0^2}{m_t^2}\right)^\epsilon \left\{\frac{1}{\epsilon} + \frac{4}{3}\right\}$$

* m_t in $\overline{\text{MS}}$ scheme:

$$m_{t,0} = \overline{m}_t(\mu_t) - \delta\overline{m}_t \,,$$
$$\frac{\delta\overline{m}_t}{\overline{m}_t(\mu_t)} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_0^2}{\mu_t^2}\right)^\epsilon \frac{1}{\epsilon}$$

with

[Gray eal,'90;Chetyrkin,Steinhauser,'99 Melnikov,Ritbergen,'99]

$$\overline{m}_t(m_t) = \frac{m_t}{1 + \frac{4}{3}\frac{\alpha_s(m_t)}{\pi} + K_2\left(\frac{\alpha_s(m_t)}{\pi}\right)^2 + K_3\left(\frac{\alpha_s(m_t)}{\pi}\right)^3}$$

and $K_2 \approx 10.9$ and $K_3 \approx 107.11$

scale dependence of $\overline{\text{MS}}$ mass treated at N^{3}LL:

$$\overline{m}_t\,(\mu_t) = \overline{m}_t\,(m_t)\,\frac{c\,[\alpha_s\,(\mu_t)/\pi]}{c\,[\alpha_s\,(m_t)/\pi]}$$

with

$$c(x) = \left(\frac{7}{2}x\right)^{\frac{4}{7}} [1 + 1.398x + 1.793\,x^2 - 0.6834\,x^3]$$

[Tarasov,'82;Chetyrkin,'97]

Uncertainties Related to Top Mass

* transform $m_t \rightarrow \overline{m}_t(\mu)$ ($\overline{\text{MS}}$) -> modification of mass counterterm

* use m_t, $\overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q$ -> uncertainty = envelope:

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}\Big|_{Q=300\ \text{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \text{fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}\Big|_{Q=400\ \text{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \text{fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}\Big|_{Q=600\ \text{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \text{fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}\Big|_{Q=1200\ \text{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \text{fb/GeV}$$

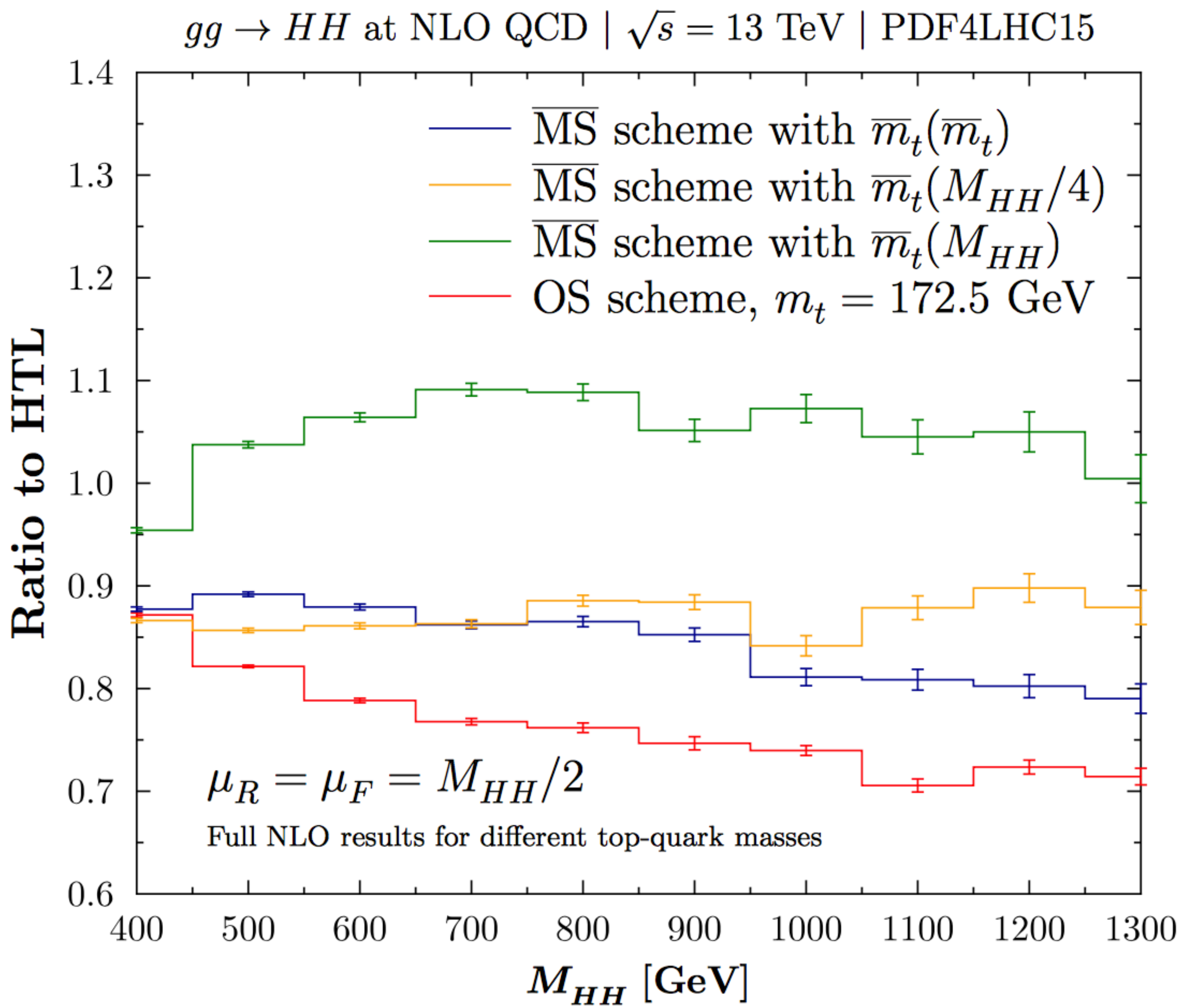
* bin-by-bin interpolation: $\sigma(gg \rightarrow HH) = 32.81^{+4\%}_{-18\%}$ fb

