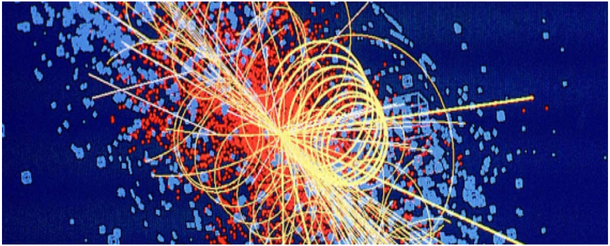
Zero-jettiness soft and beam functions at NNLO to higher orders in epsilon

Daniel Baranowski | 09.06.20

TTP

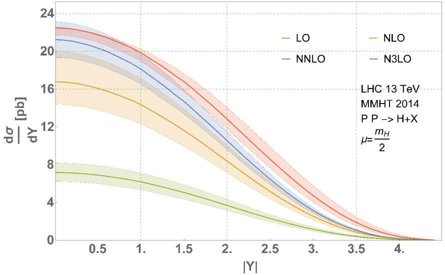


Outline

Motivation

- *Fully differential* colour singlet production $pp \to V + \mathrm{jets}$ currently known at N^{2}LO
- Few processes already known at N^{3}LO: Higgs rapidity distribution, fully inclusive cross section for gg$\to$ H , pp$\to \gamma^*$ and bb$\to$ H [Dulat, Mistlberger, Peloni'18]

  [Mistlberger'18][Duhr,Dulat,Mistlberger'20][Duhr,Dulat,Hirschi,Mistlberger'20]



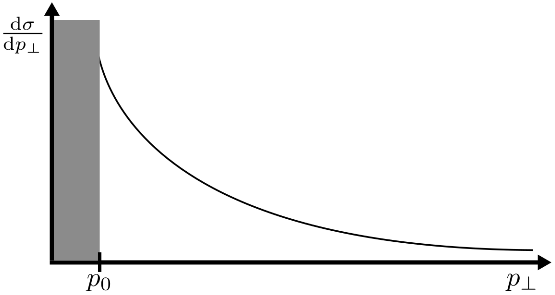
[Dulat, Mistlberger, Peloni'18]

- High experimental precision requires knowledge of N^{3}LO corrections to interesting processes: Higgs boson production, Drell-Yan, ...

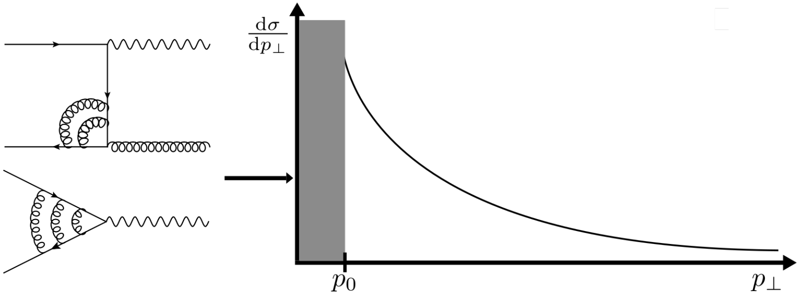
Motivation

- Problem:
  - Soft, collinear and virtual singularities have to cancel each other $\rightarrow$ spread across phase-spaces with different jet multiplicity
  - Systematic approach to compensate singularities(subtraction schemes) only known at N^{2}LO $\rightarrow$ not at N^{3}LO
- Solution:
  - Concentrate the most offending singularities in one phase-space region through smart choice of an observable
- Example:
  - Vector boson production $pp \rightarrow V + \mathrm{jets}$ with $p_\perp$ as observable

