Inclusive non-leptonic decays of heavy quarks: completing the $\mathcal{O}(1/m_Q^3)$ corrections

A talk by

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Outline I

Aim

Aim: Compute the Darwin (ρ_D) term contribution ($1/m_b^3$ correction) to the inclusive non-leptonic B-hadron decay rate originated by the flavor changing transition $b \to c\bar{q}_1 q_2$, where $q_1 = u, c$ and $q_2 = d, s$.

Key ingredients for the computation:

- Optical Theorem (OT).
- Heavy Quark Expansion (HQE).
- Local expansion of the quark propagator in the external gluon field/background field method (BFM).
- Dimensional regularization (DR) in $D = 4 - 2\epsilon$.
- $\overline{\text{MS}}$ renormalization scheme: $\bar{\mu}^{-2\epsilon} = \mu^{-2\epsilon}(e^{\gamma_E}/4\pi)^{-\epsilon}$

Motivation

Motivation: Experimental precision is so high that it is sensitive to the ρ_D contribution. Even higher precision seems to be achievable from LHCb and ATLAS. The theory precision should live up to these experimental advancements.

The current (2019) experimental averages obtained by the Heavy Flavor Averaging Group (HFLAV) of the B-hadron lifetime ratios are[1]:

$$\left.\frac{\tau(B_s)}{\tau(B_d)}\right|^{\rm exp} = 0.994 \pm 0.004\,, \quad \left.\frac{\tau(B^+)}{\tau(B_d)}\right|^{\rm exp} = 1.076 \pm 0.004\,, \quad \left.\frac{\tau(\Lambda_b)}{\tau(B_d)}\right|^{\rm exp} = 0.969 \pm 0.006\,,$$

whereas the current status of the theoretical predictions is[2]:

$$\left.\frac{\tau(B_s)}{\tau(B_d)}\right|^{\rm th} = 1.0006 \pm 0.0025\,, \quad \left.\frac{\tau(B^+)}{\tau(B_d)}\right|^{\rm th} = 1.082^{+0.022}_{-0.026}\,, \quad \left.\frac{\tau(\Lambda_b)}{\tau(B_d)}\right|^{\rm th} = 0.935 \pm 0.054\,,$$

[1] Y. S. Amhis et al., hep-ex/1909.12524.

[2] A. Lenz, Int. J. Mod. Phys., **A**30, 1543005 (2015); M. Kirk et al., JHEP 068, (2017).

The effective electroweak Lagrangian

The effective weak interaction Lagrangian describing $b \to c\bar{q}_1 q_2$ transitions reads

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cb} V^*_{q_1 q_2} (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2) + \text{h.c} \quad (1)$$

where G_F is the Fermi constant, $V_{q_1 q_2}$ are CKM matrix elements, $C_{1,2}$ are Wilson coefficients and $\mathcal{O}_{1,2}$ are the four-quark operators

$$\mathcal{O}_1 = (\bar{b}\Gamma_\mu c)(\bar{q}_1 \Gamma^\mu q_2)\,, \quad \mathcal{O}_2 = (\bar{b}\Gamma_\mu q_2)(\bar{q}_1 \Gamma^\mu c)\,, \quad (2)$$

with $\Gamma_\mu = \gamma_\mu(1-\gamma_5)/2$. Note that

- $\mathcal{O}_2 = \mathcal{O}_1(c \leftrightarrow q_2)$.
- We consider two Cabibbo favoured decay channels, $(q_1, q_2) = (u, d)$ and $(q_1, q_2) = (c, s)$.
- The b and c-quarks have mass m_b and m_c, and the u, d, s-quarks to be massless.

HQE for the Total Decay rate

According to the **OT** ($\hat{T}^\dagger\hat{T} = 2\mathrm{Im}\,\hat{T}$) the total width for the inclusive decay of a B-hadron can be computed from the discontinuity of the forward scattering matrix element[3]

$$\Gamma(B \to X) = \frac{1}{2M_B}\langle B(p_B)|\mathrm{Im}\,\hat{T}(B \to X \to B)|B(p_B)\rangle \tag{3}$$

Γ is not tractable in perturbative QCD (all the scales are involved).

