Multi-emission Kernels for Parton Branching Algorithms[a]

Maximilian Löschner

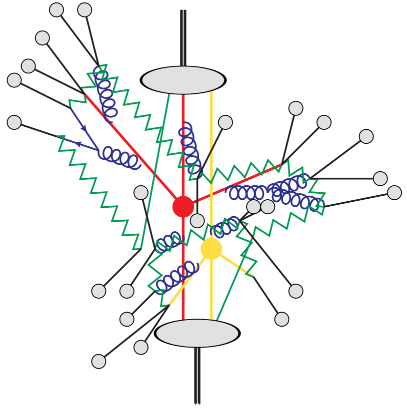
Karlsruher Institut für Technologie

Institute for Theoretical Physics (Project B1d, PI: S. Gieseke)
Erwin Schrödinger Institut (research stay in Vienna)

7 October 2020

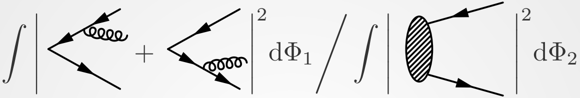
[a] in collaboration with Simon Plätzer and Emma Simpson Dore, arXiv:2010.xxxxx

- Motivation
- Current activities
- Splitting kernels
- Conclusions



$$d\sigma \simeq d\sigma_{\rm hard}(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

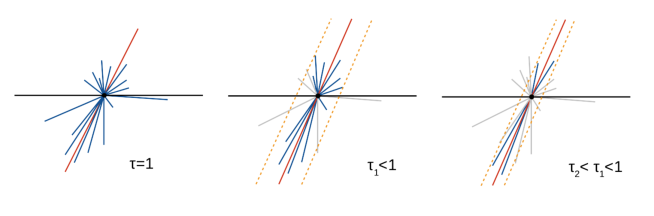
Parton shower status



- Despite pushes for higher orders in parton showers (*e.g.* [Prestel, Hoeche—Phys.Rev.D 96 (2017) 7, 074017], [Skands, Li—PLB 771 (2017) 59-66]) road to accuracy requires paradigm shift
  - **Recoil, ordering, colour, correlations** [Dasgupta, Salam—PLB 512 (2001) 323-330], [Bewick, Seymour, Richardson—JHEP 04 (2020) 019], [Forshaw, Holguin, Plätzer–JHEP 09 (2020) 014], [Ruffa, Plätzer—soon], [ML, Plätzer, Simpson—in progr.]
- **Amplitude level** sets the complexity for resolving these [Nagy, Soper], [DeAngelis, Forshaw, Plätzer—2007.09648 & JHEP 05 (2018) 044]
- Not only relevant theoretically but also in its own right to **go beyond leading-N_C resummation for complex observables**

Coherent branching

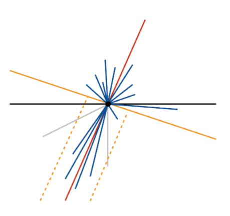
- **Coherent emission of soft large angle gluons** from systems of collinear partons
- **Angular ordering** essential for including large-angle soft contributions



- Resummation of global jet observables such as thrust τ
- NLL accurate @NLC if inclusive over secondary soft gluon emission (LC: Leading Colour)
- Leading regions analyzed in iterated $1 \to 2$ branchings

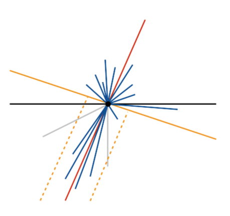
Non-global observables

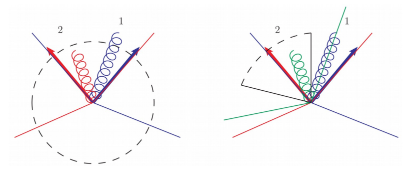
- No global measure of deviation from jet configuration: **Coherent branching fails**, full complexity of amplitudes strikes back.
- **Dipole shower**: correct LL@LC for non-global, but issues in NLL@LC and LL@NLC for global observables



Non-global observables

- No global measure of deviation from jet configuration: **Coherent branching fails**, full complexity of amplitudes strikes back.
- **Dipole shower**: correct LL@LC for non-global, but issues in NLL@LC and LL@NLC for global observables
- Require dipole-type soft gluon evolution to account for change in colour structure
- Even with a dipole approach, $1/N_C$ effects possibly become comparable to subleading logs, and intrinsically $\sim 10\%$ effects





Motivation

Study approximations in emission iterations rather than iterations of one emission approximation.