Slicing



Slicing



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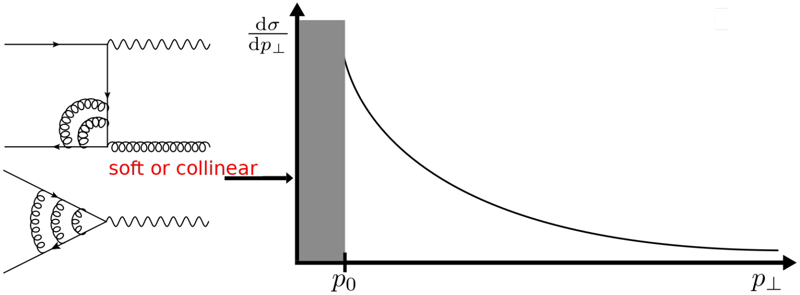
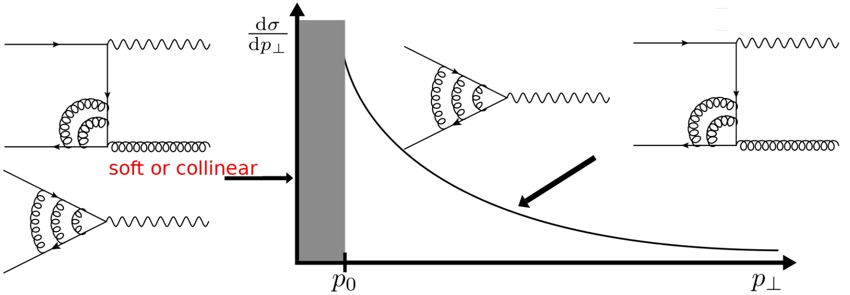
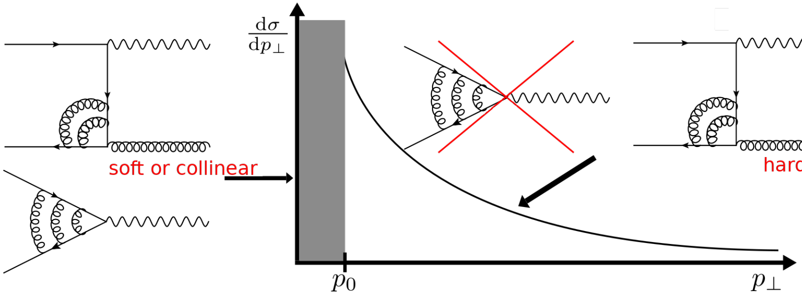


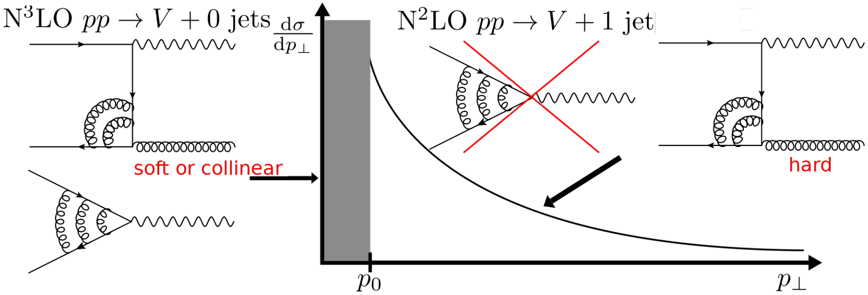
Slicing



Slicing

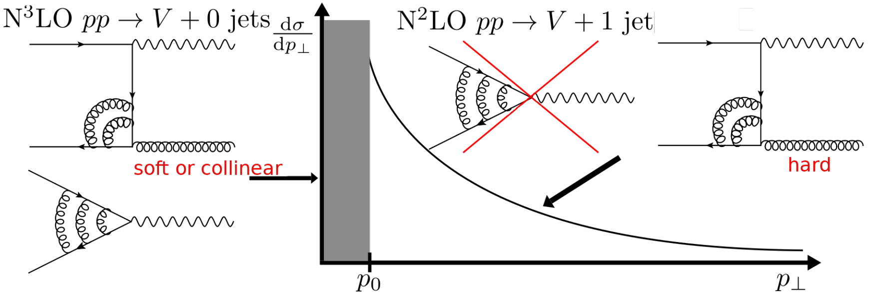


Slicing



$N^3LO\ pp \to V + 0$ jets
$\frac{d\sigma}{dp_\perp}$
$N^2LO\ pp \to V + 1$ jet
soft or collinear
hard
p_0
$p_\perp$

