# Investigation on the validity of the heavy quark expansion for charmed hadrons

A talk by

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## Outline I

# Introduction

The heavy quark expansion (HQE) provides an expansion in the inverse heavy quark mass

- has proven to be very successful for describing bottomed hadrons.
- applicability has often been questioned for charmed hadrons due to the charm quark is not so heavy.

**Aim**: We revisit the HQE for charm. In particular, we study pseudoscalar $D$-meson semileptonic (sl) and nonleptonic (nl) decay widths including available NLO QCD and subleading $1/m_c$ corrections.

# Introduction

The HQE predicts

- Decay width mainly due to the free heavy quark decay (**FHQD**) (blind to the spectator quark) $\Rightarrow \tau(M)/\tau(M') = 1$.
- Sensitivity to the spectator quark appears at $\mathcal{O}(1/m_Q^3)$ due to 4q operators, which introduce small differences between hadrons.

Look at the current experimental values for $B$-meson lifetime ratios[1]

$$\left.\frac{\tau(B_s)}{\tau(B_d)}\right|^{\text{exp}} = 0.998 \pm 0.006\,, \quad \left.\frac{\tau(B^+)}{\tau(B_d)}\right|^{\text{exp}} = 1.076 \pm 0.004\,.$$

However, for $D$-mesons, the lifetime ratios are[2]

$$\left.\frac{\tau(D^\pm)}{\tau(D^0)}\right|^{\text{exp}} = 2.563 \pm 0.017\,, \quad \left.\frac{\tau(D_s)}{\tau(D^0)}\right|^{\text{exp}} = 1.219 \pm 0.017\,,$$

which can not be undestood in the picture above (specially $D^\pm$).
**Can we trust the HQE for charm?**

[1] Values taken according to PDG and HFAG (see arXiv: 1405.3601).
[2] $D^0 \equiv D_u = c\bar{u}$, $D^+ \equiv D_d = c\bar{d}$, $D_s^+ \equiv D_s = c\bar{s}$.

# Introduction

For charm, the normal counting in the HQE is broken

- The contribution from $\mathcal{O}(1/m_Q^3)$ 4q operators can be comparable or even exceeds the contribution from the FHQD.
- $c$-quark inside $D$-mesons does not decay freely, but it is very sensitive to the spectator quark.

The reason for this to happen is[3]

- 4q operator coefficients are very large, they carry a $16\pi^2$ enhanced phase space factor.
- In $B$-mesons suppressed by $\Lambda^3/m_b^3 \sim 0.001$: highly suppressed.
- In $D$-mesons suppressed by $\Lambda^3/m_c^3 \sim 0.1$: not so highly suppressed.

**Despite of this, the HQE is able to predict correct widths, at least in some cases.** We focus on pseudoscalar $D$-mesons.

[3] The typical hadronization scale is $\Lambda =$ 500-600 MeV.

# HQE for the decay width

If $m_c \gg \Lambda$ then charm is a heavy quark and we can perform an OPE in $1/m_c$ (**HQE**) for the decay width (nl and sl)

$$\Gamma_{D\to X/X\bar{\ell}\nu_\ell} = \Gamma^0 |V_{\rm CKM}|^2 |V'_{\rm CKM}|^2 \Big[ C_0 - C_{\mu_\pi} \frac{\mu_\pi^2}{2m_c^2} + C_{\mu_G} \frac{\mu_G^2}{2m_c^2} - C_{\rho_D} \frac{\rho_D^3}{2m_c^3} - C_{\rho_{LS}} \frac{\rho_{LS}^3}{2m_c^3} + \sum_{i,q} C_{4F_i}^{(q)} \frac{\langle \mathcal{O}_{4F_i}^{(q)} \rangle}{4m_c^3} + \mathcal{O}\left(\frac{1}{m_c^4}\right) \Big] \quad (1)$$

- factorized short-distance effects, called Wilson coefficients $C_i$, which can be computed in perturbation theory.
- non perturbative effects encoded in the matrix elements of local operators over hadronic states.
  - $\mu_\pi^2$, $\mu_G^2$, $\rho_D^3$ and $\rho_{LS}^3$: matrix elements of two-quark operators.
  - $\langle \mathcal{O}_{4F_i}^{(q)} \rangle$: matrix elements of four-quark operators.

where $\Gamma^0 = G_F^2 m_c^5 / 192\pi^3$, $G_F$ is the Fermi constant and $V'_{\rm CKM} \to 1$ in $\Gamma^{sl}$.