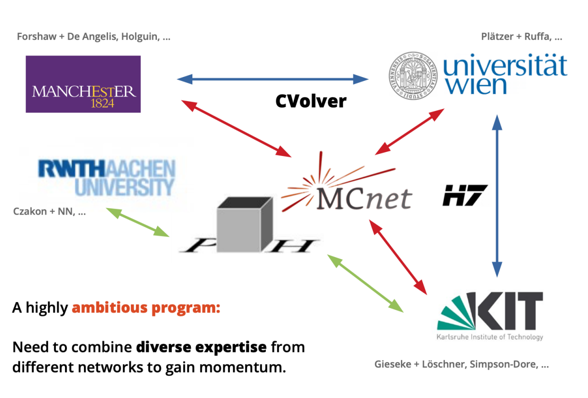
- Going beyond iterated $1 \to 2$ splittings in parton showers
- Combine with global recoil scheme
- Address non-global observables

} higher logarithmic accuracy $\Leftrightarrow$ Systematic expansion to handle uncertainties

- Include color and spin correlations
- Refine ad hoc models of MC-programs, *e.g.* azimuthal correlations
- Define language for connecting fixed order to parton showers

Team

Karlsruhe/Manchester/Vienna network with support from **SFB** drives significant parts of the development, also relating to aspects such as **color reconnection** [*e.g.* Gieseke, Kirchgaesser, Plätzer-JHEP 11 (2018) 149]



[Plätzer—Annual CRC Meeting 2019]

Current activities

1. Amplitude evolution, link to resummation in existing showers
   [Forshaw, Holguin, Plätzer— 2003.06400 & JHEP 08 (2019) 145]
2. New mappings and dipole shower improvements in Herwig
   [Holguin, Plätzer, Simpson, in progr.]
3. Virtual corrections [Ruffa, Plätzer—soon]
4. Dipole showers analytics [Gieseke, Plätzer, Schaber—in progr.]
5. Real corrections [ML, Plätzer, Emma Simpson Dore—in progr.]
   ↑ **this talk**

Goal: build a universal algorithm with well-handled accuracy

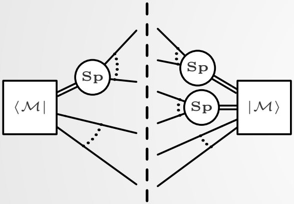
- Focus on: factorization, systematic expansion of emission contributions, recoil and its relation to factorizing evolution kernels

Splitting kernels

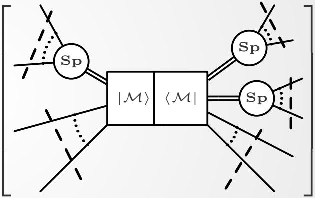
Splitting kernels from amplitudes

From the cross-section level to decomposed amplitudes:

$$\sigma = \sum_n \int \mathrm{Tr}\left[\, |\mathcal{M}(\mu)\rangle \langle \mathcal{M}(\mu)| \,\right] u(p_1, \ldots, p_n) \mathrm{d}\phi_n$$