However, if B is a heavy flavoured hadron we can write the original quark fields to effective fields within HQET[4]:

$$\begin{aligned} b(x) &= e^{-im_b v\cdot x} b_v(x) \qquad (4) \\ &= e^{-im_b v\cdot x}\left[1 + \frac{\not{\pi}_\perp}{2m_b} - \frac{(v\cdot\pi)\not{\pi}_\perp}{4m_b^2} + \frac{\not{\pi}_\perp\not{\pi}_\perp}{8m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\right] h_v(x) \end{aligned}$$

[3] Transition operator: $\hat{T} = i\int dx\,\mathcal{T}\{\mathcal{L}_{\mathrm{eff}}(x), \mathcal{L}_{\mathrm{eff}}(0)\}$
[4] $\pi_\mu = iD_\mu = i\partial_\mu + g_s A_\mu^a T^a$ and $\pi^\mu = v^\mu(v\pi) + \pi_\perp^\mu$

HQE for the Total Decay rate

That singles out a large phase factor and allows to perform an OPE in $1/m_b$ (**HQE**)

$$\Gamma(B \to X) = \Gamma^0 \Big[C_0 - C_{\mu_\pi} \frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G} \frac{\mu_G^2}{2m_b^2} - C_{\rho_D} \frac{\rho_D^3}{2m_b^3} - C_{\rho_{LS}} \frac{\rho_{LS}^3}{2m_b^3} + \sum_{i,q} C_{4F_i}^{(q)} \frac{\langle \mathcal{O}_{4F_i}^{(q)} \rangle}{4m_b^3} \Big] \quad (5)$$

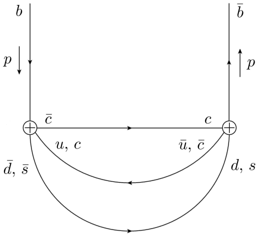
- factorized short-distance effects (Wilson coefficients, C_i) which are treatable in perturbation theory, e.g. C_{ρ_D}.
- non perturbative effects encoded in the expectation values of local operators, e.g. $\rho_D^3 = -\frac{1}{2} \frac{1}{2M_B} \langle B(p_B)|\bar{h}_v [\pi_{\perp\,\mu}, [\pi_\perp^\mu, v \cdot \pi]] h_v |B(p_B)\rangle$.

where μ_π^2, μ_G^2, ρ_D^3 and ρ_{LS}^3 are matrix elements of two-quark operators and $\langle \mathcal{O}_{4F_i}^{(q)} \rangle$ of four-quark operators.

We focus on $X =$ hadrons: $\Gamma^0 = G_F^2 m_b^5 |V_{cb}|^2 |V_{q_1 q_2}|^2 / 192\pi^3$.

Matching of two-quark operators: Generalities

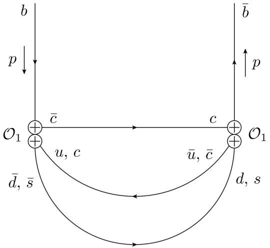
In order to compute the matching coefficients in the HQE of the decay rate we need to compute the imaginary part of 2-loop diagrams of the form:



We develop a tool based on Mathematica which makes use of the packages:

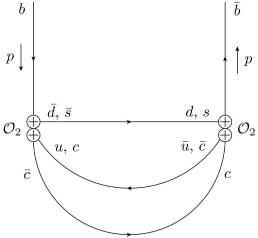
- **Tracer**: to deal with four vectors and gamma matrices.
- **LiteRed**: to reduce integrals to a small set (indeed only one) of Master Integrals (MIs).
- **HypExp**: to perform the ϵ expansion of Hypergeometric functions.