* dynamical scale Q motivated by large momentum expansion $(\hat{s} = Q^2 \gg m_t^2)$

[Davis,Mishima,Steinhauser,Wellmann]

Top-Mass Scale and Scheme Dependence

[Baglio,Campanario,Glaus,MM,Ronca,Spira]



Combination of uncertainties

* Higher-order corrections:

$$d\sigma_n = \sum_{i=0}^{n} d\sigma^{(i)}$$

$$d\sigma_n = d\sigma_{n-1} \times \left(K_{SV}^{(n)} + K_{rem}^{(n)}\right)$$

(top-mass independent) soft+virtual part is dominant for the first few orders

moderate (top-mass dependent) remainder only adds 10-15% to bulk of the corrections of 100%

$K_{SV}^{(n)}$ are basically the same for the (subleading) mass effects to all orders. These pieces are part of the HTL at all perturbative orders

=> Born-improved and FT_{approx}: reasonable approximation of the total cross section within 10-15% at NLO

=> mass effects and their uncertainties scale with dominant part of QCD corrections

Recommendation: [Baglio,Campanario,Glaus,MM,Ronca,Spira,'20]

=> at NLO and FT_{approx}
combine relative top mass scale and scheme & factorization and renormalization scale uncertainties linearly

Combination of uncertainties

Finally combine top mass scale and scheme & renormalization and factorization scale uncertainties at NNLO FT_{approx}

[Baglio,Campanario,Glaus,MM,Ronca,Spira,'20]

$$\sqrt{s} = 13 \text{ TeV}: \quad \sigma_{tot} = 31.05^{+6\%}_{-23\%} \text{ fb}$$
$$\sqrt{s} = 14 \text{ TeV}: \quad \sigma_{tot} = 36.69^{+6\%}_{-23\%} \text{ fb}$$
$$\sqrt{s} = 27 \text{ TeV}: \quad \sigma_{tot} = 139.9^{+5\%}_{-22\%} \text{ fb}$$
$$\sqrt{s} = 100 \text{ TeV}: \quad \sigma_{tot} = 1224^{+4\%}_{-21\%} \text{ fb}$$

