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On inverse moment of B_s meson distribution amplitude

Rusa Mandal
Universität Siegen, AvH Fellow

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with Alexander Khodjamirian & Thomas Mannel

UNIVERSITÄT SIEGEN

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Motivation

- B_s LCDA is required in QCDF, LCSR form factors e.g., $B_s \to K$

$$\langle 0|\bar{s}(tn)i\gamma_5 \not{n}[tn,0]h_v(0)|\bar{B}_s(v)\rangle = F_{B_s}(\mu)\int_0^\infty dk\, e^{-ikt}\phi_+^{B_s}(k,\mu)$$

- Can't extract from charged-current interaction like $B_s \not\to \ell\nu\gamma$

  $B_s \to \ell\ell\gamma$ ⇒ contaminated with nonlocal effects

- Key parameter: $\lambda_{B_{(s)}}^{-1}(\mu) = \int_0^\infty \frac{dk}{k}\,\phi_+^{(s)}(k,\mu)$

need to understand *SU(3)* violation effects— difference (if any) with λ_B

- QCD SR for $\lambda_B = 460 \pm 110\,\mathrm{MeV}$ [Braun, Ivanov, Korchemsky *'04*]

  — extend for λ_{B_s}

2-pt SR for decay constant

- correlation function: $i\int d^4x\, e^{i\omega v\cdot x}\langle 0|T\{\bar{s}(x)i\gamma_5 h_v(x)\bar{h}_v(0)i\gamma_5 q(0)\}|0\rangle$

$\bar{\Lambda} + m_{B_s} - m_B$

different duality threshold

$$\left[F_{B_s}(\mu)\right]^2 e^{-\bar{\Lambda}_s/\tau} = \frac{3}{\pi^2}\int_{m_s}^{\omega_{0s}} d\omega'\, e^{-\omega'/\tau}(\omega' + m_s)\sqrt{\omega'^2 - m_s^2}$$
$$+ \frac{3\alpha_s}{\pi^3}\int_0^{\omega_{0s}} d\omega'\, e^{-\omega'/\tau}\omega'^2\left(\frac{17}{3} + \frac{4\pi^2}{9} - 2\ln\frac{2\omega'}{\mu}\right)$$
$$- \langle\bar{s}s\rangle\left[1 + \frac{2\alpha_s}{\pi} - \frac{m_0^2}{16\tau^2}\right],$$

B_s decay constant

s-quark condensate

LO correction to perturbative spectral density ($\mathcal{O}(\alpha_s m_s)$ neglected)

SR for inverse moment

- Correlation function

$$\mathcal{P}_s(\omega,t) = i\int d^4x e^{-i\omega\, v\cdot x}\langle 0|T\left\{\bar{s}(tn)i\gamma_5\, \not{n}[tn,0]h_v(0)\bar{h}_v(x)i\gamma_5 s(x)\right\}|0\rangle$$

- Hadronic dispersion relation relates to

$$\mathcal{P}_s(\omega,t) = \frac{1}{\pi}\int_0^\infty d\omega' \frac{\mathrm{Im}\mathcal{P}_s(\omega,t)}{\omega'-\omega}$$

$$= \frac{\langle 0|\bar{s}(tn)i\gamma_5\, \not{n}[tn,0]h_v(0)|\bar{B}_s(v)\rangle\langle\bar{B}_s(v)|\bar{h}_v i\gamma_5 s|0\rangle}{2(\bar{\Lambda}_s-\omega)} + \ldots$$

$$= \frac{\left[F_{B_s}(\mu)\right]^2}{2(\bar{\Lambda}_s-\omega)}\int_0^\infty dk\, e^{-ikt}\phi_+^{B_s}(k,\mu) + \ldots ,$$

excited & continuam states

→ Apply Borel transformation to reduce sensitivity to quark-hadron duality assumption

SR for inverse moment

- OPE for correlation function for $|\omega| \gg \Lambda_{\rm QCD}$

$$\mathcal{P}_s^{\rm OPE}(\omega,t) = \mathcal{P}_s^{\rm (pert)}(\omega,t) + \mathcal{P}_s^{\rm (cond)}(\omega,t)$$

$$\frac{1}{\pi}\int_{m_s}^{\infty} d\omega' \, \frac{{\rm Im}\,\mathcal{P}_s^{\rm (pert)}(\omega,t)}{\omega'-\omega}$$

m_s → threshold for the quark loop

resulting into

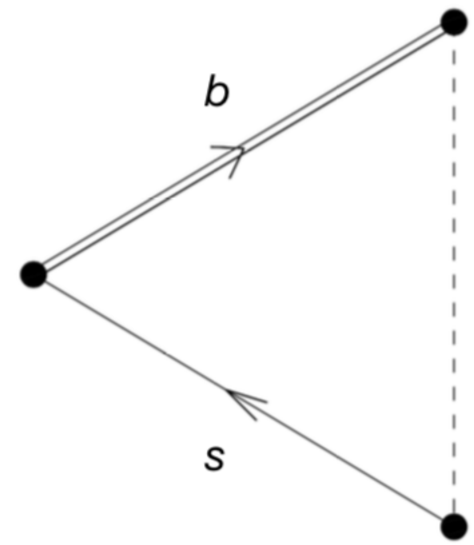
$$\frac{1}{2}\left[\lambda_{B_s}(\mu)\right]^{-1}\left[F_{B_s}(\mu)\right]^2 e^{-\bar{\Lambda}_s/\tau} = \frac{1}{\pi}\int_{m_s}^{\omega_{0s}} d\omega' \, e^{-\omega'/\tau} \int_0^{\infty} \frac{dk}{k}\,{\rm Im}\widetilde{\mathcal{P}}_s^{\rm (pert)}(\omega',k) + \int_0^{\infty} \frac{dk}{k}\,\widetilde{\mathcal{P}}_s^{\rm (cond)}(\tau,k)$$