Matching of two-quark operators: $\mathcal{O}_1 \otimes \mathcal{O}_1$



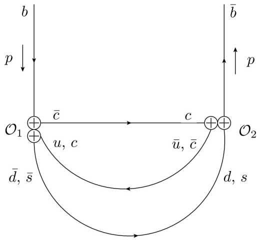
- **Semi-leptonic like**: a single gluon can only be emitted from the $\bar{b}cb$-line due to the color structure. In the BFM, only expansion of the c-quark propagator.
- **Standard**: we can compute it with our technology or take it from the literature ($b \to c\ell\bar{\nu}_\ell$).
- **IR safe**: due to the c-quark is massive.

Matching of two-quark operators: $\mathcal{O}_2 \otimes \mathcal{O}_2$



- **Semi-leptonic like**: a single gluon can only be emitted from the $\bar{b}q_2b$-line due to the color structure. In the BFM, only expansion of the q_2-quark propagator.
- **Standard**: we can compute it with our technology or take it from the literature ($b \to u\bar{\ell}\nu_\ell$).
- **IR divergent**: ρ_D coefficient is IR divergent due to q_2 is massless (expansion of propagators makes the diagram more IR).
- **Renormalization**: is required $\Rightarrow$ renormalization scheme dependence.

Matching of two-quark operators: $\mathcal{O}_1 \otimes \mathcal{O}_2$



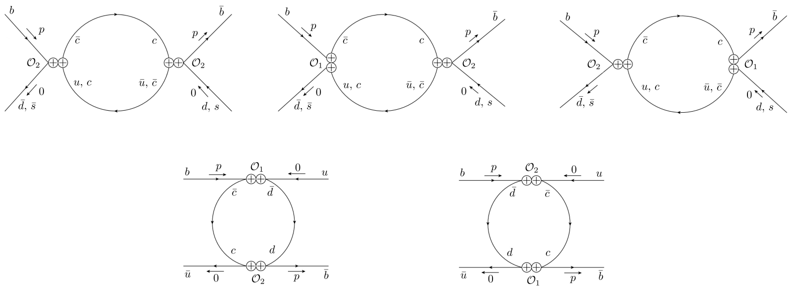
- **Non-leptonic**: color structure allow a single gluon to be emitted from all quark propagators (expansion of all propagators).
- **IR divergent**: ρ_D coefficient is IR divergent due to q_1 and q_2 are massless (expansion of propagators makes the diagram more IR).
- **Renormalization**: is required $\Rightarrow$ renormalization scheme dependence.

Matching of four-quark operators

IR poles in $C_{\rho_D} \Rightarrow$ UV mixing of $\langle \mathcal{O}_{4F_i}^{(q)} \rangle$ and ρ_D.

Renormalization of C_{ρ_D} requires knowing the coefficients of $\mathcal{O}_{4F_i}^{(q)}$.

That requires the computation of the following 1-loop diagrams.



Matching of four-quark operators

The relevant four-quark operators are

$b \to c\bar{u}d$	$b \to c\bar{c}s$
$\mathcal{O}^{(d)}_{4F_1} = (\bar{h}_v \Gamma_\mu d)(\bar{d}\Gamma^\mu h_v)$ $\mathcal{O}^{(d)}_{4F_2} = (\bar{h}_v P_L d)(\bar{d} P_R h_v)$ $\mathcal{O}^{(u)}_{4F_1} = (\bar{h}_v \Gamma^\sigma \gamma^\mu \Gamma^\rho u)(\bar{u}\Gamma_\sigma \gamma_\mu \Gamma_\rho h_v)$ $\mathcal{O}^{(u)}_{4F_2} = (\bar{h}_v \Gamma^\sigma \not{v} \Gamma^\rho u)(\bar{u}\Gamma_\sigma \not{v} \Gamma_\rho h_v)$	$\mathcal{O}^{(s)}_{4F_1} = (\bar{h}_v \Gamma_\mu s)(\bar{s}\Gamma^\mu h_v)$ $\mathcal{O}^{(s)}_{4F_2} = (\bar{h}_v P_L s)(\bar{s} P_R h_v)$