$\rightarrow \quad \mathrm{Tr}_c$



$\times\, \mathbf{P}$

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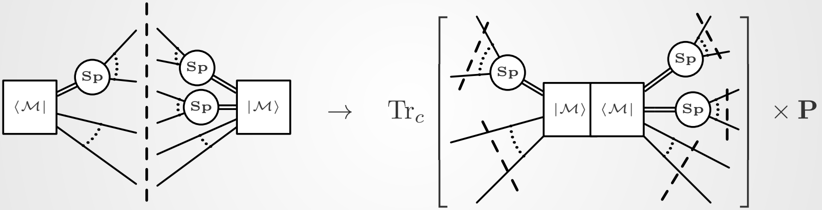
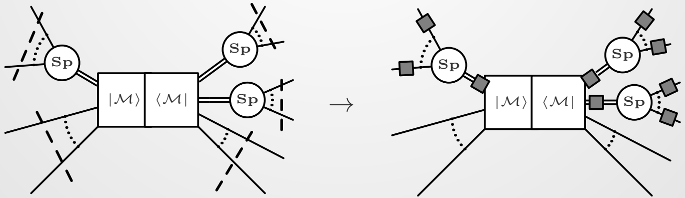
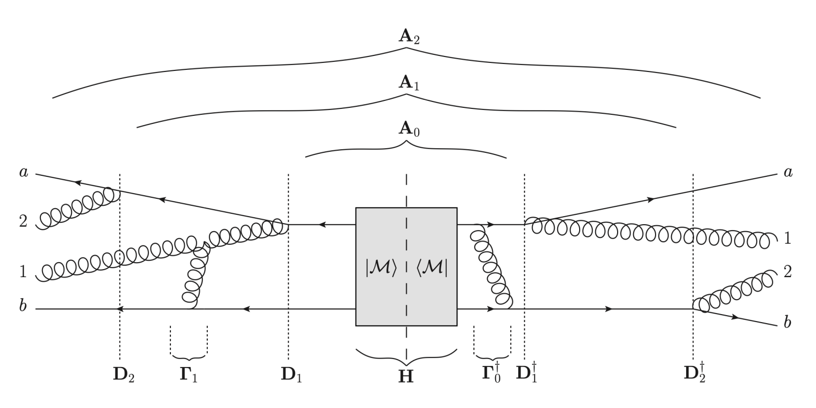




Splitting kernel iterations

Density operator language is useful for discussing emissions in iterative manner:



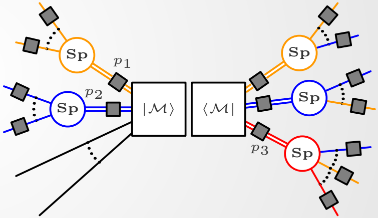
[Forshaw, Holguin, Plätzer–JHEP 09 (2020) 014]

Disentangling different collinear sectors

- Use partition of one to single out different collinear sectors

$$1 = \mathbb{P}_1^{(\mathcal{A})} + \mathbb{P}_2^{(\mathcal{A})} + \mathbb{P}_3^{(\mathcal{A})} + \ldots$$

- Decomposition in terms of set of possible collinear pairings
- **Avoid overlapping collinear singularities**
- Keep smooth interpolation over whole phase space



Partitioning for two emissions

- Example: triple collinear and coll-coll pairings
- Read
  $(i \parallel j \parallel k) : S_{ijk} = (q_i + q_j + q_k)^2 \to 0$

	$\frac{1}{S_{ij} S_{kl} S_{ijk} S_{jkl}}$
$i \parallel j \parallel k$	$S_{ij} S_{ijk}$
$i \parallel j \parallel l$	S_{ij}
$i \parallel k \parallel l$	S_{kl}
$j \parallel k \parallel l$	$S_{kl} S_{jkl}$
$(i \parallel j), (k \parallel l)$	$S_{ij} S_{kl}$
$(i \parallel k), (j \parallel l)$	$\times$
$(i \parallel l), (j \parallel k)$	$\times$

$\Rightarrow$ Construct partitioning factors of the form

$$\mathbb{P}^{(\mathcal{A})}_{(ijk)} = \frac{S_{kl} S_{jkl}}{S_{kl} S_{jkl} + S_{ij} S_{ijk} + S_{ijk} S_{jkl} + (S_{kl} + S_{ij}) S_{ijk} S_{jkl}}$$

- **Extracts the $(i \parallel j \parallel k)$- singular behaviour**
- **Non-singular in any collinear configuration**