# Basic assumptions

Evaluation of matrix elements (**ME**) is a primary source of uncertainties[4]. We assume[5]

$$\left.\begin{aligned} 2M_{D_q}\mu_\pi^2 &= -\langle D_q(p_D)|\bar{h}_v \pi_\perp^2 h_v|D_q(p_D)\rangle \\ \dots & \end{aligned}\right\} \begin{array}{c}\text{2q op.}\\ q\text{-indep.}\end{array}$$

$$\left.\begin{aligned} \langle \mathcal{O}_{4F_2}^{(q')}\rangle = \frac{1}{2M_{D_q}}\langle D_q|(\bar{h}_v q'_L)(\bar{q}'_L h_v)|D_q\rangle &= \frac{1}{8} f_{D_q}^2 M_{D_q}\delta_{qq'} \\ \dots & \end{aligned}\right\} \begin{array}{c}\text{4q op.}\\ \text{VSA}\end{array}$$

[4]Since they can not be computed in PT, only in lattice, using sum rules, modelling...

[5]$M_{D_q}$ is the $D_q$ meson mass carrying spectator quark $q$, $f_{D_q}$ is its decay constant, $h_v$ is the HQET field, $q_L$ is a left-handed relativistic quark field and $\pi_\mu$ stands for cov. derivative.

# Basic assumptions

Other assumptions are

- electron, muon, up and down quarks massless.
- $m_s/m_c$ corrections only included in the LO term.
- We add up to $\lambda^2$-Cabibbo suppressed channels[6] i.e charm decays with flavour structure
  - sl: $c \to q_3 \bar{\ell} \nu_\ell$ with $q_3 = s,\, d$.
  - nl: $c \to q_3 \bar{q}_1 q_2$ with $(q_3, \bar{q}_1, q_2) = (s, \bar{d}, u),\, (s, \bar{s}, u),\, (d, \bar{d}, u),\, (d, \bar{s}, u)$.
- Coefficients of operators up to dim 5 included at NLO (exept for $\mu_G^2$ in the nl width).
- Coefficients of dim 6 operators included at LO.
- Coefficients of dim 7 four-quark operators included at LO only for the Cabibbo-favoured channel: $(q_3, \bar{q}_1, q_2) = (s, \bar{d}, u)$.

[6] $\lambda = 0.2257^{+0.0009}_{-0.0010}$ is a Wolfenstein expansion parameter in the CKM matrix.

# Phenomenological analysis of $\Gamma^{sl,e}(D_q)$

The theory expression for the semileptonic width to electrons is

$$\begin{aligned}\Gamma^{sl,\,e}(D_q) &= \Gamma(c \to s e^+ \nu_e) + \Gamma(c \to d e^+ \nu_e) \\ &= \Gamma^0 \Bigg[ \left(1 - 8(1-\lambda^2)\frac{m_s^2}{m_c^2} + \frac{4}{3}\frac{\alpha_s(\mu_c)}{8\pi}(25 - 4\pi^2)\right)\left(1 - \frac{\mu_\pi^2}{2m_c^2}\right) \\ &\quad - \left(3 - \frac{\alpha_s(\mu_c)}{72\pi}\left(3(124 - 8\pi^2) - \frac{4}{3}(91 + 20\pi^2)\right)\right)\left(\frac{\mu_G^2}{2m_c^2} - \frac{\rho_{LS}^3}{2m_c^3}\right) \\ &\quad + \left(15 + 16\ln\left(\frac{\mu^2}{m_c^2}\right)\right)\frac{\rho_D^3}{2m_c^3}\Bigg] + \mathcal{O}(\lambda^4)\,, \end{aligned} \tag{2}$$

which is aimed to explain the experimental data

$$\begin{aligned}\Gamma^{sl,\,e}_{exp}(D^\pm) &= (1.02 \pm 0.02)\cdot 10^{-13}\ \text{GeV}\,, \\ \Gamma^{sl,\,e}_{exp}(D^0) &= (1.04 \pm 0.02)\cdot 10^{-13}\ \text{GeV}\,, \\ \Gamma^{sl,\,e}_{exp}(D_s) &= (0.85 \pm 0.05)\cdot 10^{-13}\ \text{GeV}\,. \end{aligned} \tag{3}$$

Note that $\Gamma^{sl,\,e}(D^\pm) \approx \Gamma^{sl,\,e}(D^0) > \Gamma^{sl,\,e}(D_s)$.