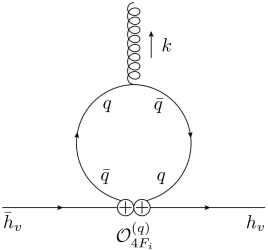
General remarks:

- Coefficients are computed in $D = 4 - 2\epsilon$ dimensions.
- Coefficients are functions of C_1, C_2, $r = m_c^2/m_b^2$, D and μ.
- The pole in $C^B_{\rho_D}$ is proportional to a combination of $C^{(q)}_{4F_i}$.
- In principle, $\mathcal{O}^{(u)}_{4F_i}$ can not be reduced to the form of e.g. $\mathcal{O}^{(d)}_{4F_i}$ in D dimensions unlike in $D = 4$.
- Still we can reduce to this basis by introducing evanescent operators $\Rightarrow$ additional scheme dependence (choice is not unique).

Renormalization

The IR singularity in C_{ρ_D} is canceled by the UV pole coming from the operator mixing

$$\langle \mathcal{O}_{4F_i}^{(q)} \rangle^R = \langle \mathcal{O}_{4F_i}^{(q)} \rangle^B + \gamma_i^{(q)} \frac{1}{\epsilon} \rho_D$$



where $\gamma_i^{(q)}$ is the anomalous dimension. The counterterm of C_{ρ_D} in the $\overline{\text{MS}}$ renormalization scheme for $b \to c\bar{u}d$ and $b \to c\bar{c}s$, respectively

$$\delta C_{\rho_D}^{\overline{\text{MS}}}(\mu) = \left(C_{4F_1}^{(d)} - \frac{1}{2} C_{4F_2}^{(d)} + 16 C_{4F_1}^{(u)} + 4 C_{4F_2}^{(u)} \right) \frac{1}{48\pi^2 \epsilon} \mu^{-2\epsilon} \left(\frac{e^{\gamma_E}}{4\pi} \right)^{-\epsilon}, \quad (6)$$

$$\delta C_{\rho_D}^{\overline{\text{MS}}}(\mu) = \left(C_{4F_1}^{(s)} - \frac{1}{2} C_{4F_2}^{(s)} \right) \frac{1}{48\pi^2 \epsilon} \mu^{-2\epsilon} \left(\frac{e^{\gamma_E}}{4\pi} \right)^{-\epsilon}, \quad (7)$$

where $C_{\rho_D}^B = C_{\rho_D}^{\overline{\text{MS}}}(\mu) + \delta C_{\rho_D}^{\overline{\text{MS}}}(\mu)$.

Results for the $b \to c\bar{u}d$ coefficients

$$
\begin{aligned}
C_{\rho_D}^{\overline{\mathrm{MS}}} &= C_1^2\Big[-77+88r-24r^2+8r^3+5r^4-48\ln(r)-36r^2\ln(r)\Big] \\
&\quad +\frac{2}{3}C_1C_2\Big[-53+16r+144r^2-112r^3+5r^4+96(-1+r)^3\ln(1-r) \\
&\quad -12(4-9r^2+4r^3)\ln(r)-48(-1+r)^3\ln\left(\frac{\mu^2}{m_b^2}\right)\Big] \\
&\quad +C_2^2\Big[-45+16r+72r^2-48r^3+5r^4+96(-1+r)^2(1+r)\ln(1-r) \\
&\quad +12(1-4r)r^2\ln(r)-48(-1+r)^2(1+r)\ln\left(\frac{\mu^2}{m_b^2}\right)\Big], \qquad (8)
\end{aligned}
$$

where $r = m_c^2/m_b^2$.