Power counting

- Sudakov-like decomposition of momenta:

$$q_I^\mu = \sum_{k\in I} r_{ik} = z_I\, p_i^\mu + \frac{S_I + p_{\perp,I}^2}{2 z_I\; p_i \cdot n}\, n^\mu + k_{\perp,I}^\mu \;,$$

- Decompose fermion and gluon lines (factors of $\sqrt{z_I}$ absorbed in vertices for fermions):

 $= \not{p}_i,$

 $= \dfrac{S_I + p_{\perp,I}^2}{2 z_I^2\; p_i \cdot n}\not{n},$

 $= \dfrac{\not{k}_{\perp,I}}{z_I},$

 $= d^{\mu\nu}(p_i),$

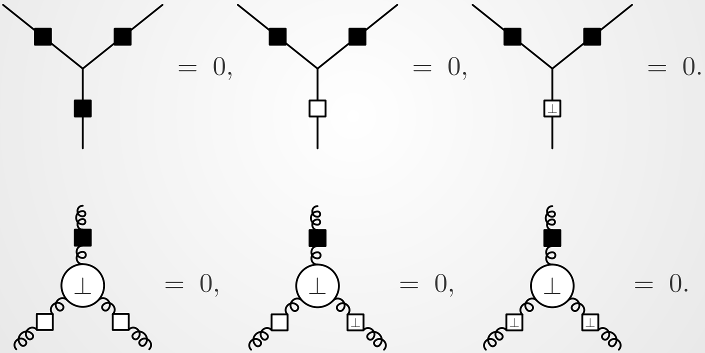
 $= \dfrac{S_I + p_{\perp,I}^2}{(z_I\, p_i \cdot n)^2}\, n^\mu n^\nu,$

 $= \dfrac{k_{\perp,I}^\mu n^\nu + n^\mu k_{\perp,I}^\nu}{z_I\; p_i \cdot n}.$

- Leads to power counting rules with potential connection to SCET

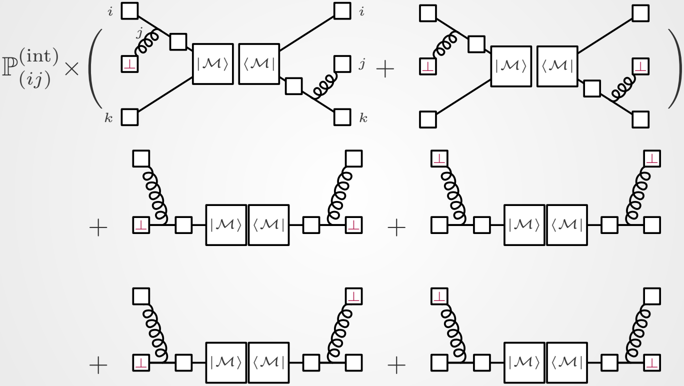
Vertex rules

- Can find vertex rules such as:



One emission example

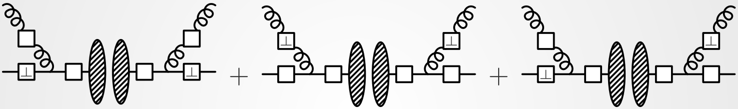
Full one emission (ij)-splitting kernel consists of



- **Exhibits factorisation to hard amplitude**
- **Smooth interpolation between soft and collinear limits**
- **Algorithmically generalizable for more emissions**

Check: One emission splitting function

- Reproduce **Splitting function** P_{qg} as a crosscheck
  (Note: interference diagrams are power-suppressed in lightcone gauge)



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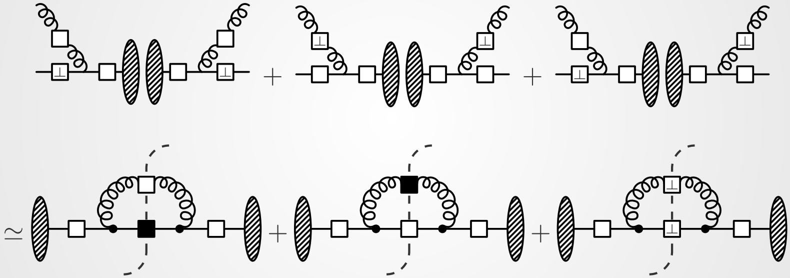