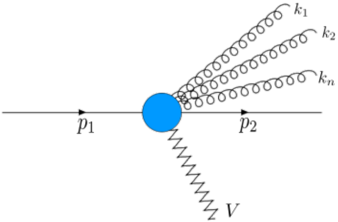
Slicing



- $p_\perp > p_0$: $\mathrm{N^2LO} \to$ known
- $p_\perp < p_0$: $\mathrm{N^3LO} \to$ unknown, however soft and collinear limits lead to simplification $\Rightarrow$ *Slicing* [Giele, Glover '92]

Zero-jettiness

- zero-jettiness τ instead $p_\perp$ as slicing variable



$$\tau = \sum_j \min_{i \in 1,2} \left[\frac{2p_i \cdot k_j}{Q_i} \right]$$

- simplification through the factorization theorem derived in SCET

$$\lim_{\tau \to 0} \mathrm{d}\sigma_{pp\to V}(\tau) = B_\tau \otimes B_\tau \otimes S_\tau \otimes H_\tau \otimes \mathrm{d}\sigma_V^{LO}$$

[Stewart, Tackmann, Waalewijn ’10]

- beam function B, describes singularities due to collinear emission of incoming particles
- soft function S, describes singularities due to soft emission
- hard function H, describes corrections to the Born-like process

Zero-jettiness

- B for $p_\perp$ known through N^{3}LO QCD

  [Luo, Yang, H. Zhu, Y. Zhu '19]

- B and S for zero-jettiness currently known through NNLO QCD

  [Gaunt, Stahlhofen, Tackmann'14] [Gaunt, Stahlhofen, Tackmann'14]
  [Boughezal, Petriello, Schubert, Xing '17][Monni, Gehrmann, Luisoni '11]
  [Kelley, Schwartz, Schabinger, Zhu '11]

- Since Yesterday: B for zero-jettiness calculated through N^{3}LO QCD

  [Ebert, Mistlberger, Vita '20]

- All quantities needed to N^{3}LO $\rightarrow$ renormalization $\rightarrow$ NNLO through higher orders in ϵ
- In case of S develop tools for N^{3}LO calculation, test them on NNLO

Calculation of B_{ij}

- B_{ij} as an integral over the collinear QCD splitting function $P_{j\to i^*\{m\}}$

$$B_{ij} \sim \sum_{\{m\}} \int \mathrm{dPS}^{(m)} P_{j\to i^*\{m\}}$$

[Ritzmann,Waalewijn'14]

- Obtain splitting function through collinear projection operator $\mathcal{P}$

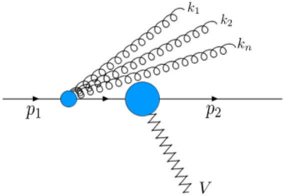
[Catani, Grazzini '00]

$$P_{j\to i^*\{m\}} \sim \mathcal{P}|M_{j\to i^*\{m\}}|^2$$

- Since QCD is charge-conjugation invariant we only need to consider the sets $(i,j) \in \{(q_i,q_j),(q_i,g),(q_i,\bar{q}_j),(g,g),(g,q_j)\}$

Phase space measure

- Consider collinear emissions off p_1



$$\tau = \sum_m \min_{i\in 1,2}\left[\frac{2p_i\cdot k_m}{Q_i}\right] \Rightarrow \sum_m \frac{2p_1\cdot k_m}{Q_1}$$

- m particle phase space now defined as

$$\int \mathrm{dPS}^{(m)} = \left(\prod_m \int \frac{\mathrm{d}^d k_m}{(2\pi)^{d-1}}\delta^+\left(k_m^2\right)\right)$$
$$\times\delta\left(2\sum_m k_m\cdot p - \frac{t}{z}\right)\delta\left(\frac{2\sum_m k_m\cdot\bar{p}}{s} - (1-z)\right)$$

Reverse unitarity, IBPs

- Use reverse unitarity to rewrite phase-space delta functions

[Anastasiou,Melnikov'02]

$$\delta\left(p^2-m^2\right)=\frac{\mathrm{i}}{2\pi}\left[\frac{1}{p^2-m^2+\mathrm{i}\epsilon}-\frac{1}{p^2-m^2-\mathrm{i}\epsilon}\right],$$

- Use Integration-by-parts machinery to reduce beam function to sum of master integrals (MIs) [Chetyrkin, Tkachov '81]
- Compute master integrals