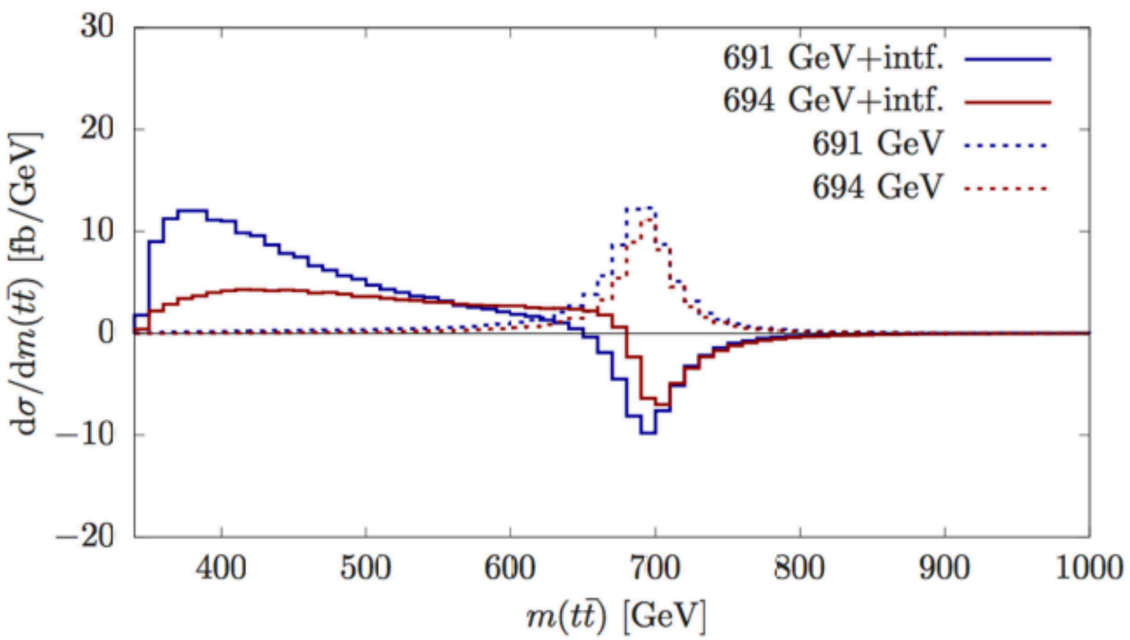
$$\kappa_\lambda = -10: \quad \sigma_{tot} = 1680^{+13\%}_{-14\%} \text{ fb}$$
$$\kappa_\lambda = -5: \quad \sigma_{tot} = 598.9^{+13\%}_{-15\%} \text{ fb}$$
$$\kappa_\lambda = -1: \quad \sigma_{tot} = 131.9^{+11\%}_{-16\%} \text{ fb}$$
$$\kappa_\lambda = 0: \quad \sigma_{tot} = 70.38^{+8\%}_{-18\%} \text{ fb}$$
$$\kappa_\lambda = 1: \quad \sigma_{tot} = 31.05^{+6\%}_{-23\%} \text{ fb}$$
$$\kappa_\lambda = 2: \quad \sigma_{tot} = 13.81^{+3\%}_{-28\%} \text{ fb}$$
$$\kappa_\lambda = 2.4: \quad \sigma_{tot} = 13.10^{+6\%}_{-27\%} \text{ fb}$$
$$\kappa_\lambda = 3: \quad \sigma_{tot} = 18.67^{+12\%}_{-22\%} \text{ fb}$$
$$\kappa_\lambda = 5: \quad \sigma_{tot} = 94.82^{+18\%}_{-13\%} \text{ fb}$$
$$\kappa_\lambda = 10: \quad \sigma_{tot} = 672.2^{+16\%}_{-13\%} \text{ fb}$$

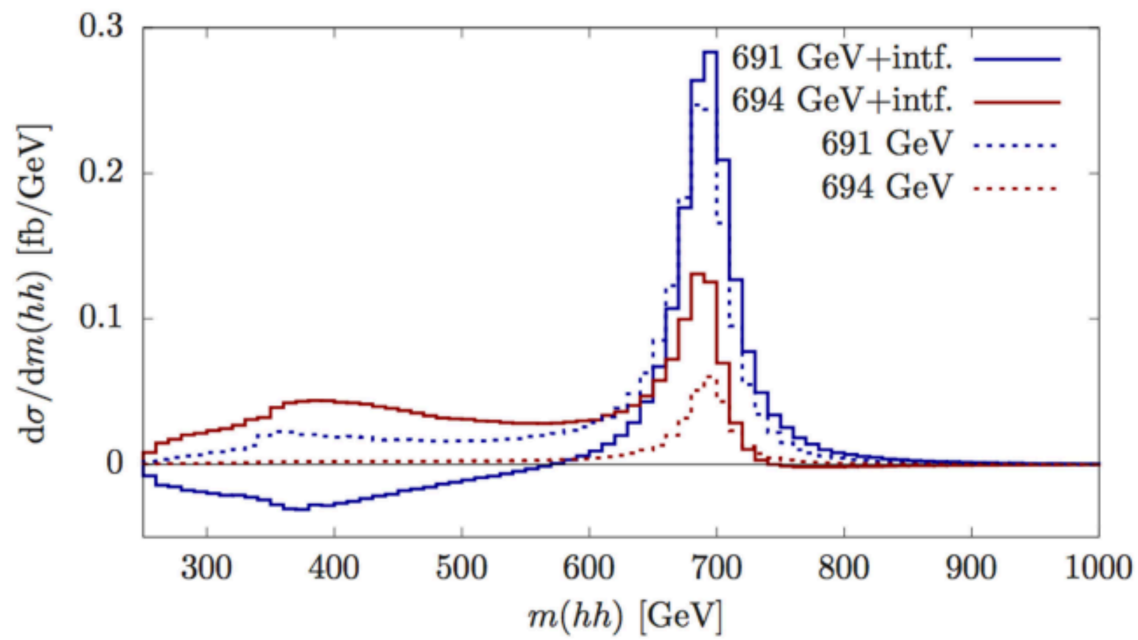
Next steps:
extension to extended Higgs sectors,
at present gg->AA