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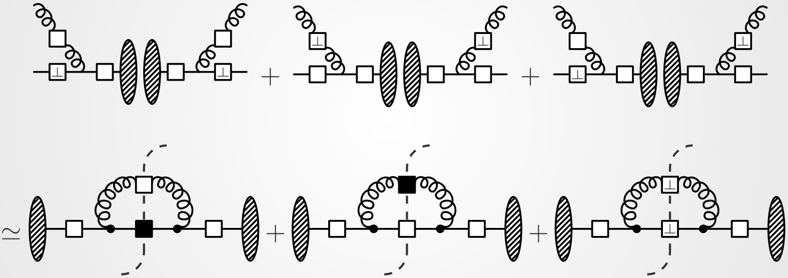
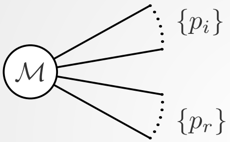


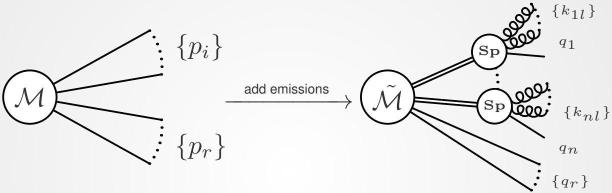
$$\rightarrow \frac{4\pi\alpha_s C_F}{\hat{\alpha}}\,\frac{1}{\lambda^2\, S_{ij}}\left\{(d-2)\alpha_i + 4\frac{(1-\alpha_i)^2}{\alpha_i} + 4(1-\alpha_i)\right\}\hat{\not{p}}_i + \mathcal{O}(\lambda^{-1}).$$

Momentum mapping
Adding emissions



- Start with **on-shell** (OS) momenta p_i (to be **emitters**) and p_r (to be **recoilers**) with overall momentum transfer $Q \equiv \sum_i p_i + \sum_r p_r$

Momentum mapping
Adding emissions



- Start with **on-shell** (OS) momenta p_i (to be **emitters**) and p_r (to be **recoilers**) with overall momentum transfer $Q \equiv \sum_i p_i + \sum_r p_r$
- Add emissions to the process with:
  1. Momentum conservation: $\sum_i q_i + \sum_{i,l} k_{il} + \sum_r q_r = Q$
  2. On-shellness of all partons
  3. Parametrization of soft & collinear behaviour for any # of emissions

Momentum mapping

$$q_r = \frac{\Lambda}{\alpha_L} p_r$$

$$k_{il} = \frac{\Lambda}{\alpha_L} \left[\alpha_{il}\, p_i + \tilde{\beta}_{il}\, n_i + \sqrt{\alpha_{il}\tilde{\beta}_{il}}\, n_{il}^{\perp} \right], \quad A_i \equiv \sum_l \alpha_{il}, \quad \tilde{\beta}_{il} = (1 - A_i)\beta_{il}$$

$$q_i = \frac{\Lambda}{\alpha_L} \left[(1 - A_i) p_i + \Big(y_i - \sum_l \tilde{\beta}_{il} \Big) n_i - \sum_l \sqrt{\alpha_{il}\tilde{\beta}_{il}}\, n_{il}^{\perp} \right]$$

- Decomposition w/ light-like momentum n_i and $n_{il}^{\perp} \cdot p_i = n_{il}^{\perp} \cdot n_i = 0$
- Need $\alpha_L^2 = (Q + N)^2 / Q^2$ for momentum conservation

$$Q = \sum_r q_r + \sum_i q_i + \sum_{i,l} k_{il} = \frac{\Lambda}{\alpha_L} \Big[\underbrace{\sum_r p_r + \sum_i}_{Q} (p_i + \underbrace{y_i n_i}_{N}) \Big]$$

- Lorentz transformation $\Lambda, \alpha_L \Rightarrow$ non-trivial **global recoil**

