Multi-emission Kernels for Parton Branching Algorithms

Maximilian Löschner[a]

Karlsruher Institut für Technologie

Institute for Theoretical Physics
Erwin Schrödinger Institut (research stay in Vienna)

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[a]in collaboration with Simon Plätzer and Emma Simpson Dore

Motivation

Project overview

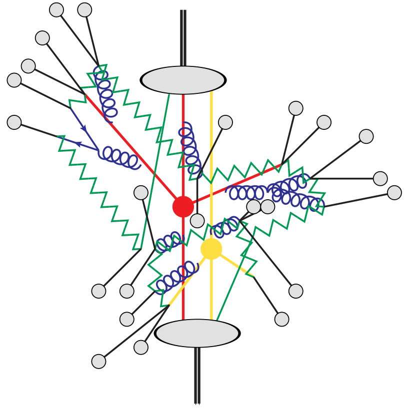
One emission

Momentum mapping

General algorithm

Two emissions

Conclusions



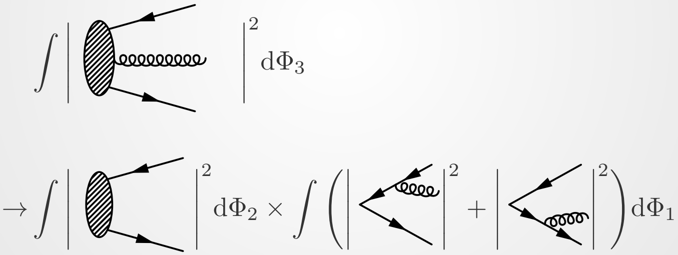
$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{hard}}(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Parton Shower

- Soft and collinear regions are of special interest ($q_{i/j}^2 = 0$):

$$S_{ij} \equiv (q_i + q_j)^2 = 2\, q_i \cdot q_j = 2 q_i^0 q_j^0 \left[1 - \cos\vartheta_{ij}\right]$$

- Amplitudes become singular/enhanced when $S_{ij} \to 0$ (origin of large logarithms)



- Probabilistic description of parton emissions
- Can iterate this process for multiple emissions

Motivation

General Idea: investigate novel routes in understanding soft and collinear dynamics in multi-parton final states.

- Going beyond iterated $1 \to 2$ splittings in parton showers
- Combine with global recoil scheme
- Address non-global observables

} higher logarithmic accuracy $\Leftrightarrow$ Systematic expansion to handle uncertainties

- Include colour and spin correlations
- Refine ad hoc models of MC-programs, *e.g.* azimuthal correlations
- Define language for connecting FO to parton showers

Project overview

Building blocks:

1. Amplitude evolution, link to resummation in existing showers [Forshaw, Holguin, SP— 2003.06400 & JHEP 08 (2019) 145]
2. Virtual corrections (Ruffa, SP)
3. Real corrections (ML, SP, Emma Simpson Dore)
   ↑ **this talk**

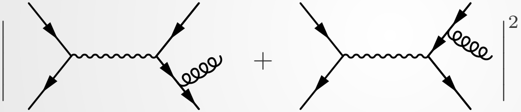
- Derive generalized splitting functions (input for parton showers)
- Only take collinear/soft limits at a late stage
- Keep interpolation over whole phase space
- Include overlapping singular regions
- Understand factorization on a diagrammatic level

One emission example

One emission example
Setup

- Leading collinear-singular behavior for gluon emission:

$$\frac{\mathrm{d}\sigma_{e^+e^-\to g+2\text{jets}}}{\mathrm{d}z\,\mathrm{d}\cos\theta} \approx \sigma_{2\text{jets}} \times C_F \frac{\alpha_S}{\pi} \frac{1}{\sin^2\theta} \hat{P}(z)$$



One emission example
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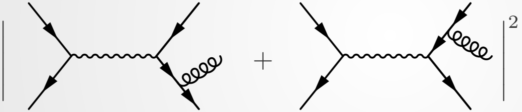