# Phenomenological analysis of $\Gamma^{sl,e}(D_q)$

Observations:

- Four-quark operators (up to dimension 7) combine in $\perp$ operators which vanish in VSA.
- The HQE predicts $\Gamma^{sl,\,e}(D_q)$ is independent of $q$.
  - Fine to explain $\Gamma^{sl,\,e}_{exp}(D^{\pm}) \approx \Gamma^{sl,\,e}_{exp}(D^0)$.
  - Problem! No theoretically simple way to explain $SU(3)$ violation i.e. $\Gamma^{sl,\,e}_{exp}(D_s) < \Gamma^{sl,\,e}_{exp}(D^{\pm}) \approx \Gamma^{sl,\,e}_{exp}(D^0)$. Possible explanations are
    - Violation of VSA.
    - $SU(3)$ violation of ME of 2q operators.
    - dim 8 4q operators.
- Four-quark operators suppressed (cancelled) by VSA
  $\Rightarrow$ semileptonic decay still dominated by FHQD, like in $B$-mesons.

# Phenomenological analysis of $\Gamma^{sl,e}(D_q)$

- We can fit experimental data for reasonable values of $m_c$ ($m_c \sim 1.65$ GeV for $D^\pm$, $D^0$ and $m_c \sim 1.6$ GeV for $D_s$).

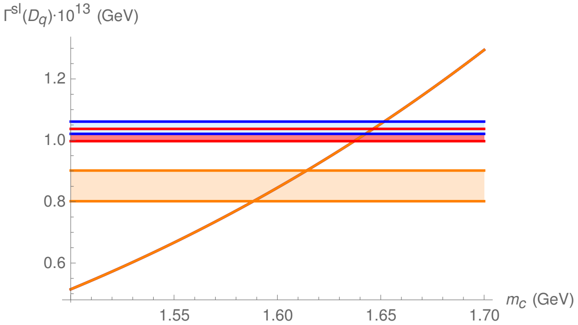



Figure: Red, blue and orange colors refer to $D^\pm$, $D^0$ and $D_s$, respectively. Bands stand for experimental data whereas curves for HQE theory predictions.

# Phenomenological analysis of $\Gamma^{nl}(D_q)$

The theory expression for the nonleptonic width is[7]

$$\Gamma^{nl}(D_q) = \Gamma(c\to s\bar{d}u) + \Gamma(c\to s\bar{s}u) + \Gamma(c\to d\bar{d}u) + \Gamma(c\to d\bar{s}u) \tag{4}$$

$$\begin{aligned} &= \Gamma^0\Bigg[\kappa(1-8(1-\lambda^2)\frac{m_s^2}{m_c^2})\left(1-\frac{\mu_\pi^2}{2m_c^2}\right)+\ldots \\ &+(C_1^2+C_2^2+6C_1C_2)16\pi^2\left((1-\lambda^2)\frac{f_D^2 M_{D^\pm}}{m_c^3}\delta_{qd}+\lambda^2\frac{f_{D_s}^2 M_{D_s}}{m_c^3}\delta_{qs}\right) \\ &-\frac{3}{2}(6C_1C_2+C_1^2+C_2^2)16\pi^2\frac{f_D^2 M_{D^\pm}}{m_c^3}\frac{2\bar{\Lambda}}{m_c}\delta_{qd}\Bigg]+\mathcal{O}(\lambda^4)\,, \end{aligned} \tag{5}$$

which is aimed to explain the experimental data

$$\Gamma^{nl}_{exp}(D^\pm) = (4.19\pm 0.06)\cdot 10^{-13}\ \text{GeV}\,, \tag{6}$$

$$\Gamma^{nl}_{exp}(D^0) = (13.91\pm 0.06)\cdot 10^{-13}\ \text{GeV}\,, \tag{7}$$

$$\Gamma^{nl}_{exp}(D_s) = (10.6\pm 0.1)\cdot 10^{-13}\ \text{GeV}\,. \tag{8}$$

Note that $\Gamma^{nl}(D^0) > \Gamma^{nl}(D_s) > \Gamma^{nl}(D^\pm)$.