[Basler,Dawson,Englert,MM, Phys.Rev.D101 (2020) 1]

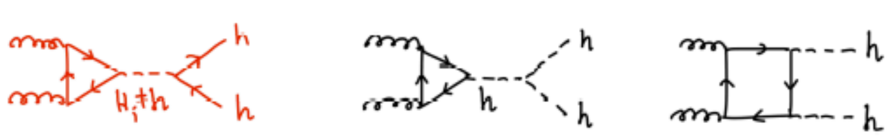
- **Di-Higgs Peaks and Top Valleys** (C2HDM) [Basler,Dawson,Englert,MM, 1909.09987]

  $gg \to H_i \to t\bar{t}$ and $gg \to H_i \to hh$ (h SM-like Higgs boson, $H_i \neq h$)





∗ Destructive signal-background interference may be correlated with constructive signal-signal interference



For interference effects, see also [Dawson,Lewis '15; Djouadi eal '19; Lewis/Carena eal/Bagnaschi eal in 1910.00012]

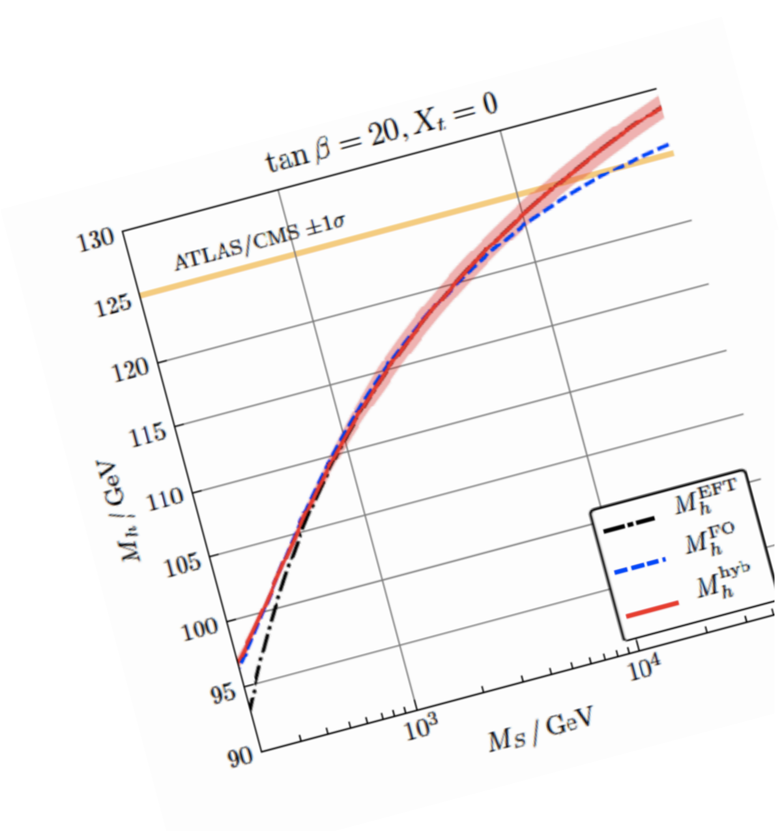
Summary

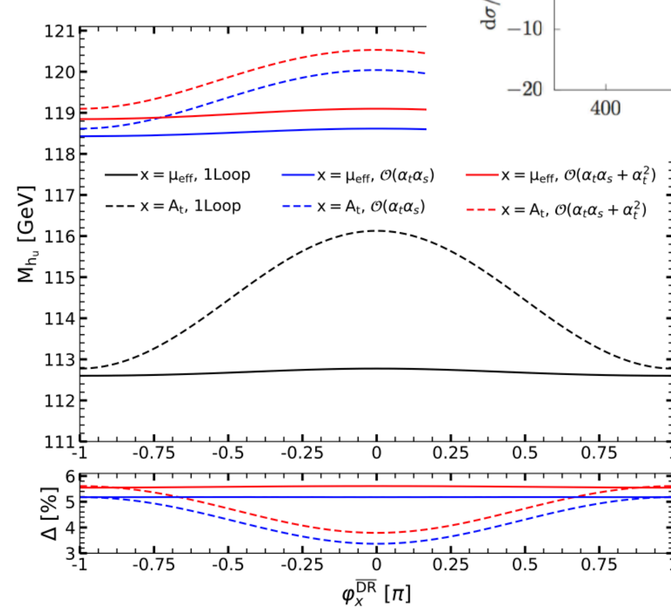
Higher-Order Corrections to MSSM Higgs masses: [Harlander,Klappert,Voigt,Eur.Phys.J.C80(2020)3]