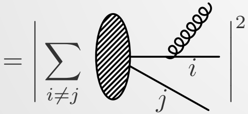

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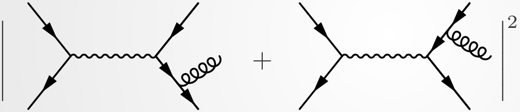
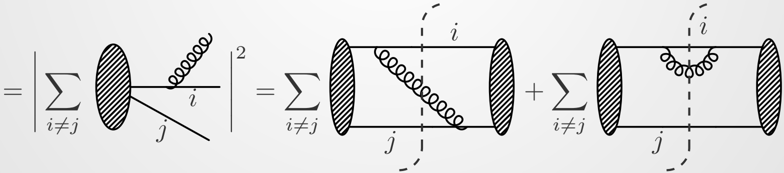
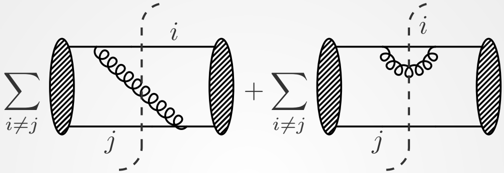




One emission example

Partitioning

- Extract IR-singular behavior ($S_{ik} \equiv 2q_i \cdot k$):



$$= \sum_{i \neq j} \tilde{\mathcal{M}}^* \left(\frac{\mathcal{N}^{\text{int}}}{S_{ik} S_{jk}} + \frac{\mathcal{N}^{\text{self}}}{S_{ik}^2} \right) \tilde{\mathcal{M}}$$

- Note: $\mathcal{N}^{\text{int/self}}$ are **operators in colour and spin space**
- Introduce **partitioning** (or **weighting factors**):

$$1 = \frac{S_{ik}}{S_{ik} + S_{jk}} + \frac{S_{jk}}{S_{ik} + S_{jk}}$$

One emission example

Partitioning

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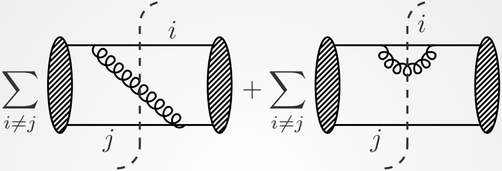

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$$1 = \frac{S_{ik}}{S_{ik} + S_{jk}} + \frac{S_{jk}}{S_{ik} + S_{jk}}$$

$$\frac{\mathcal{N}^{\mathrm{int}}}{S_{ik} S_{jk}} = \frac{1}{S_{ik}} \frac{\mathcal{N}^{\mathrm{int}}}{S_{ik} + S_{jk}} + \frac{1}{S_{jk}} \frac{\mathcal{N}^{\mathrm{int}}}{S_{ik} + S_{jk}}$$

One emission example

Splitting kernels

- Collect singular structures in **splitting kernels**

$$\mathbb{U}_{(ik)j} \equiv \frac{1}{S_{ik}} \left(\underbrace{\frac{\mathcal{N}^{\text{int}}}{S_{ik}+S_{jk}}}_{\text{non-singular in } \substack{S_{ik}\to 0 \\ \text{or} \\ S_{jk}\to 0}} + \hat{\mathcal{N}}^{\text{self}} \right)$$

- Sum of kernels smoothly approaches soft singular behavior

One emission example

Splitting kernels

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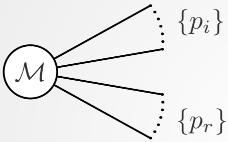
- Sum of kernels smoothly approaches soft singular behavior
- $\mathcal{N}^{\text{int/self}}$ are **operators in colour and spin-space**
  $\Rightarrow$ Keep this information in kernels
- Yields potential for tracking **colour correlations** in PS
- Aim at **Algorithmic generalization** for higher emissions
- Recover **splitting functions** and **Eikonal factors** in collinear/soft limits

Momentum mapping

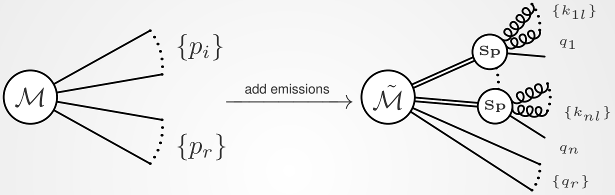
Momentum mapping

Adding emissions



- Start with **on-shell** (OS) momenta p_i (to be **emitters**) and p_r (to be **recoilers**) with overall momentum transfer $Q \equiv \sum_i p_i + \sum_r p_r$