[7] where $\kappa = 3C_1^2 + 2C_1C_2 + 3C_2^2$ and $\bar{\Lambda}$ comes from the HQE of the $D_q$-meson mass, $M_{D_q} = m_c + \bar{\Lambda} + \mathcal{O}(1/m_c)$.

# Phenomenological analysis of $\Gamma^{nl}(D_q)$

Observations:

- Contribution of dim 6 4q operators is negative.
- For $D_s$ and $D^0$ dim 6 4q operators are suppressed ($\lambda^2$-Cabbibo suppressed and zero in VSA, respectively) $\Rightarrow$ FHQD is dominating, like in $B$-mesons.
- dim 6 4q operators in $D^{\pm}$ are not suppressed and even exceed the FHQD $\Rightarrow$ leads to the $\Gamma^{nl}(D^{\pm}) < 0$ catastrophe.
- The problem is solved by adding dim 7 4q operators, whose contribution is positive, and comparable to the FHQD.
- The $D^{\pm}$ decay can not be understood as the heavy quark decaying "alone", but it decays "together with" the spectator quark.

# Phenomenological analysis of $\Gamma^{nl}(D_q)$

- Contribution from 4q operators correctly explain the hierarchy $\Gamma^{nl}(D^0) > \Gamma^{nl}(D_s) > \Gamma^{nl}(D^{\pm})$.
- Normal counting in the HQE spoiled in $D^{\pm}$, but we still get reasonable predictions.
- If the HQE is still reasonable, we can think of it as two series that converge separately, one involving 2q operators and the other involving 4q operators, with different exp. parameter[8]

$$\Gamma = \underbrace{\Gamma^0}_{\substack{\text{FHQD} \\ +\text{ dim 6} \\ +\text{ dim 7} \\ \text{4q op.}}} + \sum_{n=1}^{\infty} \Bigg[ \underbrace{a_n \left(\frac{\Lambda}{m_Q}\right)^n}_{\substack{\geq\text{3-body phase space in} \\ \text{the final state (2q op.)}}} + \underbrace{b_n 16\pi^2 \left(\frac{\Lambda}{m_Q}\right)^{n+4}}_{\substack{\text{2-body phase space in} \\ \text{the final state (4q op.)}}} \Bigg] \tag{9}$$

- The size of the 4q operators compared to the FHQD tells us how good is the HQE is the common sense, and if the reorganized version of the HQE is required.

[8] i.e. depending on the partonic phase space involved in the matching calculation.

# Phenomenological analysis of $\Gamma^{nl}(D_q)$

- We can fit experimental data for reasonable values of $m_c$ ($m_c \sim 1.63$ GeV for $D^\pm$, $m_c \sim 1.68$ GeV for $D^0$ and $m_c \sim 1.63$ GeV for $D_s$).
- Greater $SU(3)$ violation is required (inclusion of dim 7 4q operators for Cabbibo suppressed channels may improve it).
- For $D^\pm$ dim 8 4q operators could be as important as $\mu_G^2$.

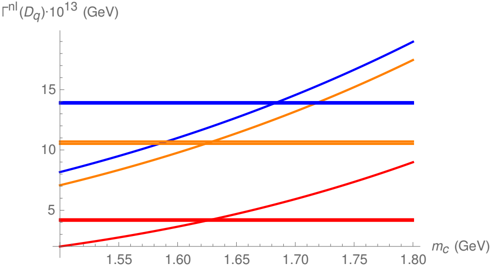



Figure: Red, blue and orange colors refer to $D^\pm$, $D^0$ and $D_s$, respectively. Bands stand for experimental data whereas curves for HQE theory predictions.

# Conclusions

- We observe that the HQE may work for pseudoscalar $D$-mesons since making very basic assumptions we can fit experimental $\Gamma^{sl}$ and $\Gamma^{nl}$ reasonably well.
- However, large sensitivity to HQE parameters (ME, $C_1$, $C_2$, $\bar{\Lambda}$, $m_c$ ...) together with large uncertainties in some of them make a quantitative study (and a definitive conclusion) very difficult.
- The HQE converges in the common sense (exept for $\Gamma^{nl}(D^{\pm})$) due to ME of 4q operators are accidentally suppressed.
- In $\Gamma^{nl}(D^{\pm})$, ME of 4q operators are dominant and exceed FHQD contribution.
- $\Gamma^{nl}(D^{\pm})$ can not be understood as the $c$-quark inside the $D$-meson decaying freely with small corrections due to spectator quark effects.
- Instead, its decay can only be understood as the $c$-quark and the spectator $d$-quark decaying "together".