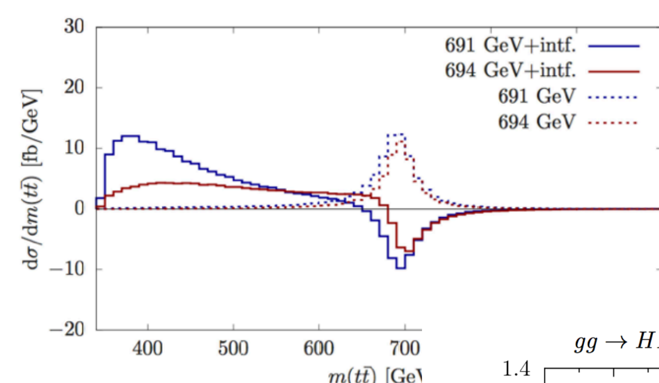
Higher-Order Corrections to NMSSM Higgs masses:
[Dao,Gröber,Krause,MM,Rzehak, JHEP08 (2019) 114]
[Dao,Gabelmann,MM,Rzehak, in preparation]

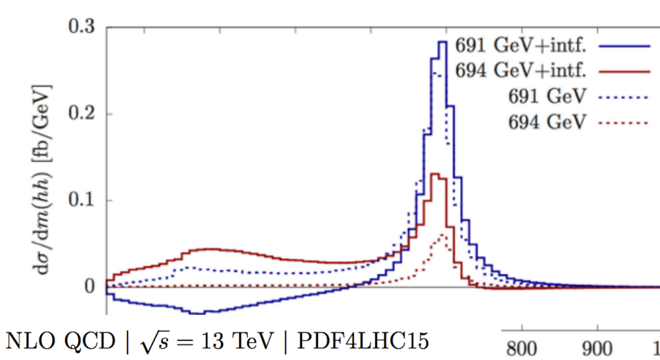
Higher-Order Corrections to Higgs pair production - details + uncertainties:
[Baglio,Campanario,Glaus,MM,Ronca,Spira,Streicher, Eur.Phys.J. C79 (2019) 6, 459; JHEP 04 (2020) 181; 2008.11626]

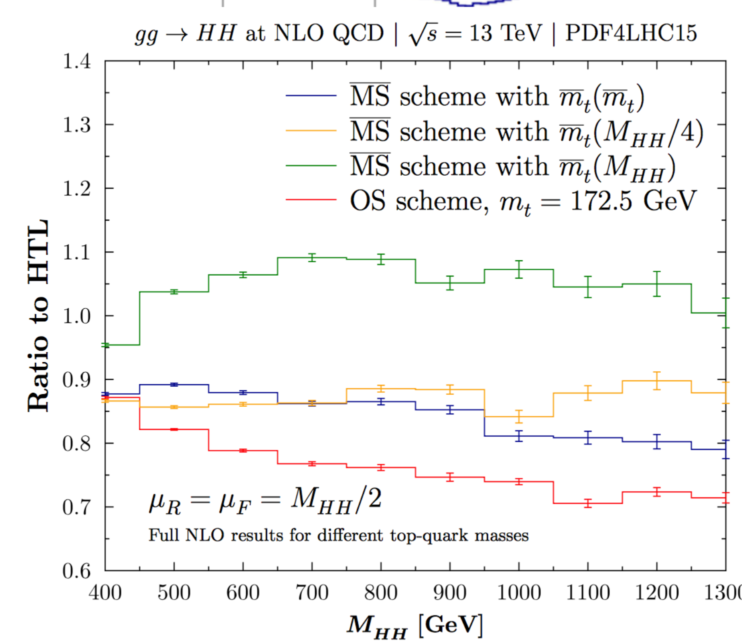
Higgs pair production - phenomenology: [Basler,Dawson,Englert,MM, Phys.Rev.D101 (2020) 1]