Momentum mapping II

- Using Λ and α_L, recoil effects are removed from considerations about factorization, due to Lorentz invariance and known mass dimension of the amplitudes:

$$|\mathcal{M}(q_1, ..., q_n)\rangle = \frac{1}{\alpha_L^{2n-4}} |\mathcal{M}(\hat{q}_1, ..., \hat{q}_n)\rangle \ .$$

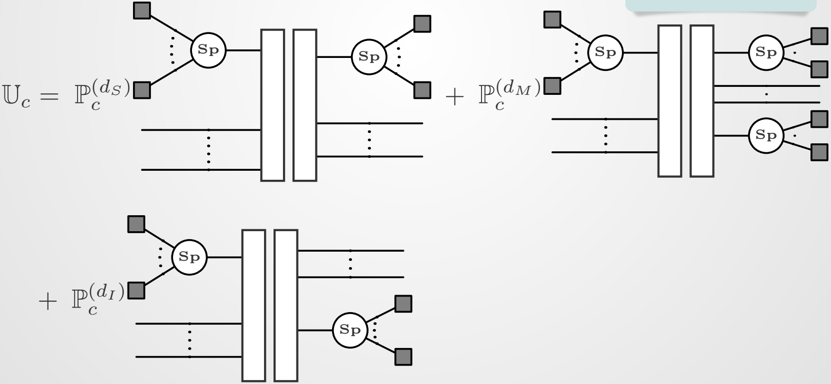
- Soft and collinear power counting possible via scaling of α_{il} and β_{il}, *i.e.* $(p_i, n_i, n_{il}^{\perp})$-components
- Can study leading singular behaviour for implementation in splitting kernels

General algorithm

- Collect leading collinear behaviour for some collinear configuration c in splitting kernels:
- Sum over configurations for full soft behaviour:

$$\mathbb{U}_c \equiv \sum_d \left[\mathbb{P}_c^{(d)} \mathcal{A}_d\right]$$

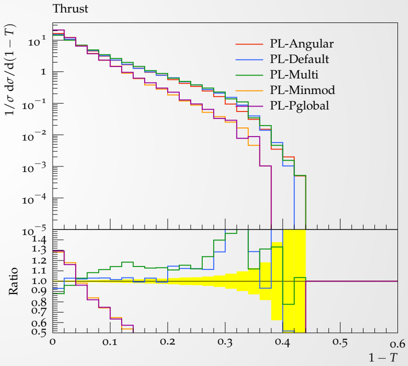
$$\mathbb{U} \equiv \sum_c \mathbb{U}_c$$



Preliminary implementations

Next to formal studies, explicit Herwig implementations are being carried out, *e.g.* momentum mappings:

[Holguin, Plätzer, Simpson, in progr.]



Conclusions

Goal: **universal algorithm for handling accuracy in multiple emissions** (for applications in parton showers and beyond)

- Momentum mapping for exposing collinear and soft factorization
- Global recoil via Lorentz transformation
- Partitioning algorithm to separate overlapping singularities
- Density-operator formalism to study iterative behaviour of emissions
- General Sudakov-like momentum decomposition for power counting rules $\Rightarrow$ simplification of amplitudes

Conclusions

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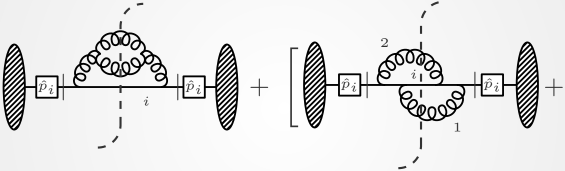
Thank you!