Momentum mapping
Adding emissions



- Start with **on-shell** (OS) momenta p_i (to be **emitters**) and p_r (to be **recoilers**) with overall momentum transfer $Q \equiv \sum_i p_i + \sum_r p_r$
- Add emissions to the process with:
  1. Momentum conservation: $\sum_i q_i + \sum_{i,l} k_{il} + \sum_r q_r = Q$
  2. On-shellness of all partons
  3. Parametrization of soft & collinear behavior for any # of emissions

Momentum mapping

$$q_r = \frac{\Lambda}{\alpha_L} p_r$$

$$k_{il} = \frac{\Lambda}{\alpha_L}\left[\alpha_{il}\, p_i + \tilde{\beta}_{il}\, n_i + \sqrt{\alpha_{il}\tilde{\beta}_{il}}\, n_{il}^{\perp}\right], \quad A_i \equiv \sum_l \alpha_{il}, \quad \tilde{\beta}_{il} = (1 - A_i)\beta_{il}$$

$$q_i = \frac{\Lambda}{\alpha_L}\left[(1 - A_i) p_i + \Big(y_i - \sum_l \tilde{\beta}_{il}\Big) n_i - \sum_l \sqrt{\alpha_{il}\tilde{\beta}_{il}}\, n_{il}^{\perp}\right]$$

- Lorentz transformation $\Lambda \Rightarrow$ non-trivial **global recoil**
- Decomposition w/ light-like momentum n_i and $n_{il}^{\perp}\cdot p_i = n_{il}^{\perp}\cdot n_i = 0$
- $k_{il}^2 = 0$ implies $(n_{il}^{\perp})^2 = -2p_i\cdot n_i$
- $q_i^2 = 0$ fixes y_i in terms of the α_{il} and β_{il}
- Need $\alpha_L^2 = (Q+N)^2/Q^2$ for momentum conservation

$$Q = \sum_r q_r + \sum_i q_i + \sum_{i,l} k_{il} = \frac{\Lambda}{\alpha_L}\Big[\underbrace{\sum_r p_r + \sum_i}_{Q} (p_i + \underbrace{y_i n_i}_{N})\Big]$$

Momentum mapping

Soft and collinear scaling

- LT Λ and scaling α_L do not spoil analysis of amplitudes
- Soft and collinear behavior studied via scaling and $\lambda \to 0$:

k_{il}	$(p_i, n_i, n_{il}^{\perp})$	$(\alpha_{il}, \beta_{il}, y_i)$
(forward) collinear	$Q(1, \lambda^2, \lambda)$	$(1, \lambda^2, \lambda^2)$
soft	$Q(\lambda, \lambda, \lambda)$	$(\lambda, \lambda, \lambda)$

- Induces collinear scaling of propagators

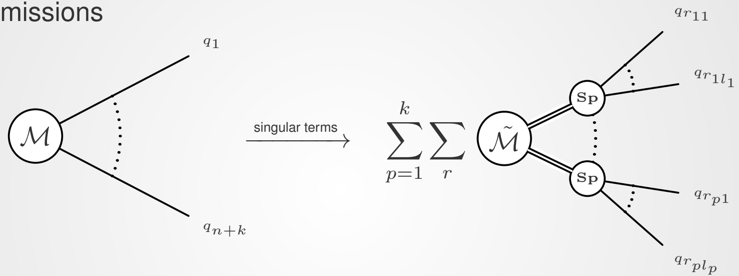
$$\frac{1}{(q_i + \sum_l k_{il})^2} = \frac{1}{y_i\, 2p_i \cdot n_i} \to \frac{1}{\lambda^2} \frac{1}{y_i\, 2p_i \cdot n_i}$$

- Take limits in splitting kernels to find leading singular behavior
- Use general set of power counting rules similar to SCET

General Algorithm and two emissions

General Algorithm
Amplitude

- Devise general setup for extracting singular behavior for k emissions



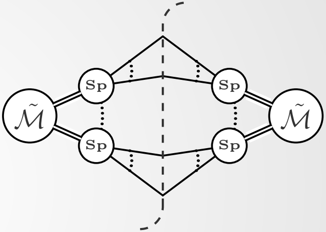
- Write amplitude in terms of splitting operators and factorized matrix element