Thank you for your attention

Renormalization

- **Renormalization conditions:** mixed $\overline{\text{DR}}$-OS scheme

$$M_{H^\pm}\ \text{input}:\quad \underbrace{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, M^2_{H^\pm}, v}_{\text{on-shell scheme}}, \underbrace{\tan\beta, |\lambda|, v_s, |\kappa|, \text{Re}A_\kappa, \varphi_\lambda, \varphi_\kappa, \varphi_u, \varphi_s}_{\overline{\text{DR}}\ \text{scheme}}$$

$$\text{Re}A_\lambda\ \text{input}:\quad \underbrace{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, v}_{\text{on-shell scheme}}, \underbrace{\tan\beta, |\lambda|, v_s, |\kappa|, \text{Re}A_\lambda, \text{Re}A_\kappa, \varphi_\lambda, \varphi_\kappa, \varphi_u, \varphi_s}_{\overline{\text{DR}}\ \text{scheme}}$$

- **Field renormalization:** $X \equiv H_d, H_u, S$

$$X \to (1 + \frac{1}{2}\delta^{(1)} Z_X + \frac{1}{2}\delta^{(2)} Z_X - \frac{1}{8}(\delta^{(1)} Z_X)^2) X \equiv (1 + \frac{1}{2}\delta^{(1)} Z_X + \frac{1}{2}\Delta^{(2)} Z_X) X$$

$*$ $\overline{\text{DR}}$ renormalization

$$\left.\frac{\partial \hat{\Sigma}^{(n)}_{ii}(p^2)}{\partial p^2}\right|_{\text{div}} = 0$$

* $\sin\theta_W$: OS (remark $p^2=0$ at two-loop); top/stop sector renormalization: $\overline{\text{DR}}$ or OS

The NLO Cross Section

$$\sigma_{\rm NLO}(pp \to HH+X) = \sigma_{\rm LO} + \Delta\sigma_{\rm virt} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar q}$$

$$\sigma_{\rm LO} = \int_{\tau_0}^1 d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat\sigma_{\rm LO}(Q^2 = \tau s)$$

quark mass dependence in LO cxn and C, d_{ij}

$$\Delta\sigma_{\rm virt} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat\sigma_{\rm LO}(Q^2 = \tau s) \; C$$

$$\Delta\sigma_{gg} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \, \hat\sigma_{\rm LO}(Q^2 = z\tau s) \left\{ -zP_{gg}(z) \log\frac{M^2}{\tau s} + d_{gg}(z) + 6[1+z^4+(1-z)^4] \left(\frac{\log(1-z)}{1-z}\right)_+ \right\}$$

$$\Delta\sigma_{gq} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q,\bar q} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \, \hat\sigma_{\rm LO}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log\frac{M^2}{\tau s(1-z)^2} + d_{gq}(z) \right\}$$

$$\Delta\sigma_{q\bar q} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar q}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \, \hat\sigma_{\rm LO}(Q^2 = z\tau s) \; d_{q\bar q}(z)$$

heavy top limit (HTL):

$$C \to \pi^2 + \frac{11}{2} + C_{\triangle\triangle}, \quad d_{gg} \to -\frac{11}{2}(1-z)^3, \quad d_{gq} \to \frac{2}{3}z^2 - (1-z)^2, \quad d_{q\bar q} \to \frac{32}{27}(1-z)^3$$

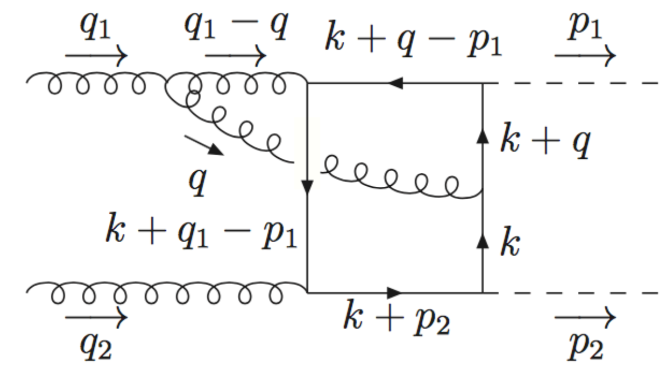
Definition of the Tensors

The two tensor structures correspond to the total angular momentum states with S_z=0 and 2:

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{q_1^\nu q_2^\mu}{(q_1 q_2)} ,$$
$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{M_H^2 q_1^\nu q_2^\mu}{p_T^2 (q_1 q_2)} - 2\frac{(q_2 p_1) q_1^\nu p_1^\mu}{p_T^2 (q_1 q_2)} - 2\frac{(q_1 p_1) p_1^\nu q_2^\mu}{p_T^2 (q_1 q_2)} + 2\frac{p_1^\nu p_1^\mu}{p_T^2}$$
$$\text{with} \quad p_T^2 = 2\frac{(q_1 p_1)(q_2 p_1)}{(q_1 q_2)} - M_H^2 ,$$

with p_T being the transverse momentum of the final-state Higgs boson

Sample Calculation - Box 39



Numerator contains regular factors, singular and higher powers in the dimensional regulator epsilon