Results for the $b \to c\bar{c}s$ coefficients

$$
\begin{aligned}
C_{\rho_D}^{\overline{\mathrm{MS}}} \quad &= C_1^2\left[(-77-2r+58r^2+60r^3)z+24(2-2r-r^2+4r^3+5r^4)\ln\left(\frac{1+z}{1-z}\right)\right] \\
&+C_2^2\left[24(-4+8r+7r^2+8r^3+5r^4)\ln\left(\frac{1+z}{1-z}\right)\right. \\
&\left.+z\Big(-45-58r+106r^2+60r^3-96\ln(r)+192\ln(z)-48\ln\left(\frac{\mu^2}{m_b^2}\right)\Big)\right] \\
&+\frac{2}{3}C_1C_2\left[24(-6+10r-5r^2+20r^3+5r^4)\ln\left(\frac{1+z}{1-z}\right)\right. \\
&\left.+z\Big(75-178r+250r^2+60r^3-96\ln(r)+192\ln(z)-48\ln\left(\frac{\mu^2}{m_b^2}\right)\Big)\right].
\end{aligned}
\tag{9}
$$

where $r = m_c^2/m_b^2$ and $z = \sqrt{1-4r}$.

Conclusions

- We completed the $1/m_b^3$ corrections to the inclusive non-leptonic width of B-hadrons T. Mannel et. al., hep-ph/2004.09485.
- C_{ρ_D} depends on the calculational scheme: renormalization scheme and treatment of the Dirac algebra in D dimensions (evanescent operators).
- After proper definition of the evanescent operators (validity of Fierz tranformation at 1-loop order), our results agree with A. Lenz et. al., hep-ph/2004.09527 where the coefficients are computed in $D = 4$.
- We obtain analytical results.
- C_{ρ_D} turns out to be sizable (gives a correction to the tree level values of $C_{4F_i}^{(q)}$ of $\sim 7\%$ for $b \to c\bar{c}s$ and of $\sim 1\%$ for $b \to c\bar{u}d$)
- Relevant for the precise determination of lifetime ratios.

Questions

Backup

Operators and non-perturbative parameters

$$\begin{aligned}
\mathcal{O}_0 &= \bar{h}_v h_v \,, && (10)\\
\mathcal{O}_v &= \bar{h}_v (v\cdot\pi) h_v \,, && (11)\\
\mathcal{O}_\pi &= \bar{h}_v \pi_\perp^2 h_v \,, && (12)\\
\mathcal{O}_G &= \frac{1}{2}\bar{h}_v [\not{\pi}_\perp, \not{\pi}_\perp] h_v = \frac{1}{2}\bar{h}_v [\gamma^\mu, \gamma^\nu] \pi_{\perp\,\mu}\pi_{\perp\,\nu} h_v \,, && (13)\\
\mathcal{O}_D &= \bar{h}_v [\pi_{\perp\,\mu}, [\pi_\perp^\mu, v\cdot\pi]] h_v \,, && (14)\\
\mathcal{O}_{LS} &= \frac{1}{2}\bar{h}_v [\gamma^\mu, \gamma^\nu]\{\pi_{\perp\,\mu}, [\pi_{\perp\,\nu}, v\cdot\pi]\} h_v \,, && (15)
\end{aligned}$$

$$\begin{aligned}
\langle B(p_B)|\bar{b}\not{v} b|B(p_B)\rangle &= 2M_B \,, && (16)\\
-\langle B(p_B)|\mathcal{O}_\pi|B(p_B)\rangle &= 2M_B\mu_\pi^2 \,, && (17)\\
C_{\rm mag}(\mu)\langle B(p_B)|\mathcal{O}_G|B(p_B)\rangle &= 2M_B\mu_G^2 \,, && (18)\\
-c_D(\mu)\langle B(p_B)|\mathcal{O}_D|B(p_B)\rangle &= 4M_B\rho_D^3 \,, && (19)\\
-c_S(\mu)\langle B(p_B)|\mathcal{O}_{LS}|B(p_B)\rangle &= 4M_B\rho_{LS}^3 \,. && (20)
\end{aligned}$$