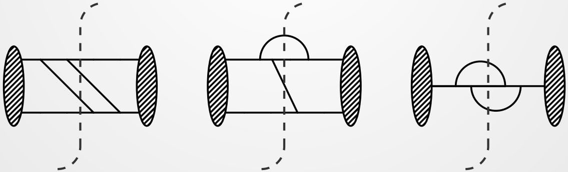
$$|\mathcal{M}_{n+k}(q_1,\ldots,q_{n+k})\rangle = \sum_{p=1}^{k} \sum_{\{r\}} \mathbf{Sp}_{(r_{11}|\ldots|r_{1\ell_1})} \ldots \mathbf{Sp}_{(r_{p1}|\ldots|r_{p\ell_p})} \, |\mathcal{M}_n(q_1,\ldots,q_{(r_{11}|\ldots|r_{1\ell_1})},\ldots,q_{(r_{p1}|\ldots|r_{p\ell_p})},\ldots,q_{n+k})\rangle$$

General Algorithm

Amplitude squared

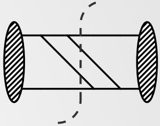
- Study **iterative behavior** of emissions
- Single out **topologies with leading singular behavior** (via # of unresolved partons)
- Examples for two emissions:





Two emissions

- For a given number of partons, find categorization of singular configs
- Read:
  $(i \parallel j \parallel k) \simeq S_{ijk} = (q_i + q_j + q_k)^2 \to 0$
- Triple collinear and double-soft contributions



	$\propto \frac{1}{S_{ij}S_{kl}S_{ijk}S_{jkl}}$
$i \parallel j \parallel k$	$S_{ij}S_{ijk}$
$i \parallel j \parallel l$	S_{ij}
$i \parallel k \parallel l$	S_{kl}
$j \parallel k \parallel l$	$S_{kl}S_{jkl}$
$(i \parallel j), (k \parallel l)$	$S_{ij}S_{kl}$
$(i \parallel k), (j \parallel l)$	$\times$
$(i \parallel l), (j \parallel k)$	$\times$

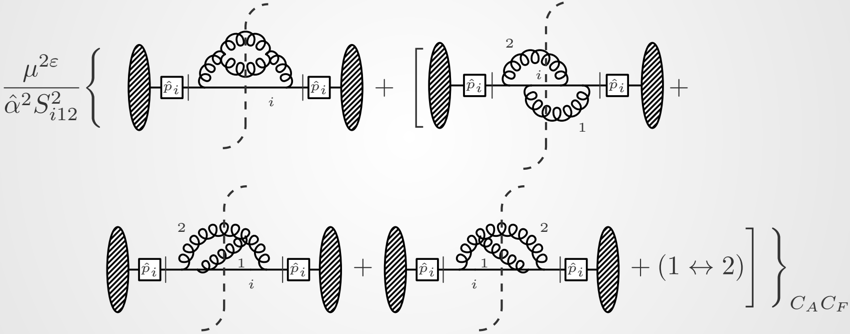
Construct partitioning factors from

$$1 = \frac{M^2(S_{kl}S_{jkl} + S_{ij}S_{ijk} + S_{ijk}S_{jkl}) + (S_{kl} + S_{ij})S_{ijk}S_{jkl}}{M^2(S_{kl}S_{jkl} + S_{ij}S_{ijk} + S_{ijk}S_{jkl}) + (S_{kl} + S_{ij})S_{ijk}S_{jkl}}$$

$\Rightarrow$ **non-singular in any configuration**

Cross Check: Two Emission Splitting Function

Reproduced from general two-emission kernel which includes soft-limit too (Here: in lightcone-gauge)



$$= \left(\frac{8\pi\alpha_S}{\hat{\alpha} S_{i12}}\mu^{\varepsilon}\right)^2 C_A C_F \, \langle \hat{P}_{ggq}^{\text{(non-Ab)}} \rangle \hat{\not{p}}_i + \mathcal{O}\left(\beta_{il}^{-3/2}\right).$$

Conclusions

Goal: study leading singular behavior for multiple emissions
(for applications in parton showers and beyond)

- Momentum mapping for exposing collinear and soft factorization
- Global recoil via Lorentz transformation
- Partitioning algorithm to separate overlapping singularities
- Comprehensive framework for organizing singular behavior
- General Sudakov-like momentum decomposition for power counting rules $\Rightarrow$ simplification of amplitudes

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Thank you!