Denominator:
$$N(\vec{x}) = 1 + \rho_s xzr\Big\{xz + (1-x)[zs + (1-s)t]\Big\}$$
$$-\rho_t x\Big\{z(1-y-r) + (y-z)[z + (1-x)(1-s)(t-z)]\Big\}$$
$$+\rho_u xzr\Big\{xz + (1-x)[zs + (1-s)y]\Big\}$$
$$-\rho_H\Big\{[xz + (1-x)y][1 - xz - (1-x)y] - x(1-x)s(y-z)^2\Big\}$$

with definitions $\rho_s = \hat{s}/m_t^2 = Q^2/m_t^2$, $\rho_t = (\hat{t} - M_H^2)/m_t^2$, $\rho_u = (\hat{u} - M_H^2)/m_t^2$ and $\rho_H = M_H^2/m_t^2$.

singularity for s->0 in integrals isolated through endpoint subtraction

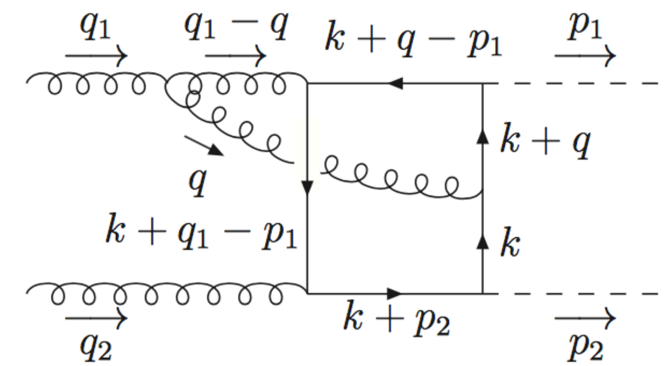
stabilization of integrals above threshold through integrations by parts w.r.t. Feynman parameter z:

$$\int_0^1 dz\, \frac{H_i(\vec{x})}{N^3} = \frac{1}{\Delta}\left\{\left[\frac{2a+b}{2N^2}H_i(\vec{x}) + \frac{\partial_z H_i(\vec{x})}{2N}\right]\Bigg|_{z=1} - \left[\frac{b}{2N^2}H_i(\vec{x}) + \frac{\partial_z H_i(\vec{x})}{2N}\right]\Bigg|_{z=0}\right.$$
$$\left. + \int_0^1 dz\, \left[\frac{3a}{N^2}H_i(\vec{x}) - \frac{\partial_z^2 H_i(\vec{x})}{2N}\right]\right\}$$

$N=az^2+bz+c$
$\Delta=(2az+b)^2$

and analogously for integrals involving additional power of log N factors in the numerator
=> integrals stable for values of $\bar{\epsilon}$ down to 0.05. Perform Richardson extrapolation down to 0

Sample Calculation - Box 39



$$A_{39}^{\mu\nu} = \frac{3}{16}\,\frac{\alpha_s}{\pi}\,(4\pi)^4\,B_{39}^{\mu\nu}\,,$$

$$B_{39}^{\mu\nu}=\int \frac{d^nk d^nq}{(2\pi)^{2n}}\frac{Tr\left\{(\not k+\not q-\not p_1+m_t)(\not k+\not q+m_t)\gamma^\sigma(\not k+m_t)(\not k+\not p_2+m_t)\gamma^\nu(\not k+\not q_1-\not p_1+m_t)\gamma^\rho\right\}}{[(k+q)^2-m_t^2][(k+q-p_1)^2-m_t^2][(k+p_2)^2-m_t^2][(k+q_1-p_1)^2-m_t^2]}$$

$$\times\ \frac{g_{\rho\sigma}(2q-q_1)^\mu - g_\rho^\mu(q-2q_1)_\sigma - g_\sigma^\mu(q+q_1)_\rho}{(k^2-m_t^2)(q-q_1)^2q^2}$$

with tensor definition $$\mathcal{M}(g^a g^b \to HH) = -i\,\frac{G_F\alpha_s(\mu_R)Q^2}{2\sqrt{2}\pi}\mathcal{A}^{\mu\nu}\epsilon_{1\mu}\epsilon_{2\nu}\delta_{ab}$$