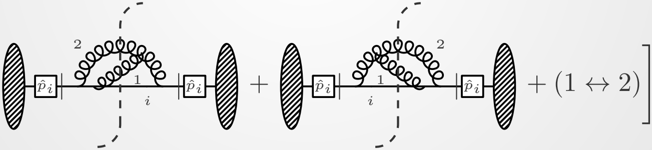
Backup slides

Check: Two Emissions

- **Reproduced from general two-emission kernel** which includes soft-limit too (here: in lightcone-gauge)

$$\frac{\mu^{2\varepsilon}}{\hat{\alpha}^2 S_{i12}^2} \Bigg\{$$

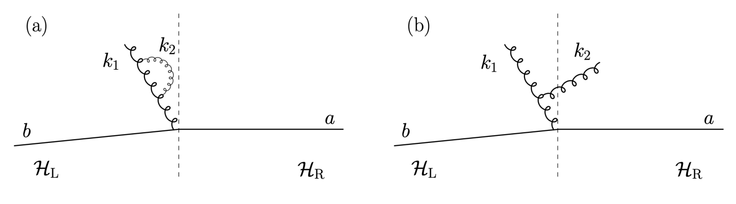




$$+ (1 \leftrightarrow 2) \Bigg] \Bigg\}_{C_A C_F}$$

$$= \left(\frac{8\pi\alpha_S}{\hat{\alpha} S_{i12}} \mu^{\varepsilon} \right)^2 C_A C_F \, \langle \hat{P}_{ggq}^{\text{(non-Ab)}} \rangle \hat{\not{p}}_i + \mathcal{O}\left(\beta_{il}^{-3/2} \right).$$

Global and non-global observables



[Dasgupta, Salam (2001)]

- Example: heavy and light jet mass (global) vs. hemisphere jet mass (non-global)
- Cancellations between large angle-soft and virtual contributions (from k_2) not guaranteed
  $\Rightarrow$ **NLL enhancement from leftover $\alpha_S^2 L^2$ terms**

Partitioning

Amplitudes carry different singular S-invariants

$$\mathcal{A}(S_1, S_2) = \frac{\mathcal{N}(S_1, S_2)}{S_1 S_2},$$

Decomposition using partitioning factors:

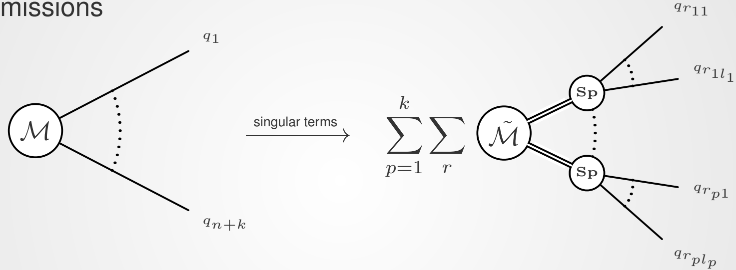
$$\mathbb{P}_{(1)}^{(\mathcal{A})} = \frac{S_2}{S_1 + S_2}, \quad \mathbb{P}_{(2)}^{(\mathcal{A})} = \frac{S_1}{S_1 + S_2},$$

we can decompose $\mathcal{A}$ into

$$\mathcal{A} = \left[\mathbb{P}_{(1)}^{(\mathcal{A})} + \mathbb{P}_{(2)}^{(\mathcal{A})}\right] \mathcal{A} = \frac{\mathcal{N}(S_1, S_2)}{S_1(S_1 + S_2)} + \frac{\mathcal{N}(S_1, S_2)}{S_2(S_1 + S_2)}.$$

General Algorithm
Amplitude

- Devise general setup for extracting singular behaviour for k emissions

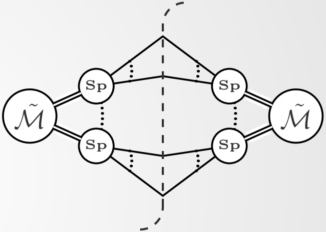


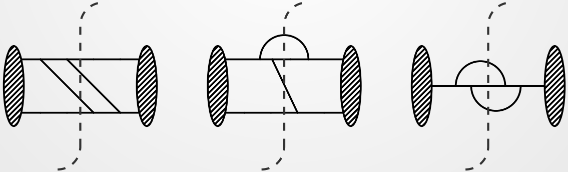
- Write amplitude in terms of splitting operators and factorized matrix element

$$|\mathcal{M}_{n+k}(q_1, \ldots, q_{n+k})\rangle = \sum_{p=1}^{k}\sum_{\{r\}} \mathbf{Sp}_{(r_{11}|\ldots|r_{1\ell_1})} \ldots \mathbf{Sp}_{(r_{p1}|\ldots|r_{p\ell_p})} \\ |\mathcal{M}_n(q_1, \ldots, q_{(r_{11}|\ldots|r_{1\ell_1})}, \ldots, q_{(r_{p1}|\ldots|r_{p\ell_p})}, \ldots, q_{n+k})\rangle$$