Relation to fixed order

A variety of *e.g.* NNLO QCD subtraction methods are available (on the cross-section level).

Relation to fixed order

Well established subtraction schemes at NLO

- Frixione-Kunst-Signer (FKS) subtraction — Frixione, Kunszt, Signer
- Catani-Seymour (CS) Dipole subtraction — Catani, Seymour
- Nagy-Soper subtraction — Nagy, Soper

Many methods available at NNLO

- Antenna subtraction — Gehrmann De Ridder, Gehrmann, Glover, Heinrich, et al.
- CoLoRFul subtraction — Del Duca, Duhr, Kardos, Somogyi, Troscanyi, et al.
- Sector-improved residue subtraction — Czakon et al.
- Nested soft-collinear subtraction — Melnikov et al.
- Local analytic sector subtraction — Magnea, Maina, Torrielli, U. et al.
- qT-slicing — Catani, Grazzini, et al.
- N-jettiness slicing — Boughezal, Petriello, et al.
- Projection to Born — Cacciari, Salam, Zanderighi, et al.
- Sector decomposition — Anastasiou, Binoth, et al.
- $\mathcal{E}$-prescription — Frixione, Grazzini
- Unsubtraction — Rodrigo et al.
- Geometric — Herzog

S. Uccirati @ Vienna CES 2019

Relation to fixed order

A variety of *e.g.* NNLO QCD subtraction methods are available (on the cross-section level).

Instead of adding to the list, want to:

- Combine real and virtual contributions differentially
- Smooth phase-space coverage
- Let a MonteCarlo do the integrals
- Still keep a bridge to the fixed order and EFT side

Backup slides

Phase space

- Can write down factorized phase space using momentum mapping

$$\mathrm{d}\phi\left(\{q_i\}_{\mathbf{S}},\{q_r\}_{\mathbf{R}},\{k_{il}\}_{\mathbf{E}_i}|Q\right)=\mathrm{d}\phi\left(\{p\}_{\mathbf{R}}|P_R\right)\alpha^{d-n_R(d-2)}(2\pi)^d\delta\left(P_S+P_R-Q\right)$$
$$\times\frac{\mathrm{d}m^2}{2\pi}[\mathrm{d}P_R]\frac{\omega(\vec{P}_R,\alpha m)}{\omega(\vec{Q}_R,m)}\Theta(Q_R^0)\prod_{i\in\mathbf{S}}[\mathrm{d}p_i]\frac{\omega(\vec{p}_i)}{\omega(\vec{q}_i)}\Theta(q_i^0)\left|\frac{\partial(\{\vec{q}\}_{\mathbf{S}},\vec{Q}_R)}{\partial(\{\vec{p}\}_{\mathbf{S}},\vec{P}_R)}\right|\prod_{l\in\mathbf{E}_i}[\mathrm{d}k_{il}]\Theta(k_{il}^0).$$

- Emission phase space:

$$\mathrm{d}^{d-1}k_{il}=|\mathcal{J}(\alpha_{il},\beta_{il},\Omega)|\,\mathrm{d}\alpha_{il}\,\mathrm{d}\beta_{il}\,\mathrm{d}^{d-3}\Omega,$$

- Computable in d dimensions:

$$[\mathrm{d}k_{il}]=\frac{1}{(2\pi)^{d-1}}\Theta(\alpha_{il})\Theta(\tilde{\beta}_{il})\frac{\alpha^{2-d}}{4}\frac{(2p_i\cdot n_i)^{\frac{d-2}{2}}}{(\alpha_{il}\tilde{\beta}_{il})^{\frac{d-4}{2}}}\mathrm{d}\alpha_{il}\,\mathrm{d}\tilde{\beta}_{il}\,\mathrm{d}\Omega^{d-3}.$$