Real-real master integral

Karlsruher Institut für Technologie

- Example of a master integral:

$$I = \int \frac{\mathrm{d}^d k_1}{(2\pi)^{d-1}} \int \frac{\mathrm{d}^d k_2}{(2\pi)^{d-1}} \, \delta^+\left(k_1^2\right) \, \delta^+\left(k_2^2\right) \delta\left(2k_{12}\cdot p - \tfrac{t}{z}\right) \\ \times \frac{\delta\left(2k_{12}\cdot \bar{p} - (1-z)\right)}{(p-k_1)^2 \, k_{12}^2 \, \bar{p}\cdot k_2}.$$

- Its solution reads

$$I = -\frac{(\Omega_{d-2})^2}{4(2\pi)^{2d-2}} t^{-1-2\epsilon} \left(\frac{1-z}{z}\right)^{-1-2\epsilon} \frac{\Gamma(1-\epsilon)^2\Gamma(-\epsilon)^2}{\Gamma(1-2\epsilon)^2} \\ \times \, {}_3\mathrm{F}_2\left(1,1,-\epsilon;1-2\epsilon,1-\epsilon,1\right)$$

Beam functions at NNLO

- We find that all five beam functions B_{q_i,q_j}, $B_{q_i,g}$, $B_{q_i,\bar{q}_j}$, $B_{g,g}$, B_{g,q_j} can be expressed through 12 master integrals
- Integrals easy enough to be straightforwardly integrated
- Many integrals can be computed in closed form in ϵ
- General structure of the result

$$I_{gg}^{(2)} = \sum_{k=0}^{5} \frac{1}{\mu^2} L_k \left(\frac{t}{\mu^2} \right) F_+^{(k)}(z) + \delta(t) F_\delta(z),$$

$$F_\delta(z) = C_{-1}\delta(1-z) + \sum_{k=0}^{5} C_k L_k(1-z) + F_{\delta,h}(z),$$

where we define the plus distribution

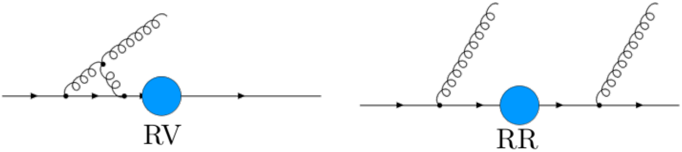
$$L_n(z) = \left[\frac{\ln^n(z)}{z} \right]_+$$

Calculation of S

- Soft function S as an integral over soft emission functions (eikonals) $\xi_{\{m\}}$

$$S \sim \sum_{\{m\}} \int \mathrm{dPS}^{(m)} \xi_{\{m\}}$$

- RV contribution simple, focus on RR



Phase space measure

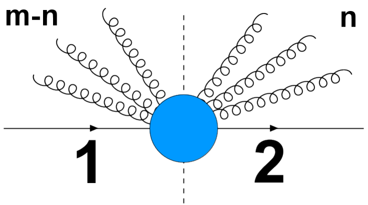
- No simplification of τ due to soft limit

$$\tau = \sum_m \min_{i\in 1,2}\left[\frac{2p_i\cdot k_m}{Q_i}\right]$$

- The m particle phase space reads

$$\int \mathrm{dPS}^{(m)} = \prod_n^m \frac{\mathrm{d}^d k_n}{(2\pi)^{d-1}}\,\delta^+\left(k_n^2\right) M_m$$

- m particle measurement function M_m splits integrand into sectors according to the emission of partons into different hemispheres



Phase space measure at NNLO

$$M_2 = \delta\left(\tau - 2p\cdot k_1 - 2p\cdot k_2\right)\theta\left(2\bar{p}\cdot k_1 - 2p\cdot k_1\right)\theta\left(2\bar{p}\cdot k_2 - 2p\cdot k_2\right) + \left(p^\mu \leftrightarrow \bar{p}^\mu\right)$$
$$+\ \delta\left(\tau - 2\bar{p}\cdot k_1 - 2p\cdot k_2\right)\theta\left(2p\cdot k_1 - 2\bar{p}\cdot k_1\right)\theta\left(2\bar{p}\cdot k_2 - 2p\cdot k_2\right) + \left(p^\mu \leftrightarrow \bar{p}^\mu\right)$$