General Algorithm
Amplitude squared

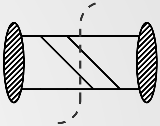
- Study **iterative behaviour** of emissions
- Single out **topologies with leading singular behaviour** (via # of unresolved partons)
- Examples for two emissions:





Two emissions

- For a given number of partons, find categorization of singular configs
- Read $(i \parallel j \parallel k) : S_{ijk} = (q_i + q_j + q_k)^2 \to 0$
- Triple collinear and double-soft contributions



	$\propto \frac{1}{S_{ij} S_{kl} S_{ijk} S_{jkl}}$
$i \parallel j \parallel k$	$S_{ij} S_{ijk}$
$i \parallel j \parallel l$	S_{ij}
$i \parallel k \parallel l$	S_{kl}
$j \parallel k \parallel l$	$S_{kl} S_{jkl}$
$(i \parallel j), (k \parallel l)$	$S_{ij} S_{kl}$
$(i \parallel k), (j \parallel l)$	$\times$
$(i \parallel l), (j \parallel k)$	$\times$

Construct partitioning factors from

$$1 = \frac{M^2(S_{kl}S_{jkl} + S_{ij}S_{ijk} + S_{ijk}S_{jkl}) + (S_{kl} + S_{ij})S_{ijk}S_{jkl}}{M^2(S_{kl}S_{jkl} + S_{ij}S_{ijk} + S_{ijk}S_{jkl}) + (S_{kl} + S_{ij})S_{ijk}S_{jkl}}$$

$\Rightarrow$ **non-singular in any configuration**

Phase space

- Can write down factorized phase space using momentum mapping

$$\mathrm{d}\phi\left(\{q_i\}_{\mathbf{S}},\{q_r\}_{\mathbf{R}},\{k_{il}\}_{\mathbf{E}_i}|Q\right)=\mathrm{d}\phi\left(\{p\}_{\mathbf{R}}|P_R\right)\alpha^{d-n_R(d-2)}(2\pi)^d\delta\big(P_S+P_R-Q\big)$$
$$\times\frac{\mathrm{d}m^2}{2\pi}\,[\mathrm{d}P_R]\,\frac{\omega(\vec{P}_R,\alpha m)}{\omega(\vec{Q}_R,m)}\Theta(Q_R^0)\prod_{i\in\mathbf{S}}[\mathrm{d}p_i]\frac{\omega(\vec{p}_i)}{\omega(\vec{q}_i)}\Theta(q_i^0)\left|\frac{\partial(\{\vec{q}\}_{\mathbf{S}},\vec{Q}_R)}{\partial(\{\vec{p}\}_{\mathbf{S}},\vec{P}_R)}\right|\prod_{l\in\mathbf{E}_i}[\mathrm{d}k_{il}]\Theta(k_{il}^0).$$

- Emission phase space:

$$\mathrm{d}^{d-1}k_{il}=|\mathcal{J}(\alpha_{il},\beta_{il},\Omega)|\;\mathrm{d}\alpha_{il}\,\mathrm{d}\beta_{il}\,\mathrm{d}^{d-3}\Omega,$$

- Computable in d dimensions:

$$[\mathrm{d}k_{il}]=\frac{1}{(2\pi)^{d-1}}\Theta(\alpha_{il})\Theta(\tilde{\beta}_{il})\frac{\alpha^{2-d}}{4}\frac{(2p_i\cdot n_i)^{\frac{d-2}{2}}}{(\alpha_{il}\tilde{\beta}_{il})^{\frac{d-4}{2}}}\mathrm{d}\alpha_{il}\,\mathrm{d}\tilde{\beta}_{il}\,\mathrm{d}\Omega^{d-3}.$$