# Conclusions

- We propose that the counting in the HQE must be redefined.
- However, it is not clear if we can rely on HQE in general, only in those cases where ME of 4q operators are suppressed (for instance, in $D^*$ they are not).
- There is no clear and simple source to explain the observed excess of $SU(3)$ violation in widths.
- More insight could be obtained after including Cabibbo suppressed dim 7 4q operators, dim 8 4q operators and NLO corrections to $\mu_G^2$ in $\Gamma^{nl}$.

# Questions

# Backup

## Operators and non-perturbative parameters

$$\begin{aligned}
\mathcal{O}_0 &= \bar{h}_v h_v \,, & (10)\\
\mathcal{O}_v &= \bar{h}_v (v\cdot\pi) h_v \,, & (11)\\
\mathcal{O}_\pi &= \bar{h}_v \pi_\perp^2 h_v \,, & (12)\\
\mathcal{O}_G &= \frac{1}{2}\bar{h}_v [\not{\pi}_\perp, \not{\pi}_\perp] h_v = \frac{1}{2}\bar{h}_v [\gamma^\mu, \gamma^\nu] \pi_{\perp\,\mu} \pi_{\perp\,\nu} h_v \,, & (13)\\
\mathcal{O}_D &= \bar{h}_v [\pi_{\perp\,\mu}, [\pi_\perp^\mu, v\cdot\pi]] h_v \,, & (14)\\
\mathcal{O}_{LS} &= \frac{1}{2}\bar{h}_v [\gamma^\mu, \gamma^\nu] \{\pi_{\perp\,\mu}, [\pi_{\perp\,\nu}, v\cdot\pi]\} h_v \,, & (15)
\end{aligned}$$

$$\begin{aligned}
\langle B(p_B)|\bar{b}\not{v} b|B(p_B)\rangle &= 2M_B \,, & (16)\\
-\langle B(p_B)|\mathcal{O}_\pi|B(p_B)\rangle &= 2M_B \mu_\pi^2 \,, & (17)\\
C_{\rm mag}(\mu)\langle B(p_B)|\mathcal{O}_G|B(p_B)\rangle &= 2M_B \mu_G^2 \,, & (18)\\
-c_D(\mu)\langle B(p_B)|\mathcal{O}_D|B(p_B)\rangle &= 4M_B \rho_D^3 \,, & (19)\\
-c_S(\mu)\langle B(p_B)|\mathcal{O}_{LS}|B(p_B)\rangle &= 4M_B \rho_{LS}^3 \,. & (20)
\end{aligned}$$

# Phenomenological analysis of $\Gamma^{nl}(D_q)$

The theory expression for the nonleptonic width is[9]

$$\begin{aligned}
\Gamma^{nl}(D_q) &= \Gamma(c \to s\bar{d}u) + \Gamma(c \to s\bar{s}u) + \Gamma(c \to d\bar{d}u) + \Gamma(c \to d\bar{s}u) \qquad (21) \\
&= \Gamma^0 \Bigg[ \kappa \Bigg( (1 - 8(1-\lambda^2)\frac{m_s^2}{m_c^2}) \left(1 - \frac{\mu_\pi^2}{2m_c^2}\right) - 3\left(\frac{\mu_G^2}{2m_c^2} - \frac{\rho_{LS}^3}{2m_c^3}\right) + \left(15 + 16\ln\left(\frac{\mu^2}{m_c^2}\right)\right)\frac{\rho_D^3}{2m_c^3} \Bigg) \\
&+ \left(1 - \frac{\mu_\pi^2}{2m_c^2}\right)\frac{\alpha_s}{\pi}\left(2(C_1^2 + C_2^2)\left(\frac{31}{4} - \pi^2\right) - \frac{4}{3}C_1C_2\left(\frac{7}{4} + \pi^2 + 6\ln\left(\frac{\mu^2}{m_c^2}\right)\right)\right) \\
&- 32C_1C_2\left(\frac{\mu_G^2}{2m_c^2} - \frac{\rho_{LS}^3}{2m_c^3} + \left(\frac{7}{6} + \ln\left(\frac{\mu^2}{m_c^2}\right)\right)\frac{\rho_D^3}{2m_c^3}\right) \\
&+ (C_1^2 + C_2^2 + 6C_1C_2)16\pi^2\left((1-\lambda^2)\frac{f_D^2 M_{D^\pm}}{m_c^3}\delta_{qd} + \lambda^2\frac{f_{D_s}^2 M_{D_s}}{m_c^3}\delta_{qs}\right) \\
&- \frac{3}{2}(6C_1C_2 + C_1^2 + C_2^2)16\pi^2\frac{f_D^2 M_{D^\pm}}{m_c^3}\frac{2\Lambda}{m_c}\delta_{qd}\Bigg] + \mathcal{O}(\lambda^4) \qquad (22)
\end{aligned}$$