- Integrand symmetric under exchange of p and $\bar{p} \rightarrow$ only need to consider two configurations A and B
- Partons emitted into the same hemisphere

$$M_A(k_1,k_2) = \delta\left(\tau - 2p\cdot k_1 - 2p\cdot k_2\right)\theta\left(2\bar{p}\cdot k_1 - 2p\cdot k_1\right)\theta\left(2\bar{p}\cdot k_2 - 2p\cdot k_2\right)$$

- Partons emitted into different hemispheres

$$M_B(k_1,k_2) = \delta\left(\tau - 2\bar{p}\cdot k_1 - 2p\cdot k_2\right)\theta\left(2p\cdot k_1 - 2\bar{p}\cdot k_1\right)\theta\left(2\bar{p}\cdot k_2 - 2p\cdot k_2\right)$$

Treatment of θ functions

- We would like to use IBP through reversed unitarity. But what do with θ functions?
- Use the identity

$$\theta(b-a) = \int_0^1 \mathrm{d}z\, \delta\left(z\, b - a\right)\; b$$

which holds for $a, b \in [0, \infty)$.

- $M_{A,B}$ now functions of auxillary parameters z_1 and z_2

$$M_A = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2\, \delta\left(\tau - 2p\cdot k_1 - 2p\cdot k_2\right) \delta\left(2z_1 \bar{p}\cdot k_1 - 2p\cdot k_1\right) 2\bar{p}\cdot k_1 \\ \times \delta\left(2z_2\bar{p}\cdot k_2 - 2p\cdot k_2\right) 2\bar{p}\cdot k_2$$

- Commute z_i and k_i integration, use reverse unitarity and IBPs to obtain master integrals

Master integrals

- Only 9 MIs that can be solved in closed form in ϵ
- For example

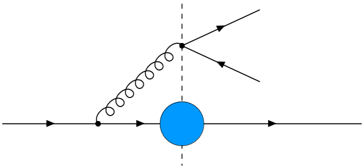
$$I = \left(\prod_n^2 \int \frac{\mathrm{d}^d k_n}{(2\pi)^{d-1}} \, \delta^+ \left(k_n^2\right) \right) \delta\left(\tau - 2p\cdot k_1 - 2p\cdot k_2\right) \; \delta(2p\cdot k_1 - z_1 2\bar{p}\cdot k_1)$$

$$\times \, \delta(2p\cdot k_2 - z_2 2\bar{p}\cdot k_2) \, \left(p\cdot k_1 - \frac{\tau z_1}{2(z_1 - z_2)}\right)^{-1} (k_1\cdot k_2)^{-1}$$

$$= -\frac{\tau^{-2-4\epsilon}}{(z_1 z_2)^{1-\epsilon}} \frac{\Gamma^2(-2\epsilon)}{\Gamma(-4\epsilon)} z_2 \frac{z_1 - z_2}{z_1} \; {}_2F_1\left(1, 1+\epsilon, 1-\epsilon, \frac{z_2}{z_1}\right)$$

$$\times \; {}_2F_1\left(1, -2\epsilon, -4\epsilon, \frac{z_1 - z_2}{z_1}\right).$$

Integration over auxillary parameters

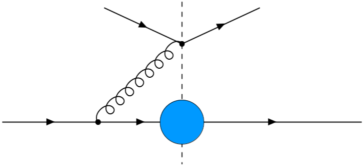


- Integrate over z_1 and z_2

$$S^{(2)}_{q\bar{q},A} = 2\int_0^1 \mathrm{d}z_1 \int_0^{z_1} \mathrm{d}z_2 \Bigg[\frac{32\epsilon(2\epsilon-1)z_1 z_2}{\tau^2(z_1-z_2)^2}\, I^{q\bar{q},3}_{00} + \frac{8\epsilon(2\epsilon-1)z_1 z_2(z_1+z_2)}{\tau(z_1-z_2)^3}\, I^{q\bar{q},3}_{10} + \frac{8(z_1+z_2)\left(16\epsilon^3 z_1 z_2 - \epsilon^2(z_1+z_2)^2 + \epsilon\left(z_1^2 - 6z_1 z_2 + z_2^2\right) + z_1 z_2\right)}{(4\epsilon-1)(z_1-z_2)^4}\, I^{q\bar{q},3}_{01} + \frac{8\tau z_1 z_2\left(\epsilon^2(z_1+z_2)^2 - z_1 z_2\right)}{(z_1-z_2)^5}\, I^{q\bar{q},3}_{11} \Bigg]$$