with $$\mathcal{A}^{\mu\nu} = F_1T_1^{\mu\nu} + F_2T_2^{\mu\nu}\,,$$

$$F_1 = C_\triangle F_\triangle + F_\Box\,, \qquad F_2 = G_\Box$$

$$C_\triangle = \frac{\lambda_{H^3}v}{Q^2-M_H^2+iM_H\Gamma_H}$$

introduce Feynman parameters x_i, symmetrize n-dimensional k integration, perform substitutions and shift in q; after projection on the two form factors => integral

full numerator

$$\Delta F_i = \frac{\alpha_s}{\pi}\,\Gamma(1+2\epsilon)\left(\frac{4\pi\mu_0^2}{m_t^2}\right)^{2\epsilon}\int_0^1 d^6x\,\frac{x^\epsilon(1-x)^\epsilon s^{-1-\epsilon}H_i(\vec{x})}{N^{3+2\epsilon}(\vec{x})}$$

Motivation of Dynamical Scale Q

* dynamical scale Q motivated by large momentum expansion $(\hat{s} = Q^2 \gg m_t^2)$ -> 2 form factors

[Davis,Mishima,Steinhauser,Wellmann]

pole mass m_t:

$$\Delta F_{1,mass} \to \frac{\alpha_s}{\pi}\left\{2F_{1,LO}\log\frac{m_t^2}{\hat{s}} + \frac{m_t^2}{\hat{s}}G_1(\hat{s},\hat{t})\right\},$$

$$\Delta F_{2,mass} \to \frac{\alpha_s}{\pi}\left\{2F_{2,LO}\log\frac{m_t^2}{\hat{s}} + \frac{m_t^2}{\hat{s}}G_2(\hat{s},\hat{t})\right\}$$

$\overline{\text{MS}}$ mass $\overline{m}_t(\mu_t)$:

$$\Delta F_{1,mass} \to \frac{\alpha_s}{\pi}\left\{2F_{1,LO}\left[\log\frac{\mu_t^2}{\hat{s}} + \frac{4}{3}\right] + \frac{\overline{m}_t^2(\mu_t)}{\hat{s}}G_1(\hat{s},\hat{t})\right\},$$

$$\Delta F_{2,mass} \to \frac{\alpha_s}{\pi}\left\{2F_{2,LO}\left[\log\frac{\mu_t^2}{\hat{s}} + \frac{4}{3}\right] + \frac{\overline{m}_t^2(\mu_t)}{\hat{s}}G_2(\hat{s},\hat{t})\right\}$$

=> scale $\mu_t \sim Q$ preferred at large Q

The Model - CP-Violating NMSSM

- Higgs potential: complex/CP-violating

$$
\begin{aligned}
V_H &= (|\lambda S|^2 + m_{H_d}^2) H_d^\dagger H_d + (|\lambda S|^2 + m_{H_u}^2) H_u^\dagger H_u + m_S^2 |S|^2 \\
&\quad + \frac{1}{8}(g_2^2 + g_1^2)(H_d^\dagger H_d - H_u^\dagger H_u)^2 + \frac{1}{2} g_2^2 |H_d^\dagger H_u|^2 \\
&\quad + |-\epsilon^{ij} \lambda H_{d,i} H_{u,j} + \kappa S^2|^2 + \left[-\epsilon^{ij} \lambda A_\lambda S H_{d,i} H_{u,j} + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.} \right]
\end{aligned}
$$

- Higgs fields after electroweak symmetry breaking:

$$
H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + h_d + i a_d) \\ h_d^- \end{pmatrix}, \quad H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{1}{\sqrt{2}}(v_u + h_u + i a_u) \end{pmatrix}, \quad S = \frac{e^{i\varphi_s}}{\sqrt{2}}(v_s + h_s + i a_s)
$$

- Tadpoles (zero at tree level):

$$
(\boldsymbol{t})_\Phi = t_\Phi = \frac{\partial V_H}{\partial \Phi}, \;\; \Phi = h_d, h_u, h_s, a_d, a_u, a_s
$$

- Independent input parameters: 2 options
  - $M_{H^\pm}$ input

$$
\{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, M_{H^\pm}^2, v, s_{\theta_W}, e, \tan\beta, |\lambda|, v_s, |\kappa|, \text{Re}A_\kappa, \varphi_\lambda, \varphi_\kappa, \varphi_u, \varphi_s\}
$$

  - ReA_λ input

$$
\{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, v, s_{\theta_W}, e, \tan\beta, |\lambda|, v_s, |\kappa|, \text{Re}A_\lambda, \text{Re}A_\kappa, \varphi_\lambda, \varphi_\kappa, \varphi_u, \varphi_s\}
$$