Perturbative part

- LO contribution

$$\mathrm{Im}\widetilde{\mathcal{P}}_s^{(\mathrm{pert,LO})}(\omega',k) = \frac{3}{4\pi}\theta\big(k_{\max}(\omega')-k\big)\theta\big(k-k_{\min}(\omega')\big)(k+m_s)$$

$$k_{\max,\,\min}(\omega') = \omega' \pm \sqrt{\omega'^2 - m_s^2}\,.$$



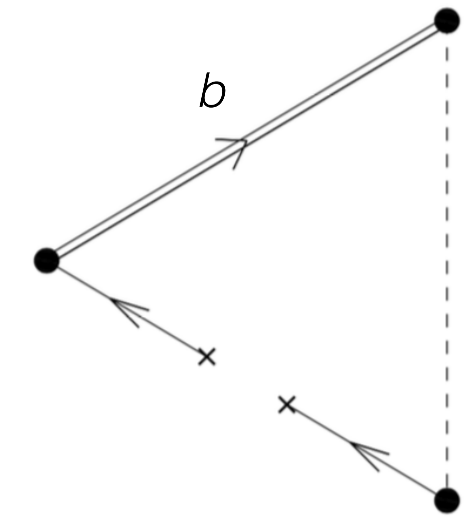
$\mathcal{O}(m_s)$ effect
not suppressed by m_b

$$[\lambda_{B_s}(\mu)]^{-1}[F_{B_s}(\mu)]^2 e^{-\bar\Lambda_s/\tau} = \frac{3}{2\pi^2}\int_{m_s}^{\omega_{0s}} d\omega'\, e^{-\omega'/\tau}\left[2\sqrt{\omega'^2-m_s^2} + m_s\log\frac{\omega'+\sqrt{\omega'^2-m_s^2}}{\omega'-\sqrt{\omega'^2-m_s^2}}\right]$$
$$+\frac{\alpha_s}{\pi^3}\int_0^{\omega_{0s}} d\omega'\, e^{-\omega'/\tau}\left[\int_0^{2\omega'} dk\,\widetilde{\rho}_<(\omega',k,\mu) + \int_{2\omega'}^{2\omega_{0s}} dk\,\widetilde{\rho}_>(\omega',k,\mu) + \int_{2\omega_{0s}}^{\infty} dk\,\widetilde{\rho}_>(\omega',k,\mu)\right] + C_s(\tau)\,,$$

NLO part from [Braun, Ivanov, Korchemsky '04]
($\mathcal{O}(\alpha_s m_s)$ neglected)

Non-perturbative Part

- Quark condensate contributions $\sim \int_0^\infty \frac{dk}{k} \langle \bar{s}s \rangle \delta(k)$



→ divergent

Stronger singularities for higher dimension condensates

Remedy → use nonlocal condensate ansatz [Mikhailov, Radyushkin '92]

$$\langle 0|\bar{s}(x)[x,0]s(0)|0\rangle = \langle \bar{s}s\rangle \int_0^\infty d\nu \, e^{\nu x^2/4} \mathcal{F}(\nu)$$

1 $\delta(\nu - m_0^2/4)\,,$

2 $\frac{\lambda^{p-2}}{\Gamma(p-2)} \nu^{1-p} e^{-\lambda/\nu}\,, \quad p = 3 + \frac{4\lambda}{m_0^2}$

Model parameters are fixed using two moments

$$\int_0^\infty d\nu \, \mathcal{F}(\nu) = 1\,,$$

$$\int_0^\infty d\nu \, \nu \mathcal{F}(\nu) = \frac{m_0^2}{4}$$

Results

- Ratios of decay constants

$$F_{B_s}(\mu = 1\,\mathrm{GeV})/F_B(\mu = 1\,\mathrm{GeV}) = 1.16 \pm 0.08$$

$\sim 15\,\%$ effect in agreement with lattice & full QCD predictions

[FLAG '20] [Gelhausen *et. al* '15]

- Differentiating the decay constant SR w.r.t. Borel parameter & dividing with the original one eliminates one input

⇒ $\omega_{0(s)}$ duality threshold is fixed for $\bar{\Lambda} \approx 0.55\,\mathrm{GeV}$ [Gambino *et. al* '17]

$$\omega_0 = 1.00 \pm 0.12\,\mathrm{GeV}, \quad \omega_{0s} = 1.10 \pm 0.13\,\mathrm{GeV}$$

Results

	$\left[\lambda_{B_s}(\mu = 1\ \text{GeV})\right]^{-1}$	$\left[\lambda_{B}(\mu = 1\ \text{GeV})\right]^{-1}$
Perturbative	1.34 ± 0.15	1.17 ± 0.05
Model 1	0.66 ± 0.25	1.00 ± 0.24
Model 2	1.23 ± 0.51	1.88 ± 0.56
Total	2.00 ± 0.29	2.17 ± 0.24
	2.57 ± 0.53	3.05 ± 0.56

$$\lambda_{B_s} = 438 \pm 150\ \text{MeV}\,, \quad \lambda_B = 383 \pm 153\ \text{MeV}$$

Large uncertainty due to model dependence

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