- Rescale $z_2 = t z_1 \rightarrow z_1$ completely factors out
- Remaining integral singular in $t = 1 \rightarrow$ endpoint subtraction

Integration over auxillary parameters



$$S^{(2)}_{q\bar{q},B} = 2\int_0^1 \mathrm{d}z_1 \int_0^{z_1} \mathrm{d}z_2 \Bigg[\frac{32\epsilon(4\epsilon-1)z_1 z_2}{\tau^2(z_1 z_2 - 1)^2}\; I^{q\bar{q},2}_{00}$$
$$- \frac{8\tau z_2\left(\epsilon^2(z_2+1)\left(z_1^2 z_2^2 - 1\right) + \epsilon(z_1-1)z_2(z_1 z_2+1) - z_1(z_2-1)z_2\right)}{(z_2-1)^3(z_1 z_2-1)^3}\; I^{q\bar{q},4}_{11}$$
$$+ \Bigg(\frac{16 z_1 z_2(z_1(-z_2)+z_1+z_2-1)}{(z_1-1)^2(z_2-1)^2(z_1 z_2-1)^2}$$
$$+ \epsilon\frac{8\left(z_1^3(-(z_2-3))z_2^2 + z_1^2 z_2\left(3z_2^2 - 11 z_2 + 6\right) + z_1\left(6z_2^2 - 11z_2 + 3\right) + 3z_2 - 1\right)}{(z_1-1)^2(z_2-1)^2(z_1 z_2-1)^2}$$
$$+ \epsilon^2\frac{32\left(z_1^3 z_2^2 + z_1^2 z_2\left(z_2^2 - 4z_2 + 2\right) + z_1\left(2z_2^2 - 4z_2 + 1\right) + z_2\right)}{(z_1-1)^2(z_2-1)^2(z_1 z_2-1)^2}\Bigg)\; I^{q\bar{q},2}_{01} + \ldots \Bigg]$$

- Emission into different hemispheres $\rightarrow$ "finite" $\rightarrow$ expand in ϵ

Results for the soft function

- The final result has the following form

$$S^{(2)} = \tau^{-1-4\epsilon}\left(C_a^2\, S_A^{(2)} + C_a T_F n_f\, S_B^{(2)} + C_A C_a\, S_C^{(2)}\right)$$

- For example

$$S_A^{(2)} = -\frac{8}{\epsilon^3} + \frac{16\pi^2}{3\epsilon} + 128\zeta(3) + \epsilon\frac{16\pi^4}{5} + \epsilon^2 \left(1536\zeta(5) - \frac{256\pi^2\zeta(3)}{3}\right) + \epsilon^3\left(\frac{2528\pi^6}{945} - 1024\zeta(3)^2\right)$$

Conclusion and Outlook

- Simplification of already existing NNLO calculations due to advanced tools
- Beam functions can be calculated as phase space integrals over matrix elements squared in the collinear limit
- Soft function can be calculated as phase space integrals over eikonal functions
- The identity $\theta(b-a) = \int_0^1 \mathrm{d}z \, \delta\left(z\, b - a\right) \, b$ allows for the use of multi-loop technology
- At NNLO all five partonic beam functions B_{q_i,q_j}, $B_{q_i,g}$, $B_{q_i,\bar{q}_j}$, $B_{g,g}$, B_{g,q_j} in terms of 12 MIs, soft function in terms of 9 MIs
- All quantities successfully calculated to order ϵ^2, and $\mathcal{O}(\epsilon^0)$ results checked against literature [DB'20]
- Outlook: N3LO beam function calculations [Ebert et al.'20][Behring et al.'19]
  N3LO soft function calculation ongoing...