which is aimed to explain the experimental data

$$\Gamma^{nl}_{exp}(D^\pm) = (4.19 \pm 0.06)\cdot 10^{-13}\ \text{GeV} \qquad (23)$$

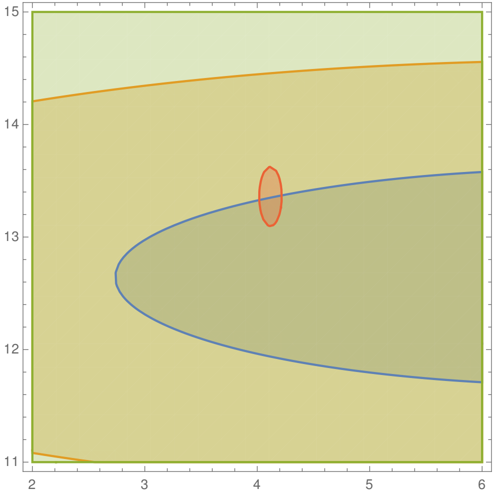
$$\Gamma^{nl}_{exp}(D^0) = (13.91 \pm 0.06)\cdot 10^{-13}\ \text{GeV} \qquad (24)$$

$$\Gamma^{nl}_{exp}(D_s) = (10.6 \pm 0.1)\cdot 10^{-13}\ \text{GeV} \qquad (25)$$

Note that $\Gamma^{nl}(D^0) > \Gamma^{nl}(D_s) > \Gamma^{nl}(D^\pm)$.

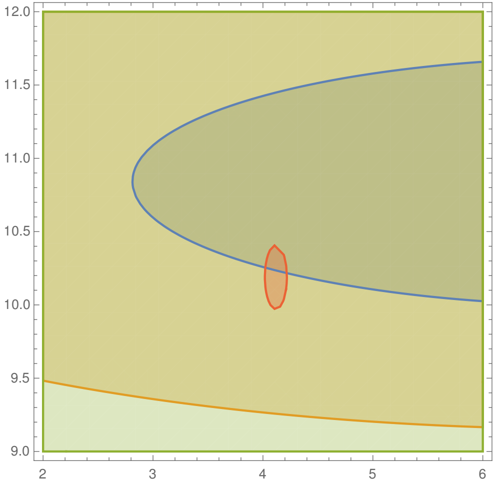
[9] where $\kappa = 3C_1^2 + 2C_1C_2 + 3C_2^2$ and $\Lambda$ comes from $M_D = m_c + \Lambda + \mathcal{O}(1/m_c)$.

## Ratios




Figure

# Ratios




Figure

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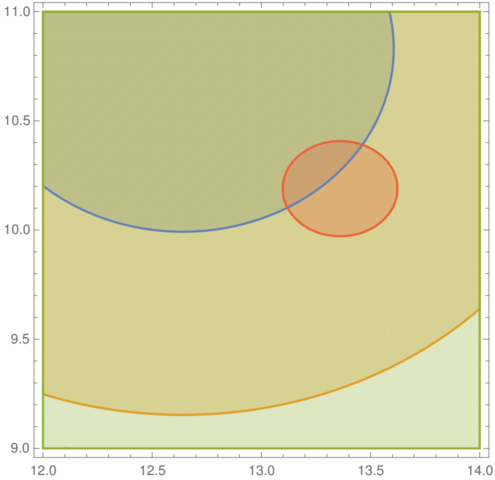


Figure